Predicting Solar Eclipses

This ***Numerit*** program (`seclipse`) can be used to predict the local circumstances of solar eclipses relative to an observer anywhere on the Earth. The source ephemeris for this program is SLP96. This software provides the eclipse type, the universal times and topocentric coordinates of the Moon at the beginning and end of the penumbra contacts, and the time and coordinates at maximum eclipse.

This program uses a combination of one-dimensional minimization and root-finding to solve this classic problem. The objective function used in these calculations involves the selenocentric separation angle between the axis of the shadow cast by the Moon and an Earth observer. The objective function used during the search for solar eclipses is given by the following expression:

$$f(t) = \cos^{-1}\left(\boldsymbol{U}_{axis} \bullet \boldsymbol{U}_{m-o}\right) - y_p \tag{1}$$

where

$\boldsymbol{U}_{axis} =$ selenocentric unit vector of the Moon's shadow
$\boldsymbol{U}_{m-o} =$ selenocentric unit position vector of the observer
$y_p =$ penumbra shadow angle

The penumbra shadow angle at the distance of the Earth observer is determined from

$$y_p = \sin^{-1}\left(\frac{r_m}{d_m}\right) + \sin^{-1}\left(\frac{r_s + r_m}{d_{m-s}}\right) \tag{2}$$

In this expression r_m is the radius of the Moon (1738 kilometers), r_s is the radius of the Sun (696,000 kilometers), d_m is the *topocentric* distance of the Moon and d_{m-s} is the distance from the Moon to the Sun.

The selenocentric position vector of the Sun is computed from the expression

$$\boldsymbol{r}_{m-s} = \boldsymbol{r}_s - \boldsymbol{r}_m \tag{3}$$

where $\boldsymbol{r}_s$ is the geocentric position vector of the Sun and $\boldsymbol{r}_s$ is the geocentric position vector of the Moon.

This software uses a special version of the ***Numerit*** minimization event prediction algorithm. Whenever the program finds a minimum it also checks to see if the topocentric elevation angle of the center of the Moon is positive. This check insures that the solar eclipse is visible to the Earth observer. If this condition is true, the software then uses Brent's root-finding algorithm to calculate the event begin and end times and the topocentric coordinates of the Moon.

The following is a typical draft output created with this program. It illustrates the local circumstances of the solar eclipse of May 10, 1994. It is based on the example given on page 203 of the book *Astronomy on the Personal Computer*, Second Edition by Oliver Montenbruck and Thomas Pfleger.

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begin penumbral phase of solar eclipse

calendar date              May 10, 1994
universal time             17 h 50 m 46.4522 s
UTC Julian date            2449483.243593196

lunar azimuth angle        280 d 49 m 9.55257 s
lunar elevation angle      15 d 58 m 41.317 s

greatest eclipse conditions

calendar date              May 10, 1994
universal time             18 h 58 m 33.9913 s
UTC Julian date            2449483.290671196

lunar azimuth angle        289 d 27 m 56.576 s
lunar elevation angle      2 d 51 m 50.4514 s

end penumbral phase of solar eclipse

calendar date              May 10, 1994
universal time             19 h 59 m 18.4396 s
UTC Julian date            2449483.33285231

lunar azimuth angle        297 d 56 m 2.06632 s
lunar elevation angle      -8 d 14 m 35.518 s

event duration              2 h 8 m 31.9875 s
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