

# Converting Classical Orbital Elements to an SGP4 Compatible Two Line Element Set (TLE)

This Windows XP/NT/2000 application (`coe2tle.exe`) can be used to convert user-defined classical orbital elements to an SGP4 compatible Two Line Element Set (TLE).

The NORAD orbit propagators are popular, accurate and easy to use. They are based on the work of Dirk Brouwer, Lane and Cranford, and others. The Two Line Element (TLE) sets required by these propagators are widely distributed on the Internet and maintained by NORAD. Each algorithm is documented in the classic SpaceTrack Report No. 3, "Models for Propagation of NORAD Element Sets" by Felix R. Hoots and Ronald L. Roehrich. This document also contains Fortran source code and test cases for each propagator.

The following is a typical TLE for the NOAA 14 spacecraft:

```
NOAA 14
1 23455U 94089A 97320.90946019 .00000140 00000-0 10191-3 0 2621
2 23455 99.0090 272.6745 0008546 223.1686 136.8816 14.11711747148495
```

The *mean* orbital elements contained in this data are ECI coordinates with respect to the true equator of date and the mean equinox of date. They do not include the effect of nutation.

The following is a brief description of the data contained in each line of a Two Line Element Set. Each item must appear in its row and column field in exactly the format specified. Most TLE databases actually contain *three* lines of data for each space object.

## Line 1

The first line is a twenty-four character satellite name. Software which reads a database of TLEs will look for this name in order to find the correct data.

## Line 2

Column	Description
01	line number of element data
03-07	satellite number
08	classification (u=unclassified)
10-11	international designator (last two digits of launch year)
12-14	international designator (launch number of the year)
15-17	international designator (piece of the launch)
19-20	epoch year (last two digits of year)
21-32	epoch (day of the year and fractional portion of the day)
34-43	first time derivative of the mean motion
45-52	second time derivative of mean motion (decimal point assumed)
54-61	bstar drag term (decimal point assumed)
63	ephemeris type
65-68	element number
69	checksum (modulo 10) (letters, blanks, periods, plus signs = 0; minus signs = 1)

### Line 3

Column	Description
01	line number of element data
03-07	satellite number
09-16	orbital inclination (degrees)
18-25	right ascension of the ascending node (degrees)
27-33	orbital eccentricity (decimal point assumed)
35-42	argument of perigee (degrees)
44-51	mean anomaly (degrees)
53-63	mean motion (orbits per day)
64-68	revolution number at epoch (orbits)
69	checksum (modulo 10)

*All other columns are blank or fixed.*

This algorithm uses the SGP4 propagator and should not be used to process orbits with periods greater than 225 minutes.

The initial guess for the SGP4 compatible *mean* orbital elements (subscript *sgp*) are set to the classical *osculating* orbital elements (subscript *osc*) as follows:

$$n_{sgp} = n_{osc} = \text{mean motion}$$

$$e_{sgp} = e_{osc} = \text{eccentricity}$$

$$i_{sgp} = i_{osc} = \text{inclination}$$

$$\omega_{sgp} = \omega_{osc} = \text{argument of perigee}$$

$$\Omega_{sgp} = \Omega_{osc} = \text{right ascension of ascending node}$$

$$M_{sgp} = M_{osc} = \text{mean anomaly}$$

This computer program solves the following vector system of nonlinear equations

$$\mathbf{r}_{osc} - \mathbf{r}_{sgp} = 0 \tag{1}$$

$$\mathbf{v}_{osc} - \mathbf{v}_{sgp} = 0$$

subject to the eccentricity inequality constraint  $e_{sgp} \geq 0$ . In this system of nonlinear equations  $\mathbf{r}$  is the Earth-centered inertial position vector and  $\mathbf{v}$  is the velocity vector.

The inequality constraint ensures that the orbital eccentricity computed by the algorithm will always be zero or positive.

The rectangular components of the satellite's osculating position vector are determined from

$$\begin{aligned}
r_x &= r [\cos \Omega \cos (\omega + \nu) - \sin \Omega \cos i \sin (\omega + \nu)] \\
r_y &= r [\sin \Omega \cos (\omega + \nu) + \cos \Omega \cos i \sin (\omega + \nu)] \\
r_z &= r \sin i \sin (\omega + \nu)
\end{aligned}
\tag{2}$$

The components of the osculating velocity vector are computed using the following equations:

$$\begin{aligned}
v_x &= -\sqrt{\frac{\mu}{p}} [\cos \Omega \{\sin (\omega + \nu) + e \sin \omega\} + \sin \Omega \cos i \{\cos (\omega + \nu) + e \cos \omega\}] \\
v_y &= -\sqrt{\frac{\mu}{p}} [\sin \Omega \{\sin (\omega + \nu) + e \sin \omega\} - \cos \Omega \cos i \{\cos (\omega + \nu) + e \cos \omega\}] \\
v_z &= \sqrt{\frac{\mu}{p}} [\sin i \{\cos (\omega + \nu) + e \cos \omega\}]
\end{aligned}
\tag{3}$$

The SGP position and velocity vectors are determined from the SGP4 algorithm. The mathematics of this algorithm are described in SpaceTrack Report 3.

Essentially, this numerical method is trying to minimize the difference between the state vector determined from the classical orbital elements input by the user and the state vector computed by the SGP4 algorithm using current values for the SGP4 orbital elements. Since the two state vectors are not dramatically different, the algorithm converges in a reasonable CPU time.

Since we do not know the value of the first time derivative of the mean motion, the second time derivative of mean motion and the *bstar* drag term, these SGP4 orbital elements are set to zero for all computations. This information could be computed by processing many state vectors and performing some type of *differential correction* or orbit determination process.

The *simulation definition* is input to the program using the following main screen:

TLE date and time		classical orbital elements	
date (mm, dd, yyyy)	3,21,1999	semimajor axis (km)	8000
UT (hr, min, sec)	10,20,30	eccentricity	0.015
create TLE data file?		inclination (degrees)	28.5
<input type="radio"/> yes		argument of perigee	200
<input checked="" type="radio"/> no		RAAN (degrees)	100
data file name	sate1.tle	true anomaly (degrees)	45
OK		Quit	

This screen allows the user to define such things as the initial calendar date, universal time and classical orbital elements. The user can also elect to save the computed TLE to a simple ASCII data file. After the user clicks on the OK button, the software will display a data screen that summarizes the orbital elements and epoch. The following is the screen for this example.

calendar date	03/21/1999		
universal time	10:20:30.00		
semimajor axis (km)	eccentricity (non-dimensional)	orbital inclination (degrees)	argument of perigee (degrees)
0.8000000000D+04	0.1500000000D-01	0.2850000000D+02	0.2000000000D+03
RAAN (degrees)	true anomaly (degrees)	argument of latitude (degrees)	orbital period (minutes)
0.1000000000D+03	0.4500000000D+02	0.2450000000D+03	0.1186846396D+03
OK		Quit	

Finally, the software will display a data screen showing the computed TLE. The following is the Two Line Element Set output screen for this example.

```

1 XXXXXU XXXXXXXX 99080.43090278 .00000000 00000-0 00000-0
2 XXXXX 28.5080 99.9801 0148100 199.9797 43.8243 12.13842236
  
```

OK

Quit

A good Internet source for the latest Two Line Element Sets is Dr. Thomas Kelso's site:

<http://celestrak.com>

You can also find a Postscript and PDF version of SpaceTrack Report No. 3 at this location.

*Warning: Do not use TLEs with other orbit propagators. They are only compatible with the SGP4, SDP4 and other algorithms developed by NORAD.*