Impulsive Hyperbolic Injection from a Circular Park Orbit

This document is the user's guide for a Windows XP/Vista Fortran computer program named `hyper1.exe` which can be used to determine the characteristics of a single impulsive maneuver from a circular park orbit to a departure hyperbola. The algorithm in this program is based on the equations derived in Chapter 4 of Richard Battin's classic text, *Astronautical Guidance*, and Chapter 11 of *An Introduction to the Mathematics and Methods of Astrodynamics*, also written by Professor Battin and published by the American Institute of Aeronautics and Astronautics.

The Earth departure trajectory for interplanetary missions is usually defined by a "targeting specification" which consists of twice the specific (per unit mass) orbital energy C_3, and the right ascension α_∞ and declination δ_∞ of the outgoing asymptote. These numbers may be supplied by a spacecraft customer or determined with a patched-conic or more sophisticated trajectory analysis computer program that solves Lambert's problem for an interplanetary mission.

The `hyper1` software determines the orbital elements and state vectors of the park orbit and departure hyperbola at injection, and the injection delta-v vector and magnitude. This information can be used as initial guesses for other trajectory simulations.

This computer program assumes that the hyperbolic targets and orbital characteristics are in the same Earth-centered-inertial (ECI) coordinate system. For example, targeting specs are often provided or computed in an Earth mean equator and equinox of J2000 coordinate system (EME2000). For this situation, the state vectors and orbital elements computed by this code will also be with respect to the EME2000 coordinate system.

Running the software

The `hyper1` program will interactively prompt the user for the park orbit altitude and orbital inclination, and the departure hyperbola characteristics. These prompts appear as follows;

```
please input the altitude of the circular park orbit (kilometers)?
185.2

please input the orbital inclination of the park orbit (degrees)?
(0 <= inclination <= 180)
28.5

please input the C3 of the departure hyperbola [(km/sec)**2]?
(C3 > 0)
9.28

please input the right ascension of the outgoing asymptote (degrees)?
(0 <= right ascension <= 360)
352.59

please input the declination of the outgoing asymptote (degrees)?
(|declination| <= park orbit inclination)
2.27
```

Please note the proper units and valid data range for each input.

Program Output

The following is the program output for this example.

```
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Interplanetary Injection from a Circular Park Orbit
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departure hyperbola characteristics
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c3                            9.28000000000000      km**2/sec**2

asymptote right ascension     352.590000000000      degrees

asymptote declination         2.27000000000000      degrees

orbital elements and state vector of park orbit at injection - opportunity #1
-----------------------------------------------------------------------------

     sma (km)        eccentricity      inclination (deg)    argper (deg)
 0.6563340000D+04    0.0000000000D+00    0.2850000000D+02    0.0000000000D+00

    raan (deg)     true anomaly (deg)     arglat (deg)       period (min)
 0.1767767337D+03    0.2507477991D+02    0.2507477991D+02    0.8819562703D+02

     rx (km)            ry (km)             rz (km)           rmag (km)
 -.6072821513D+04    -.2106348521D+04    0.1327240269D+04    0.6563340000D+04

     vx (kps)           vy (kps)            vz (kps)          vmag (kps)
 0.2948681176D+01    -.6379088912D+01    0.3368063861D+01    0.7793032157D+01

orbital elements and state vector of hyperbola at injection - opportunity #1
----------------------------------------------------------------------------

     sma (km)        eccentricity      inclination (deg)    argper (deg)
 -.4295264009D+05    0.1152804111D+01    0.2850000000D+02    0.2507477991D+02

    raan (deg)     true anomaly (deg)     arglat (deg)       period (min)
 0.1767767337D+03    0.3600000000D+03    0.2507477991D+02    0.1666666667D+98

     rx (km)            ry (km)             rz (km)           rmag (km)
 -.6072821513D+04    -.2106348521D+04    0.1327240269D+04    0.6563340000D+04

     vx (kps)           vy (kps)            vz (kps)          vmag (kps)
 0.4326433915D+01    -.9359678097D+01    0.4941770522D+01    0.1143427743D+02

injection delta-v vector and magnitude - opportunity #1
-------------------------------------------------------

x-component of delta-v        1377.75273908206      meters/second

y-component of delta-v       -2980.58918468226      meters/second

z-component of delta-v        1573.70666162370      meters/second

delta-v magnitude             3641.24527527765      meters/second
```

```
orbital elements and state vector of park orbit at injection - opportunity #2
-----------------------------------------------------------------------------

     sma (km)          eccentricity       inclination (deg)     argper (deg)
 0.6563340000D+04    0.0000000000D+00    0.2850000000D+02    0.0000000000D+00

    raan (deg)       true anomaly (deg)     arglat (deg)        period (min)
 0.3484032663D+03    0.2145979019D+03    0.2145979019D+03    0.8819562703D+02

     rx (km)             ry (km)             rz (km)             rmag (km)
 -.5950748689D+04    -.2122224678D+04    -.1778253190D+04    0.6563340000D+04

     vx (kps)            vy (kps)            vz (kps)            vmag (kps)
 0.3201396036D+01    -.6411955694D+01    -.3060921069D+01    0.7793032157D+01
```

```
orbital elements and state vector of hyperbola at injection - opportunity #2
----------------------------------------------------------------------------

     sma (km)          eccentricity       inclination (deg)     argper (deg)
 -.4295264009D+05    0.1152804111D+01    0.2850000000D+02    0.2145979019D+03

    raan (deg)       true anomaly (deg)     arglat (deg)        period (min)
 0.3484032663D+03    0.3600000000D+03    0.2145979019D+03    0.1666666667D+98

     rx (km)             ry (km)             rz (km)             rmag (km)
 -.5950748689D+04    -.2122224678D+04    -.1778253190D+04    0.6563340000D+04

     vx (kps)            vy (kps)            vz (kps)            vmag (kps)
 0.4697228205D+01    -.9407901676D+01    -.4491117193D+01    0.1143427743D+02
```

```
injection delta-v vector and magnitude - opportunity #2
-------------------------------------------------------

x-component of delta-v       1495.83216868269      meters/second

y-component of delta-v      -2995.94598183915      meters/second

z-component of delta-v      -1430.19612353262      meters/second

delta-v magnitude            3641.24527527765      meters/second
```

Trajectory Graphics

The following is the display for this example. This display is labeled with an Earth centered, inertial coordinate system. The x-axis of this system is red, the y-axis is green and the z-axis is blue. The outgoing asymptote is colored magenta, the park orbit traces are red, and the hyperbolic trajectories are black. Please note the units for each coordinate axis are Earth radii (ER).

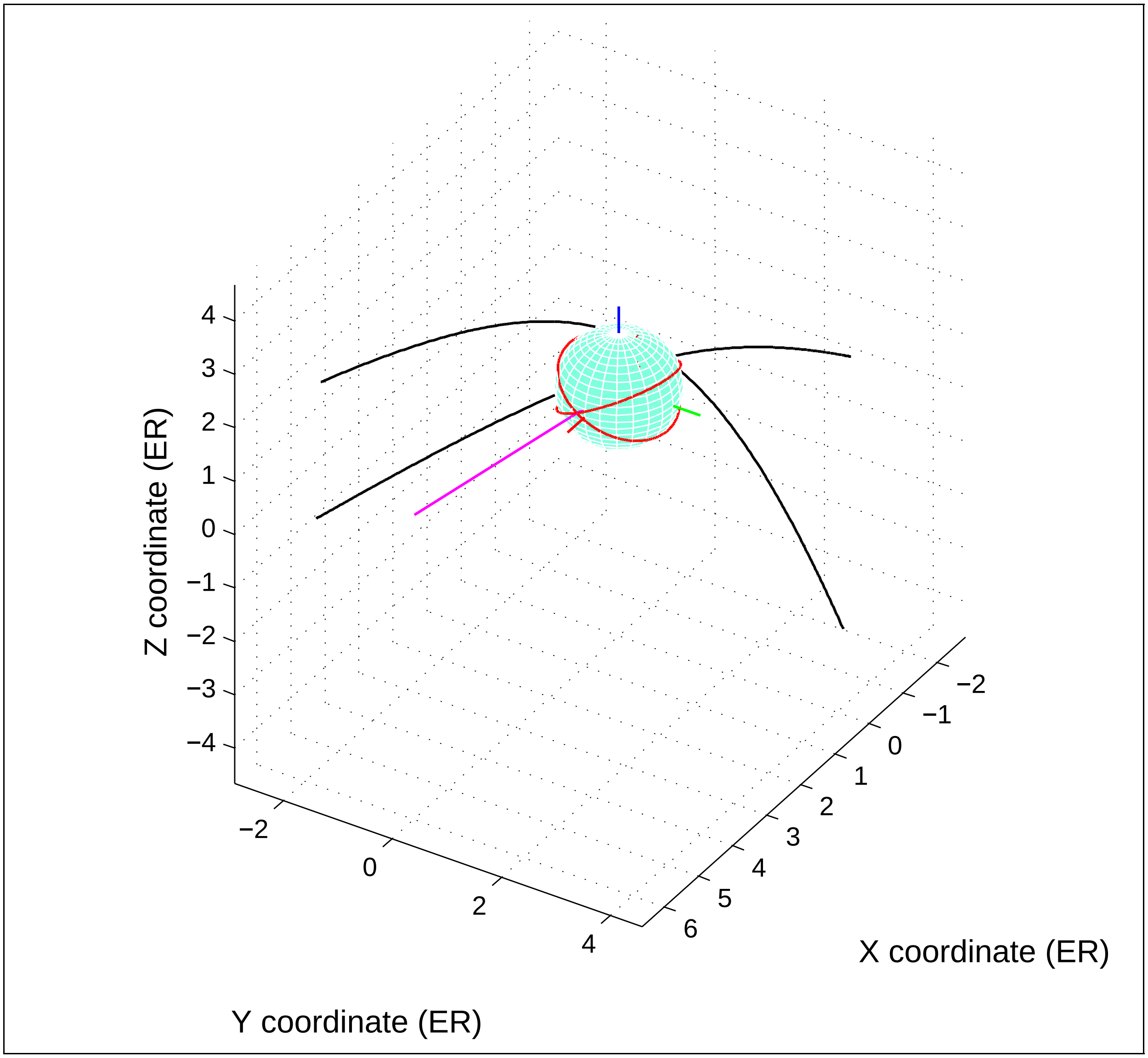

Figure 1 Park Orbits and Departure Hyperbolas

Technical Discussion

The numerical algorithm used in this program is valid for geocentric orbit inclinations that satisfy the following geometric constraint

$$i \ge |\delta_\infty|$$

where i is the orbital inclination of the park orbit. The algorithm also assumes a circular Earth park orbit and that injection occurs impulsively at the perigee of the departure hyperbola. Whenever $i > |\delta_\infty|$, there will be two opportunities to establish a departure hyperbola that will satisfy the energy and orientation of the outgoing asymptote. One injection opportunity will occur while the spacecraft is ascending and the other while the spacecraft is descending along the park orbit. For the case where $i = |\delta_\infty|$, there will be a single injection opportunity.

Orientation of the park orbit and departure hyperbola

This section summarizes the equations used to determine the right ascension of the ascending node (RAAN) of the park orbit and the injection true anomaly on the park orbit.

A unit vector is the direction of the departure asymptote is given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where

$\alpha_\infty =$ right ascension of departure asymptote

$\delta_\infty =$ declination of departure asymptote

The angle between the outgoing asymptote and the spin axis of the Earth is given by

$$\beta = \cos^{-1}\left(\hat{\mathbf{s}} \bullet \hat{\mathbf{z}}\right)$$

where $\hat{\mathbf{z}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. Note that $\beta = 90^\circ - \delta_\infty$.

The park orbit right ascension of the ascending node for each opportunity can be determined from

$$\Omega_1 = 180^\circ + \alpha_\infty + \sin^{-1}\left(\frac{\cot\beta}{\tan i}\right)$$

$$\Omega_2 = 360^\circ + \alpha_\infty - \sin^{-1}\left(\frac{\cot\beta}{\tan i}\right)$$

The true anomaly on the park orbit for each injection opportunity can be determined from

$$\theta_1 = \cos^{-1}\left(\frac{\cos\beta}{\sin i}\right) - \eta$$

$$\theta_2 = -\cos^{-1}\left(\frac{\cos\beta}{\sin i}\right) - \eta$$

where

$$\eta = \sin^{-1}\left(\frac{1}{1 + r_p V_\infty^2 / \mu}\right)$$

In the last equation, r_p is the geocentric radius of the park orbit and μ is the gravitational constant of the Earth. The velocity vector at infinity V_∞ is determined from $V_\infty = \sqrt{C_3}$.

For a tangential impulsive injection maneuver that occurs at perigee of the hyperbola, the true anomaly on the hyperbola is zero. Furthermore, since the orbit transfer modeled by this software is coplanar, the right ascension of the ascending node computed above should be the same for both the park orbit and the launch hyperbola. This can be verified by examining the hyperbola's RAAN which is computed using the state vector at injection.

Departure delta-V

The velocity vector at any geocentric position vector $\mathbf{r}$ required to achieve a launch hyperbola defined by V_∞, α_∞ and δ_∞ is given by

$$\mathbf{v}_h = \left(d + \frac{1}{2}V_\infty\right)\hat{\mathbf{s}} + \left(d - \frac{1}{2}V_\infty\right)\hat{\mathbf{r}}$$

where

$$d = \sqrt{\frac{\mu}{(1+\cos\psi)\,r_p} + \frac{V_\infty^2}{4}}$$

and ψ is the angle between the spacecraft's position vector and the departure asymptote unit vector which can be computed using

$$\cos\psi = \hat{\mathbf{s}} \bullet \hat{\mathbf{r}}$$

The injection $\Delta\mathbf{v}$ vector can be determined from the following expression

$$\Delta\mathbf{v} = \mathbf{v}_h - \mathbf{v}_p$$

where $\mathbf{v}_p$ is the inertial velocity vector in the park orbit prior to injection and $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$.

Finally, the scalar injection delta-v is $\Delta v = |\Delta\mathbf{v}|$.

Delta-v penalty for off-nominal injection

The velocity-required equation given above can also be used to access the delta-v penalty for off-nominal injection. Such things as ignition timing errors and other spacecraft contingencies may result in an injection maneuver that does not occur at the optimal true anomaly on the park orbit. The following plot illustrates how the injection delta-v penalty changes as the true anomaly at injection is displaced from the optimal.

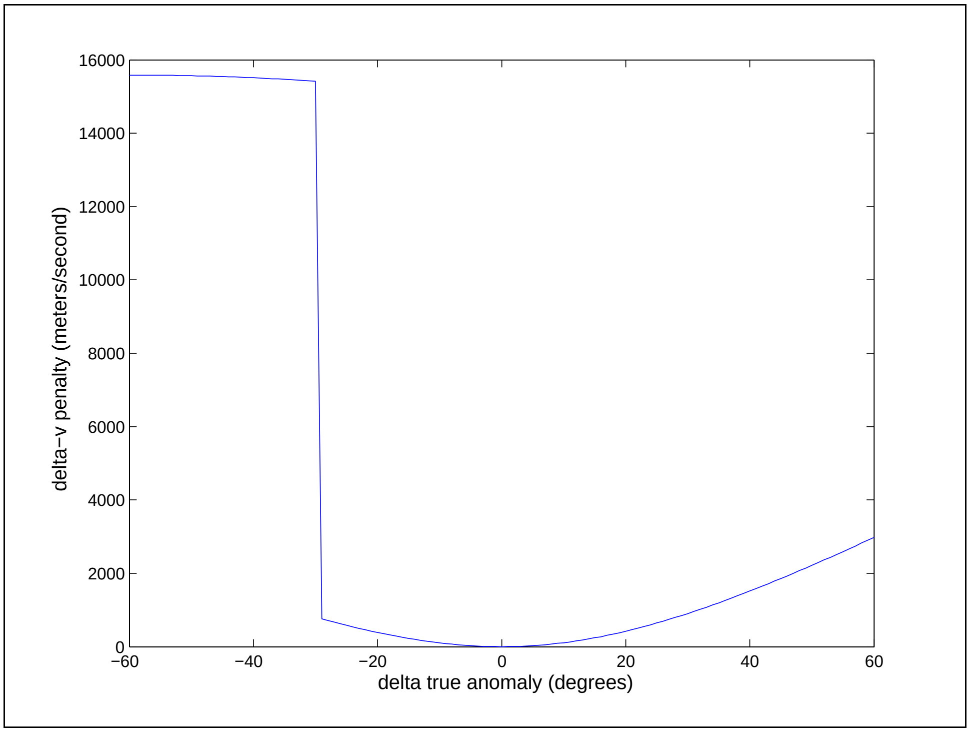

Figure 2 Delta-V Penalty for Off-nominal Injection