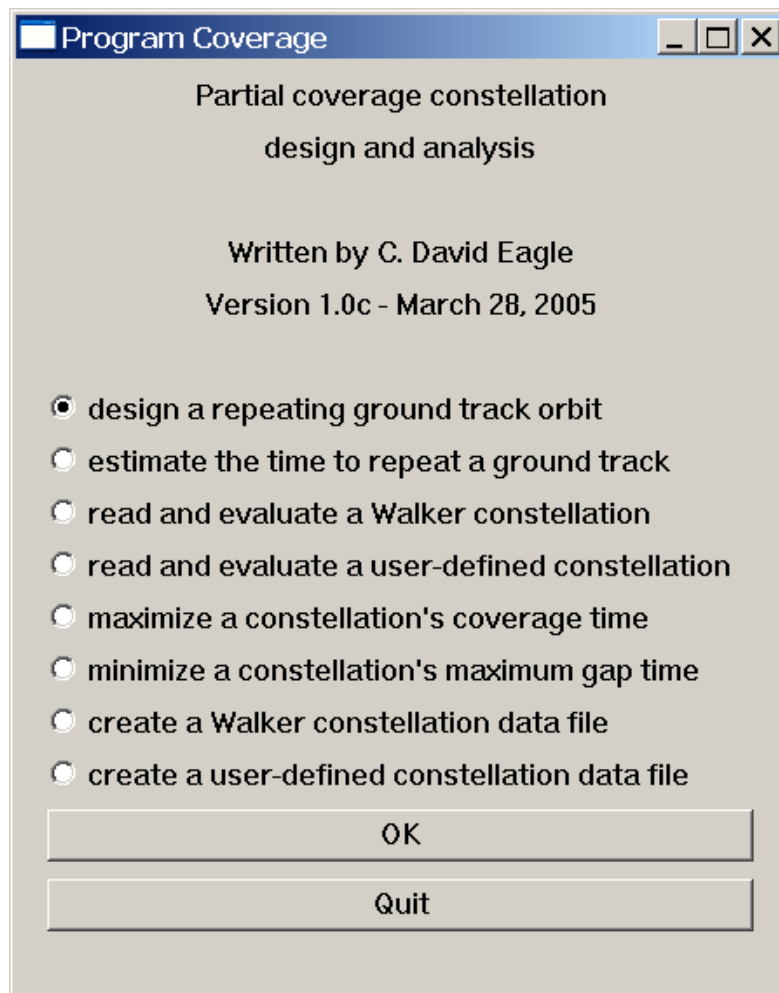


Partial Coverage Constellation Design and Analysis

This document is the user's manual for an interactive Windows XP/Vista computer program (*coverage.exe*) that can be used to analyze partial coverage satellite constellations. The user may provide a Walker or "custom" constellation, as well as a point target or circular area-of-interest (AOI) target. The software will evaluate the coverage characteristics (number of accesses and gaps, total coverage and gap time, etc.) of the constellation. The algorithm can also optimize the constellation for maximum coverage time or gap time.

The software assumes that all satellites in a Walker constellation are in circular orbits with a common altitude and inclination. The software also assumes that satellites in a user-defined constellation have a common altitude, inclination, orbital eccentricity and argument of perigee.

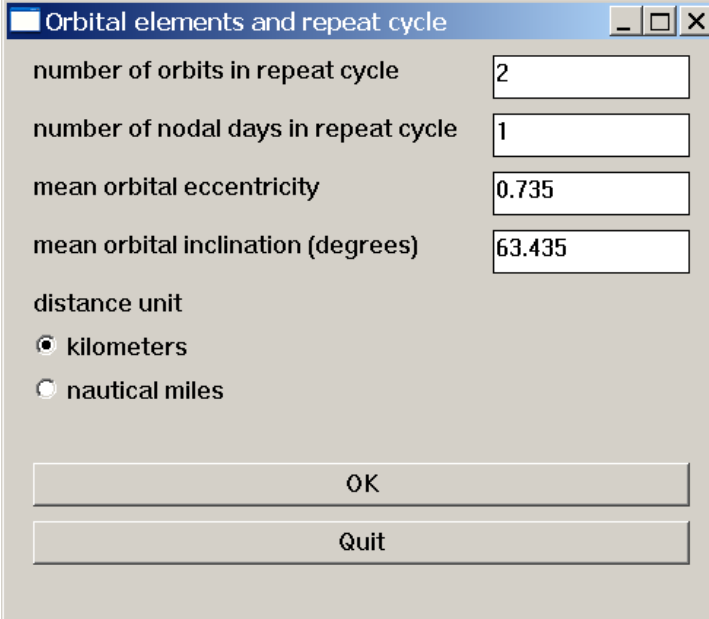
The software begins by displaying the following *main menu*:



Any program option is activated by simply selecting its radio button and clicking on the OK button. The program can be terminated at any time by clicking the Quit button. The following is a brief description of each menu item.

design a repeating ground track orbit

This program option will allow the user to determine the mean semimajor axis required for a repeating ground track orbit. The program will ask you for several items of information with the following interactive screen:

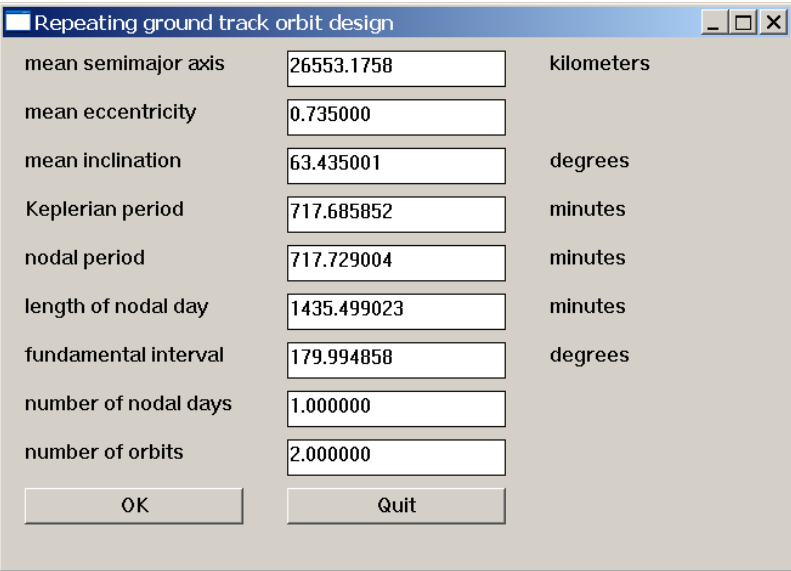


The screenshot shows a dialog box titled "Orbital elements and repeat cycle". It contains several input fields and radio buttons. The "number of orbits in repeat cycle" is set to 2, "number of nodal days in repeat cycle" is 1, "mean orbital eccentricity" is 0.735, and "mean orbital inclination (degrees)" is 63.435. The "distance unit" is set to "kilometers". There are "OK" and "Quit" buttons at the bottom.

Parameter	Value
number of orbits in repeat cycle	2
number of nodal days in repeat cycle	1
mean orbital eccentricity	0.735
mean orbital inclination (degrees)	63.435
distance unit	kilometers

The first two items are integers, the orbital eccentricity is a non-dimensional number greater than or equal to zero and less than one (elliptic orbits), and the orbital inclination should be a number between 0 and 180 degrees. The distance unit radio button determines the units of the semimajor axis calculated by this program option.

The following is a typical output screen produced by this program option:



The screenshot shows an output dialog box titled "Repeating ground track orbit design". It displays calculated values for various orbital parameters. The "mean semimajor axis" is 26553.1758 kilometers, "mean eccentricity" is 0.735000, "mean inclination" is 63.435001 degrees, "Keplerian period" is 717.685852 minutes, "nodal period" is 717.729004 minutes, "length of nodal day" is 1435.499023 minutes, "fundamental interval" is 179.994858 degrees, "number of nodal days" is 1.000000, and "number of orbits" is 2.000000. There are "OK" and "Quit" buttons at the bottom.

Parameter	Value	Unit
mean semimajor axis	26553.1758	kilometers
mean eccentricity	0.735000	
mean inclination	63.435001	degrees
Keplerian period	717.685852	minutes
nodal period	717.729004	minutes
length of nodal day	1435.499023	minutes
fundamental interval	179.994858	degrees
number of nodal days	1.000000	
number of orbits	2.000000	

The Keplerian period is the unperturbed period of the constellation, and the nodal period is the time between ascending node crossings including the first order oblateness effect (J_2) of the Earth. The fundamental interval is the longitudinal separation between sequential orbit ground tracks at the equator. The nodal period in seconds is determined with the following equation:

$$\tau_n = \frac{2\pi}{(\tilde{n} + \dot{\omega})}$$

where \tilde{n} is the perturbed mean motion and $\dot{\omega}$ is the argument of perigee perturbation due to oblateness.

The nodal day is the time required for the Earth to make one complete revolution with respect to the orbital plane. The length of the nodal day in seconds can be determined from

$$T_n = \frac{2\pi}{(\omega_e - \dot{\Omega})}$$

where ω_e is the inertial rotation rate of the Earth and $\dot{\Omega}$ is the regression (or precession) of the orbit's ascending node due to oblateness.

estimate the time to repeat a ground track

This program option will allow the user to estimate the time required for an orbit to repeat its ground track. The program will ask you for several items of information with the following interactive screen:

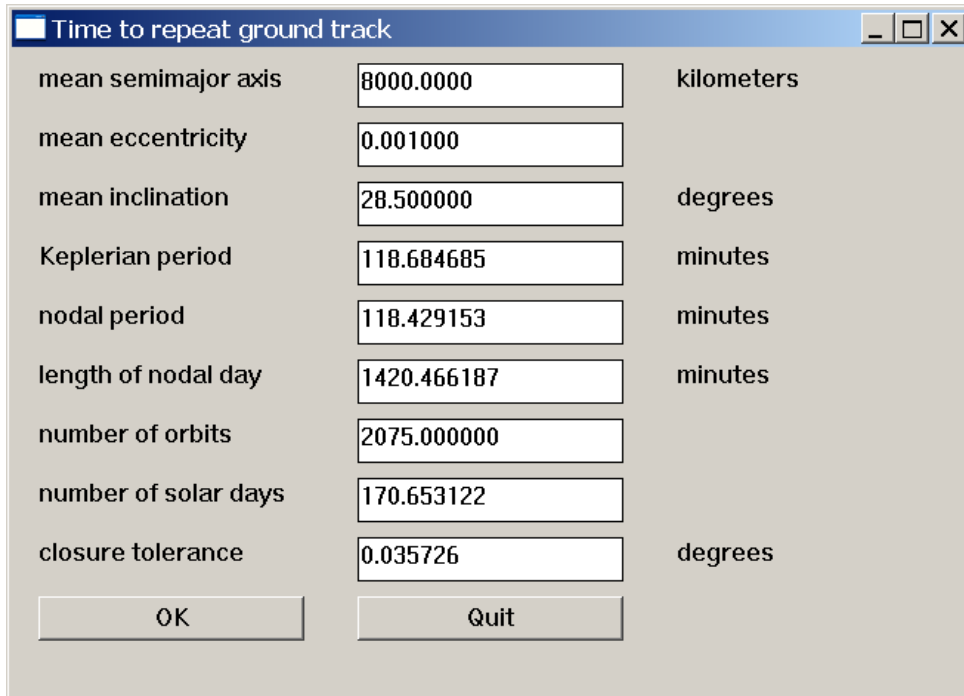
The screenshot shows a dialog box with the title "Orbital elements and closure tolerance". It contains the following fields and options:

- mean semimajor axis: 8000
- mean orbital eccentricity: 0.001
- orbital inclination (degrees): 28.5
- closure tolerance (degrees): 0.1
- distance unit:
 - kilometers
 - nautical miles

At the bottom of the dialog box are two buttons: "OK" and "Quit".

The closure tolerance determines how well the program estimates the time to repeat. A closure tolerance of 0.1 degrees is recommended. The semimajor axis input by the user should be consistent with the distance unit selection.

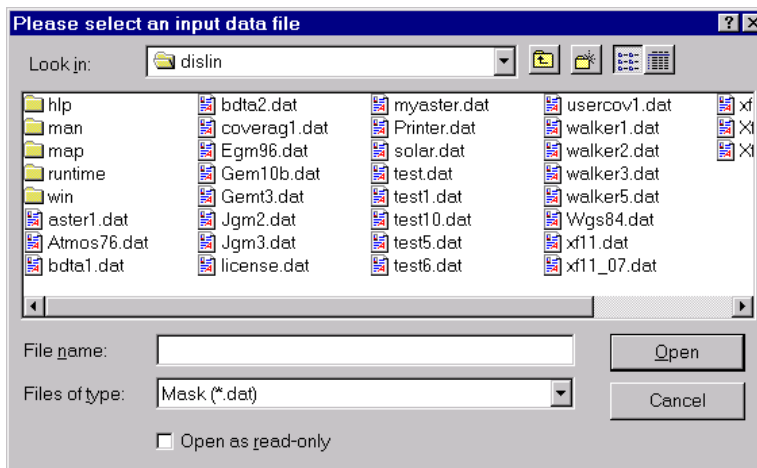
The following is a typical output screen created with this program option:



read and evaluate a Walker constellation

This program option will allow the user to read and evaluate the coverage characteristics of a Walker satellite constellation. The program determines such things as total number of accesses, minimum coverage time, average coverage time, maximum coverage time, total coverage time, total number of gaps, minimum gap time, average gap time, maximum gap time and total gap time. The software will automatically determine the mean anomalies and longitudes of ascending node for each satellite in the Walker constellation.

This main menu selection will display a file selection window similar to the following:



You can select a Walker constellation input file by double clicking on its name or by typing the name in the File name: entry field. The default file name mask for input files is *.dat. However, any compatible input file can be selected. The following is the format of a typical Walker data file:

```
Walker T/P/F configuration
7,7,4

distance unit (kilometers or nautical miles)
kilometers

constellation semimajor axis
6865.222d0

constellation inclination (degrees)
38.0d0

ground site latitude (degrees)
30.0d0

ground site longitude (degrees)
0.0d0

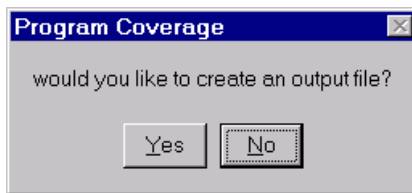
ground site altitude (meters or feet)
0.0d0

minimum elevation angle constraint (degrees)
5.0d0

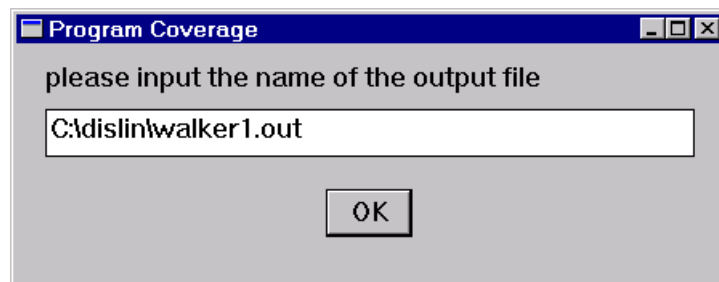
radius of area-of-interest
0.0d0

simulation duration (days)
1.0d0
```

After selecting an input file, the program will ask if you would like to create an ASCII output file with the following prompt:



If you click on the Yes button the software will ask you for the name of the output file with



After reading the input data file, the program will display a *simulation definition* screen similar to the following:

Parameter	Value	Unit
T/P/F configuration	7.7.4	
semimajor axis	6865.2222	kilometers
eccentricity	0.0000	
inclination	38.0000	degrees
argument of perigee	0.0000	degrees
target latitude	30.0000	degrees
target longitude	0.0000	degrees
minimum elevation angle	5.0000	degrees
radius of area-of-interest	0.0000	kilometers
simulation duration	1.0000	days

Buttons: OK, Quit

Finally, this option will display a *coverage statistics* screen similar to the following:

Statistic	Value	Unit
total number of accesses	45	
minimum coverage time	3.3010	minutes
average coverage time	8.4505	minutes
maximum coverage time	9.5876	minutes
total coverage time	380.2719	minutes
total number of gaps	46	
minimum gap time	1.8544	minutes
average gap time	23.0376	minutes
maximum gap time	29.8402	minutes
total gap time	1059.7281	minutes

Buttons: OK, Quit

The following is the format of a typical Walker constellation output file:

```
Walker constellation analysis

T/P/F configuration          7,7,4

semimajor axis              6865.222000 kilometers

eccentricity                0.000000

argument of perigee        0.000000 degrees

inclination                 38.000000 degrees

satellite      east longitude of      mean anomaly
number         ascending node (deg)    (degrees)

  1              0.000000              0.000000
  2              51.428571             205.714286
  3             102.857143              51.428571
  4             154.285714             257.142857
  5             205.714286             102.857143
  6             257.142857             308.571429
  7             308.571429             154.285714

coverage statistics

target latitude             30.000000 degrees
target longitude            0.000000 degrees
minimum elevation angle     5.000000 degrees

area-of-interest radius    0.000000 kilometers

total number of accesses   45

minimum coverage time      3.300965 minutes
average coverage time      8.450487 minutes
maximum coverage time      9.587650 minutes
total coverage time        380.271903 minutes

total number of gaps       46

minimum gap time           1.854360 minutes
average gap time           23.037567 minutes
maximum gap time           29.840157 minutes
total gap time             1059.728097 minutes

simulation duration        1.000000 days
```

read and evaluate a user-defined constellation

This menu option will allow the user to read and evaluate the coverage characteristics of a user-defined satellite constellation. The program determines such things as total number of accesses, minimum coverage time, average coverage time, maximum coverage time, total coverage time, total number of gaps, minimum gap time, average gap time, maximum gap time and total gap time.

The format of a typical user-defined constellation ASCII input file is as follows:

```
number of satellites
6

distance unit (kilometers or nautical miles)
nautical miles

initial orbital elements - satellite #1
3707.265d0
0.0d0
39.0d0
0.0d0
348.17d0
0.0d0

initial orbital elements - satellite #2
3707.265d0
0.0d0
39.0d0
0.0d0
22.83d0
201.21d0

initial orbital elements - satellite #3
3707.265d0
0.0d0
39.0d0
0.0d0
57.5d0
42.42d0

initial orbital elements - satellite #4
3707.265d0
0.0d0
39.0d0
0.0d0
168.57d0
180.0d0

initial orbital elements - satellite #5
3707.265d0
0.0d0
39.0d0
0.0d0
203.23d0
21.21d0

initial orbital elements - satellite #6
3707.265d0
0.0d0
39.0d0
0.0d0
237.9d0
222.42d0

ground site latitude (degrees)
30.0d0
```

ground site longitude (degrees)
0.0d0

ground site altitude (meters or feet)
0.0d0

minimum elevation angle constraint (degrees)
5.0d0

radius of area-of-interest
10.0d0

simulation duration (days)
1.0d0

The order of the orbital elements is semimajor axis, eccentricity, inclination, argument of perigee, east longitude of the ascending node and mean anomaly. All angular elements are in degrees. Please note that the semimajor axis, observer altitude and radius of area-of-interest must be consistent with the distance unit field.

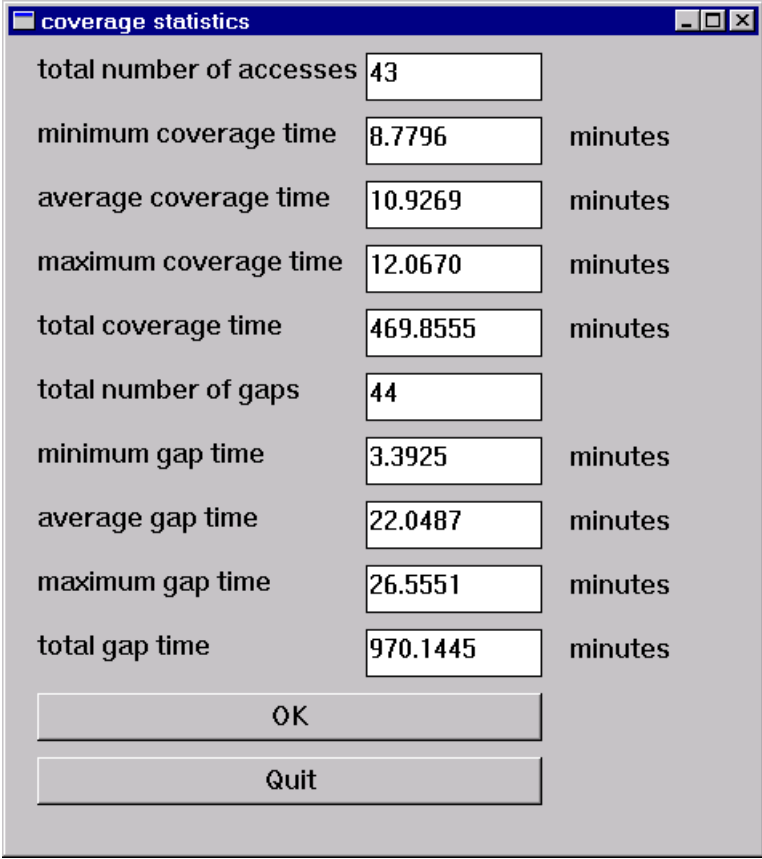
The simulation definition screen for this software option is similar to the following:

Parameter	Value	Unit
total number of satellites	6	
semimajor axis	3707.2649	nm
eccentricity	0.0000	
inclination	39.0000	degrees
argument of perigee	0.0000	degrees
target latitude	30.0000	degrees
target longitude	0.0000	degrees
minimum elevation angle	-0.1658	degrees
radius of area-of-interest	10.0000	nm
simulation duration	1.0000	days

OK

Quit

The coverage statistics screen is similar to



The screenshot shows a dialog box titled "coverage statistics" with the following data:

Metric	Value	Unit
total number of accesses	43	
minimum coverage time	8.7796	minutes
average coverage time	10.9269	minutes
maximum coverage time	12.0670	minutes
total coverage time	469.8555	minutes
total number of gaps	44	
minimum gap time	3.3925	minutes
average gap time	22.0487	minutes
maximum gap time	26.5551	minutes
total gap time	970.1445	minutes

At the bottom of the dialog box are two buttons: "OK" and "Quit".

maximize a constellation's coverage time

This option will allow the user to determine the mean anomalies and longitudes of ascending node which will maximize the coverage time of either a Walker or user-defined satellite constellation.

minimize a constellation's maximum gap time

This option will allow the user to determine the mean anomalies and longitudes of ascending node which will minimize the maximum gap time of either a Walker or user-defined satellite constellation.

IMPORTANT NOTE!!

You must read and evaluate a constellation before attempting to optimize it. The software will allow the user to evaluate a constellation first, return to the main menu, and optimize the same constellation without reading the constellation input data file again.

create a Walker constellation data file

This program option allows the user to create a new Walker constellation input data file. The interactive screen for this option is as follows:

Walker T/P/F configuration	7.7,4
constellation semimajor axis	6865.222
constellation inclination (degrees)	38.0
ground site latitude (degrees)	30.0
ground site longitude (degrees)	0.0
ground site altitude (feet or meters)	0.0
elevation angle constraint (degrees)	5.0
radius of area-of-interest	0.0
simulation duration (days)	1
file name	walker.dat
distance unit	<input checked="" type="radio"/> kilometers <input type="radio"/> nautical miles
OK	
Quit	

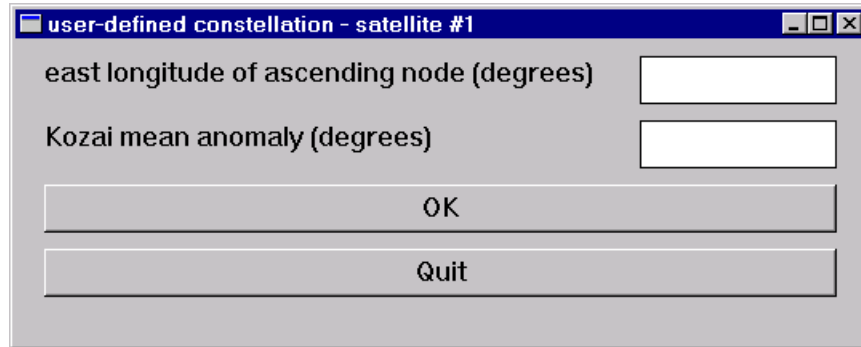
create a user-defined constellation data file

This program option allows the user to create a new user-defined constellation ASCII input data file. The interactive screen for this option is as follows:

number of constellation satellites	6	distance unit <input checked="" type="radio"/> kilometers <input type="radio"/> nautical miles
constellation semimajor axis	6865.222	
constellation eccentricity	0.0	OK
constellation inclination (degrees)	38.0	
argument of perigee (degrees)	0.0	Quit
ground site latitude (degrees)	30.0	
ground site longitude (degrees)	0.0	
ground site altitude (feet or meters)	0.0	
elevation angle constraint (degrees)	5.0	
radius of area-of-interest	50.0	
simulation duration (days)	1	
file name (include extension)	coverag.dat	

Simply edit the fields of this screen to define a new input file and click on the Ok button to save the information to disk under the user-defined file name.

The software will interactively request the east longitude of ascending node and mean anomaly of each satellite of the constellation with the following prompt:



The input for each of these two fields should be a number between 0 and 360 degrees.

Input files and program notes

This section provides additional information about input files, program conventions and calculations.

For a Walker configuration, T is the total number of satellites in the constellation, P is the number of individual orbit planes, and F is the phasing of the satellites within their orbit planes. The user must also specify the semimajor axis and orbital inclination of the constellation. The software will automatically determine the mean anomalies and east longitudes of ascending node for each satellite in the constellation.

The ground site latitude is a real number between -90 and +90 degrees, with north latitudes positive and south latitudes negative. The longitude of the ground site is measured positive east of Greenwich and should be a number between 0 and 360 degrees. The ground site altitude is measured with respect to sea level, with sites above sea level positive and those below negative. The software accounts for the flattening of the Earth during the coverage calculations.

A minimum elevation angle constraint can be enforced during all calculations by specifying a non-zero number (in degrees) for this data file item.

A circular area-of-interest (AOI) can be specified by the AOI radius in kilometers. This area-of-interest is centered at the latitude and longitude of the ground site.

IMPORTANT NOTE!!

If the AOI radius is exactly zero, the software will enforce the minimum elevation angle constraint and all calculations will be with respect to a point target. If the AOI radius is non-zero, the software will ignore the minimum elevation angle constraint and perform all calculations with respect to a circular area-of-interest with this radius.

The simulation duration data item determines how long the software will run while calculating coverage statistics. For best results it should be equal to the repeat cycle of the constellation in days.

The order, units and valid ranges of the orbital elements for each satellite of the constellation are as follows:

- semimajor axis (kilometers; $sma > 0$)
- orbital eccentricity (non-dimensional; $0 \leq \text{eccentricity} < 1$)
- orbital inclination (degrees; $0 \leq \text{inclination} \leq 180$)
- argument of perigee (degrees; $0 \leq \text{argument of perigee} \leq 360$)
- east longitude of the ascending node (degrees; $0 \leq \text{elan} \leq 360$)
- mean anomaly (degrees; $0 \leq \text{mean anomaly} \leq 360$)

Each satellite in the constellation must have a set of six orbital elements “introduced” by one annotation line of text.

Technical discussion

The `coverage.exe` software uses a J_2 form of *Kozai's method* (“The Motion of a Close Earth Satellite”, *The Astronomical Journal*, **64**, No. 1274, pp. 367-377) to propagate the orbits of all satellites in a Walker or user-defined constellation. Therefore, all orbital elements input to the program and output from the analysis are assumed to be Kozai “mean” elements. These orbital elements include the J_2 secular effects on mean anomaly, argument of perigee and longitude of ascending node. All calculations in this program are performed in an Earth-centered non-rotating (ECF) coordinate system.

According to Kozai’s method, the time evolution of the mean orbital elements is given by the following three equations:

$$\begin{aligned} M(t) &= M_0 + \tilde{n}(t - t_0) \\ \lambda(t) &= \lambda_0 + (\dot{\lambda} - \omega_e)(t - t_0) \\ \omega(t) &= \omega_0 + \dot{\omega}(t - t_0) \end{aligned}$$

where M_0 is the mean anomaly, λ_0 is the east longitude of the ascending node and ω_0 is the argument of perigee, all at the initial time t_0 . In the first expression \tilde{n} is called the *perturbed mean motion* and is equal to the time rate of change of mean anomaly. In the second expression ω_e is the inertial rotation rate of the Earth.

The perturbed mean motion can be calculated from:

$$\tilde{n} = \frac{dM}{dt} = n \left\{ 1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{p} \right)^2 \sqrt{1 - e^2} \left(1 - \frac{3}{2} \sin^2 i \right) \right\}$$

The time rate of change of the east longitude of the ascending node is determined from

$$\dot{\lambda} = \frac{d\lambda}{dt} = -\frac{3}{2} J_2 \tilde{n} \left(\frac{r_{eq}}{p} \right)^2 \cos i$$

The secular perturbation of the argument of perigee is given by:

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{3}{2} J_2 \tilde{n} \left(\frac{r_{eq}}{p} \right)^2 \left(2 - \frac{5}{2} \sin^2 i \right)$$

where

n = unperturbed or Keplerian mean motion

J_2 = Earth oblateness gravity term

r_{eq} = equatorial radius of the Earth

e = orbital eccentricity

i = orbital inclination

$p = a(1 - e^2)$ = semiparameter

a = semimajor axis

The repeating ground track orbit design algorithm is based on a numerical method by Carl Wagner.

The calculation of visibility times is based on the algorithm described in “Efficient Computation of Satellite Visibility Periods”, AAS 92-146 by M. A. Chylla and C. D. Eagle. Kepler’s equation is solved using Danby’s method and all visibility calculations include the effect of the oblate shape of the Earth. The software uses an unconstrained, multi-variable minimization algorithm for the optimization options. This numerical method is described in the book *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, by J. E. Dennis, Jr. and R. B. Schnabel.

This computer program is written in Compaq Visual Fortran version 6.6. It uses the DISLIN graphics library (www.dislin.de) for the GUI.