

# Program cae2ho

## Closest Approach between the Earth and Heliocentric Objects

This Windows compatible computer program (`cae2ho.exe`) uses a combination of Cowell's method and one-dimensional minimization to predict closest approach conditions between the Earth and objects such as asteroids, comets and spacecraft in heliocentric orbits. The software includes the *point mass* gravity perturbations due to all the planets and the Moon, and the relativistic effect of the Sun.

This computer program uses a Runge-Kutta-Fehlberg 7(8) numerical method to numerically integrate the first-order form of the orbital equations of motion. This is a variable step size method of order 7 with an 8<sup>th</sup> order error estimate which is used to dynamically change the integration step size during the simulation. This software also uses a one-dimensional minimization algorithm due to Richard Brent to solve the close approach problem. Additional information about this numerical method can be found in the book, *Algorithms for Minimization Without Derivatives*, R. Brent, Prentice-Hall, 1972. As the title indicates, this algorithm does not require derivatives of the *objective function*. This feature is important because the analytic first derivative of many objective functions may be difficult to derive. The objective function for this program is the scalar geocentric distance of the celestial body or spacecraft.

The lunar and planetary ephemeris used in this computer program is based on JPL DE421.

### Program execution

An input file created by the user can be run from the DOS command line or a simple batch file with a statement similar to the following:

```
cae2ho apophis.in
```

If the software is executed without an input file on the command line, the `cae2ho` computer program will display the following prompt:

```
please input the name of the simulation definition file
```

At this point the user should input the name of a valid input file, including the filename extension.

### Input data file

The `cae2ho` computer program is "data-driven" by a simple text file created by the user. This section describes a typical input data file. In the following discussion the actual input file contents are in *courier* font and all explanations are in *times italic* font. Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input.

The following is a typical ASCII input data file for this program. It contains the heliocentric osculating orbital elements of the asteroid Apophis. These coordinates must be relative to the J2000 equinox. The fundamental plane can be either the ecliptic or Earth mean equator. Notice that time on the Barycentric Dynamical Time (TDB) scale is used.

*The first five lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with five and only five initial text lines.*

```
*****
* input data file for cae2ho program
* apophis.in - January 22, 2011
* J2000 ecliptic orbital elements
*****
```

*The first data item in the file is the name of the heliocentric object or spacecraft.*

```
object name
-----
Apophis
```

*The next input is the calendar date at which the orbital elements are valid. Please be sure to include all digits of the calendar year.*

```
initial calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits)
-----
7, 23, 2010
```

*The next input is the TDB time at which the orbital elements are valid.*

```
initial TDB time
(0 <= hours <= 24, 0 <= minutes <= 60, 0 <= seconds <= 60)
-----
0, 0, 0
```

*The next six inputs are the heliocentric osculating orbital elements of the natural object or spacecraft.*

```
semimajor axis (astronomical units)
(semimajor axis > 0)
-----
0.9223399011158424

orbital eccentricity (non-dimensional)
(0 <= eccentricity < 1)
-----
0.191110297656661

orbital inclination (degrees)
(0 <= inclination <= 180)
-----
3.33173591830871

argument of perihelion (degrees)
(0 <= argument of perihelion <= 360)
-----
126.418616993867

longitude of the ascending node (degrees)
(0 <= longitude of the ascending node <= 360)
-----
204.4320062353886

mean anomaly (degrees)
(0 <= mean anomaly <= 360)
-----
202.4952515361516
```

*The next input is the total simulation duration in days.*

```
simulation duration (days)
-----
12000
```

*The next input is a user-defined close approach constraint in Astronomical Units. The software will ignore any close approaches that are larger than this close approach constraint.*

```
close approach constraint (astronomical units)
-----
0.1
```

*The final input is the fundamental frame of the object's orbital elements.*

```
fundamental frame
(1 = ecliptic, 2 = equator)
-----
1
```

## Program example

The following is a typical close approach screen display created by the `cae2ho` program for this example. The heliocentric orbital elements provided by the software are with respect to the Earth mean equator and equinox of J2000 (EME2000). The Barycentric Dynamical Time (TDB) scale is used.

```
program cae2ho

closest approach between the Earth and heliocentric objects
-----

object ==> apophis

initial heliocentric orbital elements
(Earth mean equator and equinox J2000)
-----

calendar date          July 23, 2010
TDB time                00:00:00.000
TDB Julian date        2455400.50000000

      sma (au)          eccentricity      inclination (deg)      argper (deg)
0.922339901116D+00    0.191110297656D+00    0.204497656781D+02    0.334511330058D+03

      lan (deg)         true anomaly (deg)      arglat (deg)          period (days)
0.356054874806D+03    0.195654481884D+03    0.170165811942D+03    0.323545171038D+03

      rx (km)           ry (km)           rz (km)           rmag (km)
-0.158353506954D+09    0.370556360809D+08    0.972222808066D+07    0.162921683789D+09

      vx (kps)          vy (kps)          vz (kps)          vmag (kps)
-0.444688346794D+01    -0.238128589868D+02    -0.897251802956D+01    0.258327912834D+02

close approach conditions
(Earth mean equator and equinox J2000)
-----

calendar date          January 9, 2013
TDB time                11:43:02.670
TDB Julian date        2456301.98822535

      sma (au)          eccentricity      inclination (deg)      argper (deg)
0.922034900273D+00    0.191299627664D+00    0.204488777710D+02    0.334378136425D+03
```

lan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.356074542094D+03	0.141150088303D+03	0.115528224728D+03	0.323384698649D+03
rx (km)	ry (km)	rz (km)	rmag (km)
-.580976534134D+08	0.136323698657D+09	0.492284976217D+08	0.156150354228D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.263657634216D+02	-.586215672559D+01	-.285369579058D+01	0.271599326572D+02
geocentric distance	9.666151989911026E-002	AU	
geocentric distance	14460357.5546626	kilometers	

close approach conditions  
(Earth mean equator and equinox J2000)

-----

calendar date	April 13, 2029		
TDB time	21:46:13.845		
TDB Julian date	2462240.40710468		
sma (au)	eccentricity	inclination (deg)	argper (deg)
0.101825436726D+01	0.223547949766D+00	0.201712367711D+02	0.306889498889D+03
lan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.355763284220D+03	0.260867949100D+03	0.207757447989D+03	0.375303702609D+03
rx (km)	ry (km)	rz (km)	rmag (km)
-.137257013911D+09	-.556037638094D+08	-.240957752218D+08	0.150039570784D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.178880982752D+02	-.227237111439D+02	-.783946372449D+01	0.299634493932D+02
geocentric distance	2.546814606760312E-004	AU	
geocentric distance	38099.8042216079	kilometers	

The simulation summary screen display contains the following information:

**TDB time** = simulation event time on the TDB time scale

**TDB Julian date** = simulation event Julian date on the TDB time scale

**sma (au)** = semimajor axis in astronomical units

**eccentricity** = orbital eccentricity (non-dimensional)

**inclination (deg)** = orbital inclination in degrees

**argper (deg)** = argument of perigee in degrees

**lan (deg)** = longitude of the ascending node in degrees

**true anomaly (deg)** = true anomaly in degrees

**arglat (deg)** = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

**period (days)** = orbital period in days

**rx (km)** = x-component of the object's position vector in kilometers

**ry (km)** = y-component of the object's position vector in kilometers

**rz (km)** = z-component of the object's position vector in kilometers

**rmag (km)** = scalar magnitude of the object's position vector in kilometers

**vx (km/sec)** = x-component of the object's velocity vector in kilometers per second

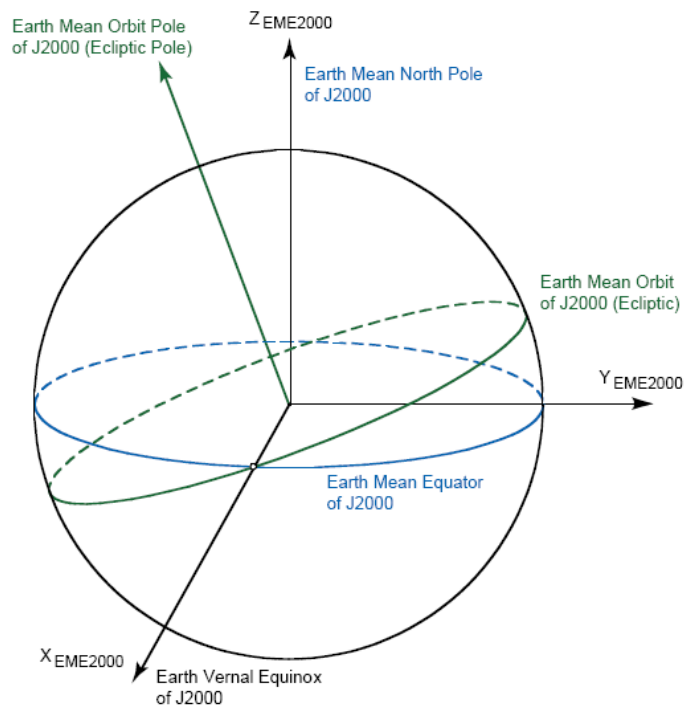
**vy (km/sec)** = y-component of the object's velocity vector in kilometers per second

**vz (km/sec)** = z-component of the object's velocity vector in kilometers per second

**vmag (km/sec)** = scalar magnitude of the object's velocity vector in kilometers per second

## Technical Discussion

In this scientific simulation, the heliocentric orbital motion of the natural object or man-made spacecraft is modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this heliocentric inertial coordinate system is the Sun and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time (TT).



**Figure 1. Earth mean equator and equinox of J2000 coordinate system**

The second-order heliocentric equations of motion of a satellite or celestial body subject to the point mass gravitational attraction of the Sun, Moon and planets are given by

$$\ddot{\mathbf{r}} = \frac{d^2 \mathbf{r}}{dt^2}(\mathbf{r}, t) = -\mu_s \frac{\mathbf{r}_{s-b}}{|\mathbf{r}_{s-b}|^3} - \sum_{i=1}^9 \mu_{p_i} \left( \frac{\mathbf{r}_{(p-b)_i}}{|\mathbf{r}_{(p-b)_i}|^3} + \frac{\mathbf{r}_{p_i}}{|\mathbf{r}_{p_i}|^3} \right)$$

where

$\mu_s$  = gravitational constant of the Sun

$\mu_{p_i}$  = gravitational constant of planet  $i$

$\mathbf{r}_p$  = position vector from the Sun to planet

$\mathbf{r}_{s-b}$  = position vector from the Sun to the body

$\mathbf{r}_{p-b}$  = position vector from the planet to the body

These position vectors are related according to

$$\mathbf{r}_{s-b} = \mathbf{r}_p + \mathbf{r}_{p-b}$$

To avoid numerical problems, use is made of Richard Battin's  $f(q)$  function given by

$$f(q_k) = q_k \left[ \frac{3 + 3q_k + q_k^2}{1 + (\sqrt{1 + q_k})^3} \right]$$

where

$$q_k = \frac{\mathbf{r}^T (\mathbf{r} - 2\mathbf{s}_k)}{\mathbf{s}_k^T \mathbf{s}_k}$$

The point-mass acceleration due to  $n$  gravitational bodies can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^n \frac{\mu_k}{d_k^3} [\mathbf{r} + f(q_k) \mathbf{s}_k]$$

In these equations,  $\mathbf{s}_k$  is the vector from the primary body to the secondary body,  $\mu_k$  is the gravitational constant of the secondary body and  $\mathbf{d}_k = \mathbf{r} - \mathbf{s}_k$ , where  $\mathbf{r}$  is the position vector of the spacecraft relative to the primary body. The derivation of the  $f(q)$  functions is described in Section 8.4 of "An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition", by Richard H. Battin, AIAA Education Series, 1999.

The additional relativistic effect of the Sun is given by

$$\ddot{\mathbf{r}} = \frac{d^2 \mathbf{r}}{dt^2}(\mathbf{r}, \mathbf{v}, t) = -\mu_s \frac{\mathbf{r}_{s-b}}{|\mathbf{r}_{s-b}|^3} + \frac{\mu_s}{|\mathbf{r}_{s-b}|^3} \left\{ \left( 4 \frac{\mu_s}{|\mathbf{r}_{s-b}|} - \frac{v^2}{c^2} \right) \mathbf{r}_{s-b} + 4 \frac{(\mathbf{r}_{s-b} \bullet \mathbf{v}_{s-b})}{c^2} \right\}$$

where  $\mathbf{v}_{s-b}$  is the heliocentric velocity vector of the body,  $v$  is the scalar heliocentric speed of the body and  $c$  is the speed of light. An excellent discussion of this effect can be found in “Relativistic Effects on the Motion of Asteroids and Comets”, B. Shahid-Saless and D. Yeomans, *The Astronomical Journal*, Volume 107, Number 5, May 1994.

For orbital elements in the J2000 ecliptic coordinate system, the ecliptic state vector (position and velocity vectors) must be transformed to the equatorial frame for use within the `cae2ho` computer program. The required transformation is given by

$$\mathbf{r}_{eq} = \begin{bmatrix} 1 & -0.000000479966 & 0 \\ 0.000000440360 & 0.917482137087 & 0.397776982902 \\ -0.000000190919 & -0.397776982902 & 0.917482137087 \end{bmatrix} \mathbf{r}_{ec}$$

where  $\mathbf{r}_{ec}$  is the position vector in the ecliptic frame and  $\mathbf{r}_{eq}$  is the position vector in the equatorial frame. The same transformation is applied to the velocity vector.

### Useful links

<http://newton.dm.unipi.it/neodys/index.php?pc=1.1.0&n=Aphophis>

[http://neo.jpl.nasa.gov/apophis/Aphophis\\_PUBLISHED\\_PAPER.pdf](http://neo.jpl.nasa.gov/apophis/Aphophis_PUBLISHED_PAPER.pdf)

<http://neo.jpl.nasa.gov/apophis/>