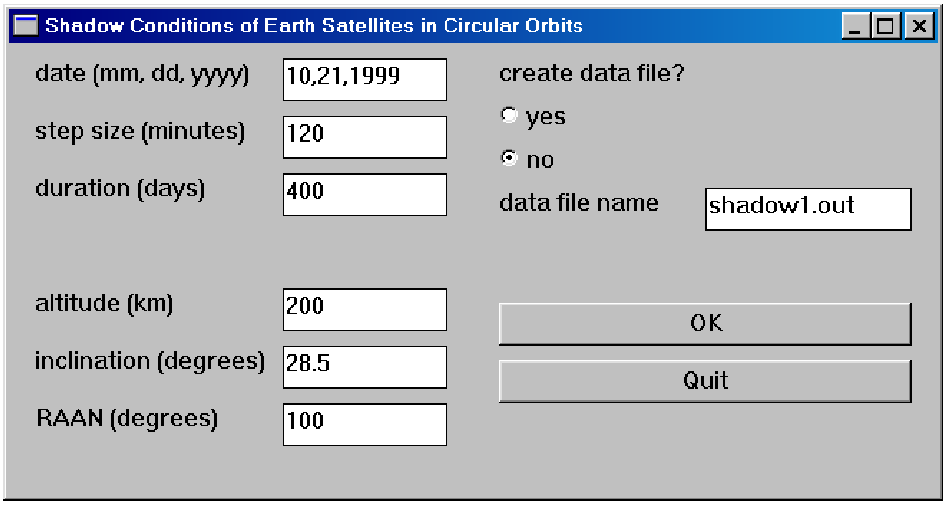
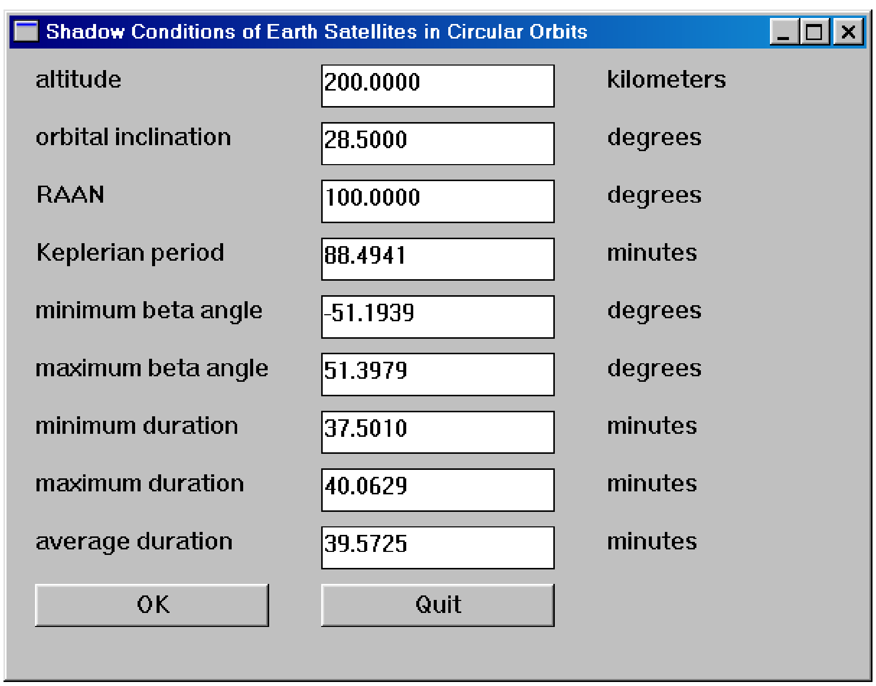
# Shadow Conditions of Earth Satellites in Circular Orbits

This Windows XP/Vista application (`shadow1.exe`) can be used to determine the eclipse duration and beta angle for Earth satellites in circular orbits. This computer program is written in Fortran and uses the DISLIN graphics library to display graphics of the shadow conditions.

The *simulation definition* is input to the program using the following main screen:



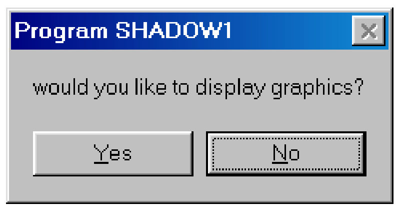

This screen allows the user to define such things as the initial calendar date and orbital elements, and the simulation step size and duration. After the user clicks on the OK button, the software will display a data screen that summarizes the characteristics of the shadow conditions. The following is the program output for this example.



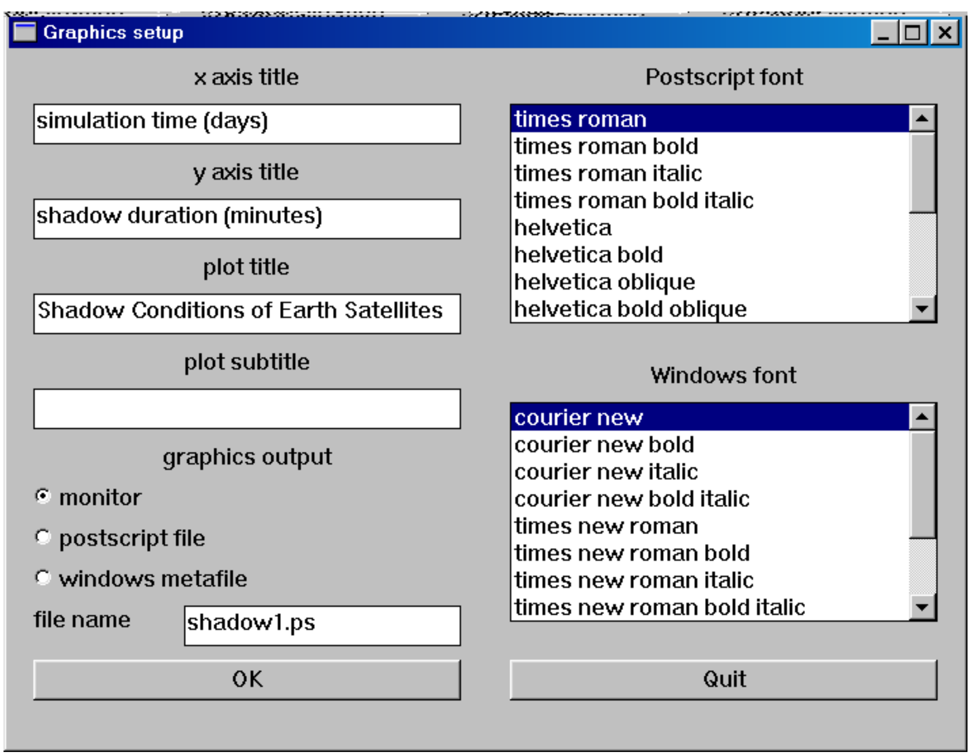
The user can also elect to create a simple ASCII data file of the shadow conditions. The following is part of the data file for this example.

```
#     time          duration       beta angle
#    (days)         (minutes)       (degrees)
     0.0000          38.9553        -37.5850
     0.0833          38.9609        -37.5082
     0.1667          38.9667        -37.4261
     0.2500          38.9730        -37.3387
     0.3333          38.9795        -37.2460
     0.4167          38.9864        -37.1481
     0.5000          38.9937        -37.0450
     0.5833          39.0012        -36.9368
     0.6667          39.0091        -36.8235
     0.7500          39.0173        -36.7052
     0.8333          39.0257        -36.5819
     0.9167          39.0345        -36.4536
     1.0000          39.0435        -36.3204
```

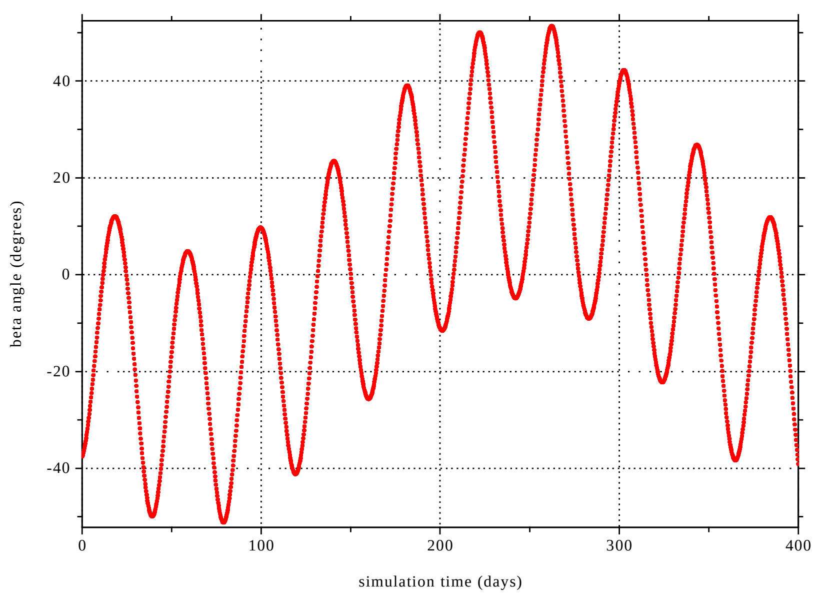
After the data display screen is created and displayed, the program will ask if you would like to create shadow conditions graphics with




If the user clicks on the Yes button, the program will display the following *graphics setup* screen.



## Shadow Conditions of Earth Satellites



## Technical Discussion

The approximate eclipse duration of a satellite in a circular orbit which penetrates an Earth shadow represented by a right circular cylinder can be calculated from

$$t_s = \left\{ \cos^{-1}\left( \frac{\sqrt{1-R^2}}{\cos\beta} \right) \right\} \frac{\tau}{\pi} \tag{1}$$

where

$R$ = radius ratio = $r_{eq} / r_{sat}$

$r_{eq}$ = equatorial radius of the Earth

$r_{sat}$ = geocentric radius of the satellite = $r_{eq} + h_{sat}$

$h_{sat}$ = altitude of the satellite

$\beta$ = Sun-orbit-plane angle

$\tau$ = orbital period of the satellite = $2\pi\sqrt{r_{sat}^3 / \mu}$

$\mu$ = gravitational constant of the Earth

Equation (1) also assumes that the Sun does not move during the eclipse. Furthermore, the time units of eclipse duration are the same as the orbital period $\tau$ provided the inverse cosine in this equation returns an angle in radians.

The part of Equation (1) represented by

$$\cos^{-1}\left( \frac{\sqrt{1-R^2}}{\cos\beta} \right)$$

is one half the true anomaly angle traversed by the satellite during the eclipse. A closer examination of this equation also reveals that satellites at altitudes which satisfy the *radius ratio* inequality given by

$$R > \sin\left|\beta_{\max}\right|$$

will have periods during which they are not eclipsed by the Earth.

The Sun-orbit plane or *beta* angle is the angle between the geocentric position vector to the Sun and the satellite's orbit plane. It is calculated from

$$\beta = \sin^{-1}\left( \hat{\mathbf{r}}_{sun} \bullet \hat{\mathbf{h}}_{sat} \right) \tag{2}$$

where $\hat{\mathbf{r}}_{sun}$ is the geocentric unit position vector of the Sun and $\hat{\mathbf{h}}_{sat}$ is the unit angular momentum vector of the satellite's orbit. The unit angular momentum vector is defined by the cross product $\hat{\mathbf{r}}_{sat} \times \hat{\mathbf{v}}_{sat}$ where $\hat{\mathbf{r}}_{sat}$ and $\hat{\mathbf{v}}_{sat}$ are the unit position and velocity vectors of the satellite, respectively.

A positive beta angle indicates that the Sun in on the positive angular momentum side of the orbit plane. The inertial orientation of this vector changes over time as the oblateness of the Earth causes the orbit plane to move. The unit angular momentum vector can also be determined from the satellite's classical orbital elements with

$$\hat{\mathbf{h}}_{sat} = \begin{Bmatrix} \sin\Omega \sin i \\ -\cos\Omega \sin i \\ \cos i \end{Bmatrix} \tag{3}$$

where $i$ is the satellite's orbital inclination and $\Omega$ is the right ascension of the ascending node (RAAN).

The unit position vector of the Sun can be determined with the expression

$$\hat{\mathbf{r}}_{sun} = \begin{Bmatrix} \cos\delta_{sun} \cos\alpha_{sun} \\ \cos\delta_{sun} \sin\alpha_{sun} \\ \sin\delta_{sun} \end{Bmatrix} \tag{4}$$

where $\alpha_{sun}$ and $\delta_{sun}$ are the geocentric, equatorial right ascension and declination of the Sun, respectively.

The beta angle in terms of the satellite's RAAN and orbital inclination is given by

$$\beta = \arcsin\left(\cos\delta_{sun} \sin i \sin\left(\Omega - \alpha_{sun}\right) + \sin\delta_{sun} \cos i\right) \tag{5}$$

Over a period of one year the beta angle varies between $-\left(i + \delta_{sun}\right) \le \beta \le +\left(i + \delta_{sun}\right)$ as the solar declination varies between about plus and minus 23.5°.

An examination of Equation (1) reveals that the maximum shadow time occurs when $\cos\beta = 1$ which corresponds to $\beta = 0$. This is the instant when the Sun unit position vector $\hat{\mathbf{r}}_{sun}$ lies in the satellite's orbit plane. The minimum shadow time will occur when $\cos\beta$ reaches its minimum value. This happens when the beta angle reaches its largest plus and minus values.

During a simulation the value of RAAN at the initial time $t_0$ is updated for the secular effect of Earth oblateness according to

$$\Omega\left(t\right) = \Omega_0 + \dot{\Omega}\left(t - t_0\right) \tag{6}$$

where

$$\dot{\Omega} = \frac{d\Omega}{dt} = -\frac{3}{2} J_2 \tilde{n} \left(\frac{r_{eq}}{p}\right)^2 \cos i$$

For more realistic shadow predictions, the radius of the Earth is increased by 2% to account for the effect of the atmosphere on the size of the shadow.