Predicting Lunar Eclipses

This ***Numerit*** program (`eclipse`) can be used to predict lunar eclipses. The source ephemeris for this program is SLP96. This software provides the eclipse type, the universal times and topocentric coordinates of the Moon at the beginning and end of the penumbra contacts, and the time and coordinates at maximum eclipse.

This program uses a combination of one-dimensional minimization and root-finding to solve this classic problem. The objective function used in these calculations involves the geocentric separation angle between the center of the Moon and the anti-Sun position vector or shadow axis, and the semidiameter and horizontal parallax of the Sun and Moon. The objective function used during the search for lunar eclipses is given by the following expression:

$$f(t) = \cos^{-1}\left(-\boldsymbol{U}_m \bullet \boldsymbol{U}_s\right) - f_1 + s_m \tag{1}$$

where

$\boldsymbol{U}_s =$ geocentric unit position vector of the Sun
$\boldsymbol{U}_m =$ geocentric unit position vector of the Moon
$f_1 = 1.02\left(p_1 + p_s + s_s\right) =$ size of penumbra shadow
$p_1 = 0.99834 p_m =$ corrected parallax
$p_m =$ horizontal parallax of the Moon
$p_s =$ horizontal parallax of the Sun
$s_m =$ semidiameter of the Moon
$s_s =$ semidiameter of the Sun

In these expressions

$$\begin{aligned} p_m &= \sin^{-1}\left(\frac{r_{eq}}{d_m}\right) & \quad p_s &= \sin^{-1}\left(\frac{r_{eq}}{d_s}\right) \\ s_m &= \sin^{-1}\left(\frac{r_m}{d_m}\right) & \quad s_s &= \sin^{-1}\left(\frac{r_s}{d_s}\right) \end{aligned} \tag{2}$$

where r_{eq} is the equatorial radius of the Earth (6378.14 kilometers), r_m is the radius of the Moon (1738 kilometers), r_s is the radius of the Sun (696,000 kilometers), d_m is the geocentric distance of the Moon and d_s is the geocentric distance of the Sun.

If we let s represent the minimum geocentric separation angle of the Moon relative to the shadow axis, a penumbral lunar eclipse occurs whenever the following geometric condition is satisfied:

$$s < 1.02\left(p_1 + p_s + s_s\right) + s_m \qquad (3)$$

A partial lunar eclipse will happen whenever the following is true:

$$s < 1.02\left(p_1 + p_s - s_s\right) + s_m \qquad (4)$$

Finally, the geometric condition for a total lunar eclipse is given by

$$s < 1.02\left(p_1 + p_s - s_s\right) - s_m \qquad (5)$$

The following is a typical draft output created with this program. It illustrates the circumstances of the penumbral lunar eclipse of March 13, 1998. The topocentric coordinates correspond to an observer located in Denver, Colorado.

```
penumbral lunar eclipse

begin penumbral phase of lunar eclipse

calendar date              March 13, 1998
universal time             2 h 14 m 16.8698 s
UTC Julian date            2450885.593250808

lunar azimuth angle        97 d 31 m 43.799 s
lunar elevation angle      14 d 44 m 12.6553 s

greatest eclipse conditions

calendar date              March 13, 1998
universal time             4 h 20 m 1.9293 s
UTC Julian date            2450885.680577886

lunar azimuth angle        121 d 37 m 55.0332 s
lunar elevation angle      36 d 37 m 56.4931 s

end penumbral phase of lunar eclipse

calendar date              March 13, 1998
universal time             6 h 25 m 49.395 s
UTC Julian date            2450885.767932812

lunar azimuth angle        159 d 57 m 20.5276 s
lunar elevation angle      51 d 31 m 55.7293 s

event duration              4 h 11 m 32.5252 s
```