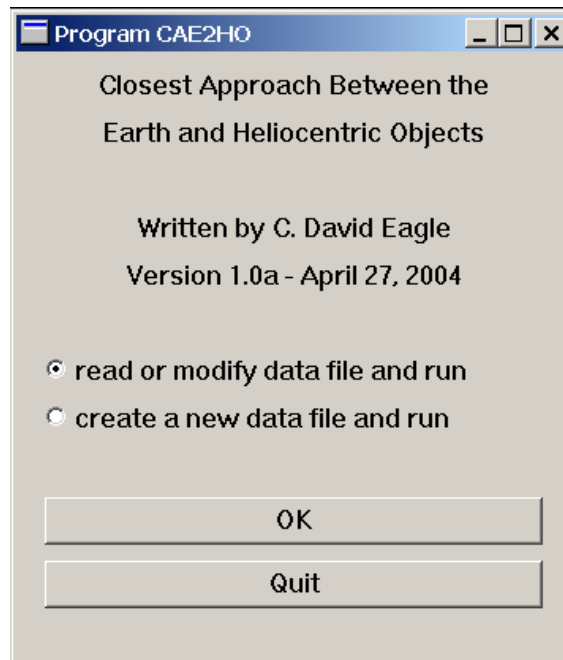


## Closest Approach Between the Earth and Heliocentric Objects

This Windows XP/2000/NT computer program (`cae2ho.exe`) uses a combination of Cowell's method and one-dimensional minimization to predict closest approach conditions between the Earth and objects such as asteroids, comets and spacecraft in heliocentric orbits. The software includes the *point mass* gravity perturbations due to the planets Mercury, Venus, Earth, Mars, Jupiter, Saturn, Neptune, Uranus and the Sun. It also includes the point-mass effect of the Moon via a combined Earth/Moon gravitational constant, and the relativistic effect of the Sun.

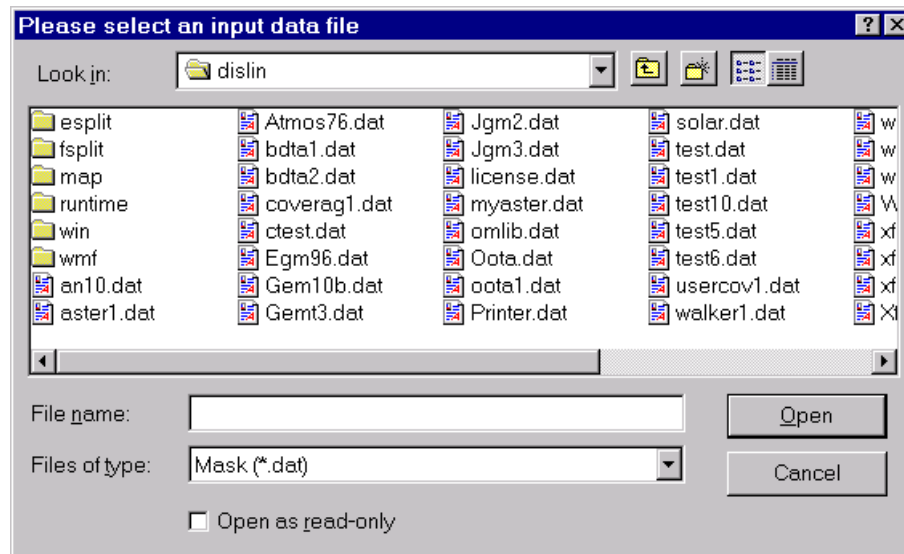
The user can elect to use either the JPL DE405 ephemeris or the SLP96 ephemeris developed by the Bureau of Longitudes in Paris for the planetary and solar ephemeris calculations. The software expects to find one or both of these binary files in the same directory as the `cae2ho.exe` executable program. Additional information about the SLP96 ephemeris can be found on the Internet at <ftp://ftp.bdl.fr/pub/ephem/sun/slp96>. Information about the DE405 ephemeris is available at the JPL Solar System Dynamics web site which is located at <http://ssd.jpl.nasa.gov>.

The following is the main menu for this program. It allows you to read, modify and run an existing input data file. It also allows you to create an input data file from scratch. Each program option is activated by simply selecting its radio button and clicking on the OK button. Clicking on the Quit button will terminate the software.



If you select the first menu item the software will display a file selection window. You can select an input file by double clicking on its name or by typing the name in the File name: entry field. The default file name mask for input files is `*.dat`. However, any `CAE2HO` compatible input file can be selected.

The following is a typical file selection screen display.



The following is a typical ASCII input data file for this program. It contains the heliocentric orbital elements of the asteroid 1997 XF 11. These coordinates must be relative to the J2000 equinox. The fundamental plane can be either the ecliptic or Earth equator. This file was created using data available on the Horizons ephemeris system which is located at <http://ssd.jpl.nasa.gov>. Additional information about Near Earth Objects can be found at <http://impact.arc.nasa.gov>.

The user can create input data files for this software using an ASCII text editor or the create a new data file and run option of the main menu. Do not change the total number of lines or the order of annotation and data in this file. The software expects to find exactly 35 lines of information in the input data file.

```
initial calendar date (1 <= month <= 12, 1 <= day <= 31, year = all digits!)
10,20,1998

initial TDB (0 <= hours <= 24, 0 <= minutes <= 60, 0 <= seconds <= 60)
0,0,0

semimajor axis (astronomical units; semimajor axis > 0)
0.1441779254846407D+01

orbital eccentricity (non-dimensional; 0 <= eccentricity < 1)
0.4836740409964595D+00

orbital inclination (degrees; 0 <= inclination <= 180)
0.2017287833484568D+02

argument of perihelion (degrees; 0 <= argument of perihelion <= 360)
0.3228014264370198D+03

longitude of the ascending node (degrees; 0 <= raan <= 360)
0.3533298230482567D+03

mean anomaly (degrees; 0 <= mean anomaly <= 360)
0.2708752703925974D+03
```

```
ephemeris source (SLP96 or DE405)
SLP96

simulation duration (days)
12000

close approach constraint (astronomical units)
0.1

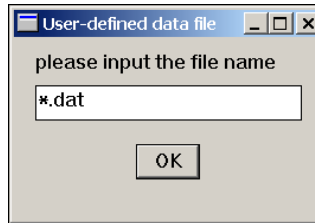
coordinate frame (ecliptic or equator)
equator
```

If you select the program option to create a new input data file, the software will display the following screen. Simply fill in all fields of this screen and click on the OK button to save it to disk and run the main calculations.

Field Label	Input Type
calendar date (mm, dd, yyyy)	Text
TDB (hours, min, sec)	Text
semimajor axis (AU)	Text
orbital eccentricity	Text
inclination (degrees)	Text
argument of perihelion (degrees)	Text
RAAN (degrees)	Text
mean anomaly (degrees)	Text
ephemeris type (DE405 or SLP96)	Text
simulation duration (days)	Text
close approach constraint (AU)	Text
coordinate frame	Text
OK	
Quit	

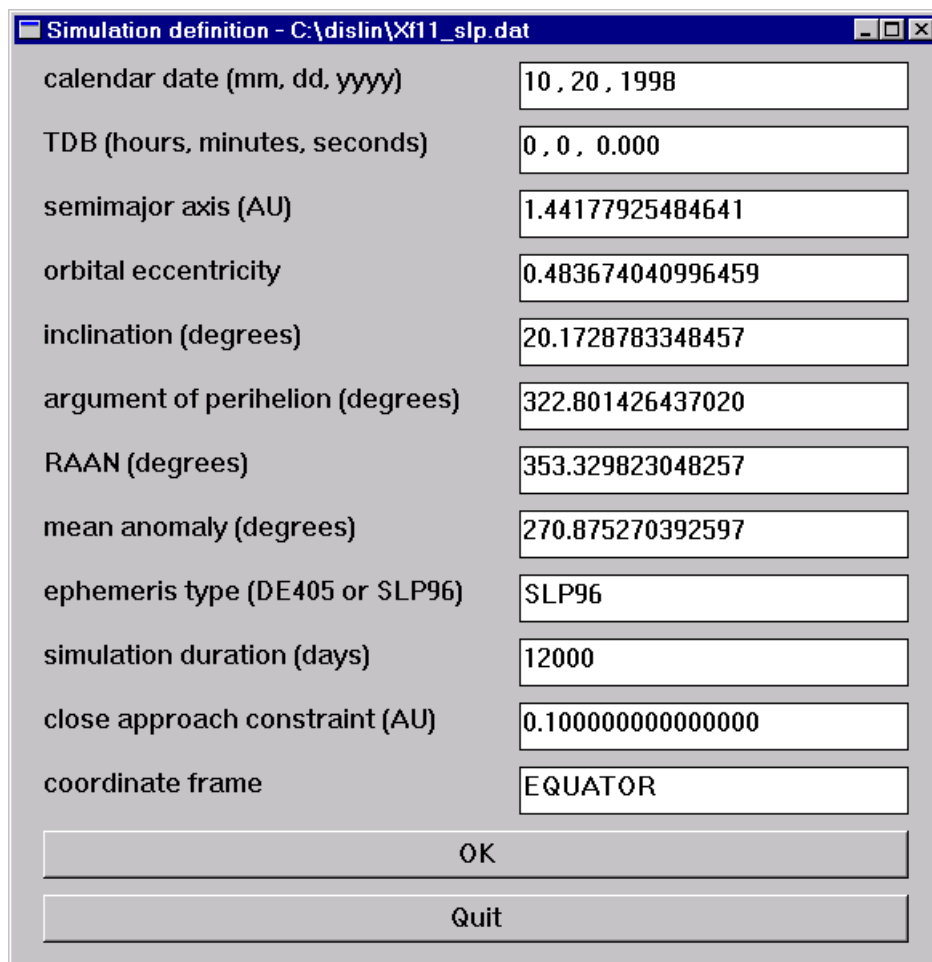
Please note that the coordinate frame field must contain either ecliptic or equator. This input is not case sensitive but must be spelled correctly. The same is true for the ephemeris type field.

After this input screen is complete, the software will prompt you for the name of the new data file with the following display:



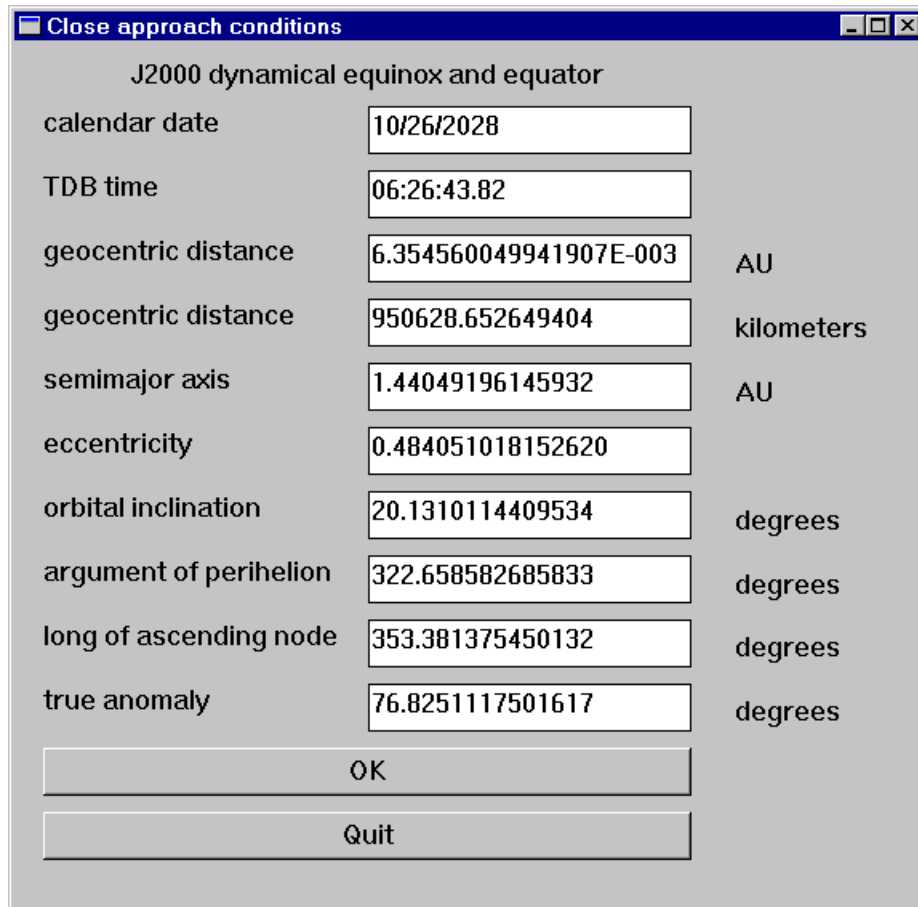
After reading the input data file the program creates a *simulation definition* screen similar to the following. The user can edit anything on this screen before clicking the OK button which runs the program with the data currently displayed in each field.

The following is a typical input screen with data read from an example input file named Xf11\_slp.dat.



The user can edit this data file and specify the ephemeris source to use during the simulation, the total simulation (search) duration in days and an Earth-to-body close approach constraint in astronomical units. The software will ignore any close approaches that are larger than this close approach constraint. Be sure to include all the digits of the calendar year and separate the calendar date items with commas. Notice that time on the Barycentric Dynamical Time (TDB) scale is used.

The following is a typical close approach screen display created by the CAE2HO program for this example.



The orbital elements in this screen are with respect to the J2000 dynamical equinox and equator when the SLP96 ephemeris is used, and the J2000/ICRF equinox and equator when the DE405 source ephemeris is used.

The JPL Solar System Dynamics XF11 web site ([http://ssd.jpl.nasa.gov/ca\\_97xf11.html](http://ssd.jpl.nasa.gov/ca_97xf11.html)) provides the following close approach table for comparison:

Date of Close Earth Approach	Approach Distance	
	(AU)	(km)
1990-Jul-05 19:16	0.244	36,480,000
1997-May-08 21:59	0.156	23,270,000
2002-Oct-31 00:33	0.064	9,510,000
2009-Aug-27 14:21	0.262	39,170,000
2016-Jun-10 15:30	0.180	26,910,000
2021-Nov-18 23:24	0.443	66,340,000
2023-May-05 07:26	0.242	36,190,000
2028-Oct-26 06:26	0.006	954,000

## Technical Discussion

The second-order heliocentric equations of motion of a satellite or celestial body subject to the point mass gravitational attraction of the Sun and planets are given by

$$\ddot{\mathbf{r}} = \frac{d^2 \mathbf{r}}{dt^2}(\mathbf{r}, t) = -\mu_s \frac{\mathbf{r}_{s-b}}{|\mathbf{r}_{s-b}|^3} - \sum_{i=1}^9 \mu_{p_i} \left( \frac{\mathbf{r}_{(p-b)_i}}{|\mathbf{r}_{(p-b)_i}|^3} + \frac{\mathbf{r}_{p_i}}{|\mathbf{r}_{p_i}|^3} \right) \quad (1)$$

where

- $\mu_s$  = gravitational constant of the Sun
- $\mu_{p_i}$  = gravitational constant of planet  $i$
- $\mathbf{r}_p$  = position vector from the Sun to planet
- $\mathbf{r}_{s-b}$  = position vector from the Sun to the body
- $\mathbf{r}_{p-b}$  = position vector from the planet to the body

These position vectors are related according to

$$\mathbf{r}_{s-b} = \mathbf{r}_p + \mathbf{r}_{p-b} \quad (2)$$

The additional relativistic effect of the Sun is given by

$$\ddot{\mathbf{r}} = \frac{d^2 \mathbf{r}}{dt^2}(\mathbf{r}, \mathbf{v}, t) = -\mu_s \frac{\mathbf{r}_{s-b}}{|\mathbf{r}_{s-b}|^3} + \frac{\mu_s}{|\mathbf{r}_{s-b}|^3} \left\{ \left( 4 \frac{\mu_s}{|\mathbf{r}_{s-b}|} - \frac{v^2}{c^2} \right) \mathbf{r}_{s-b} + 4 \frac{(\mathbf{r}_{s-b} \bullet \mathbf{v}_{s-b})}{c^2} \right\} \quad (3)$$

where  $\mathbf{v}_{s-b}$  is the heliocentric velocity vector of the body,  $v$  is the scalar heliocentric speed of the body and  $c$  is the speed of light. An excellent discussion of this effect can be found in “Relativistic Effects on the Motion of Asteroids and Comets”, B. Shahid-Saless and D. Yeomans, *The Astronomical Journal*, Volume 107, Number 5, May 1994.

This computer program uses a Runge-Kutta-Fehlberg 7(8) numerical method to numerically integrate the first-order form of the orbital equations of motion. This is a variable step size method of order 7 with an 8<sup>th</sup> order error estimate which is used to dynamically change the integration step size during the simulation.

This software also uses a one-dimensional optimization algorithm due to Richard Brent to solve the close approach problem. Additional information about this numerical method can be found in *Algorithms for Minimization Without Derivatives*, R.P. Brent, Prentice-Hall, 1972. As the title of this book indicates, this algorithm does not require derivatives of the *objective function*. This feature is important because the analytic first derivative of many objective functions is difficult to derive and code. The objective function for this program is the scalar geocentric distance of the celestial body or spacecraft.