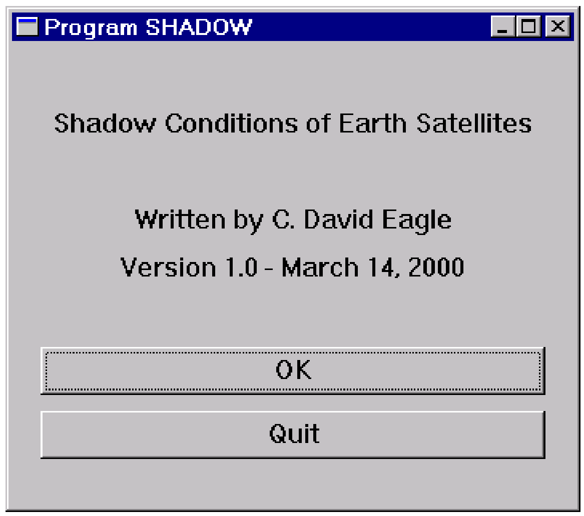
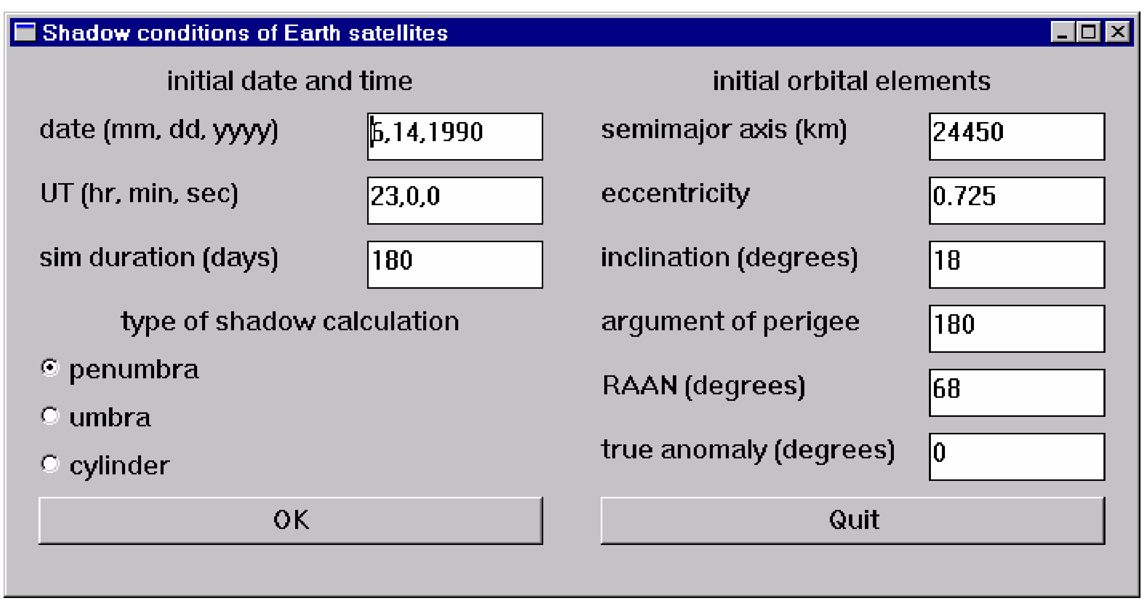
Shadow Conditions of Earth Satellites

Accurate predictions of shadow conditions for a satellite in any type of Earth orbit can be determined by using a combination of one-dimensional minimization and root-finding. The algorithm used in this Windows XP/2000/NT computer program (`shadow2.exe`) searches for minimum values of the angle between the shadow axis and the satellite's position vector as a function of time. If this angle lies within the penumbra, umbra or cylinder shadow, the algorithm uses Brent's root-finding method to look backward and forward relative to this minima time to find entrance and exit conditions.

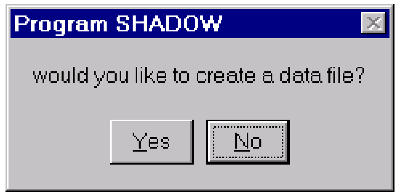
The software begins by displaying the following opening screen:



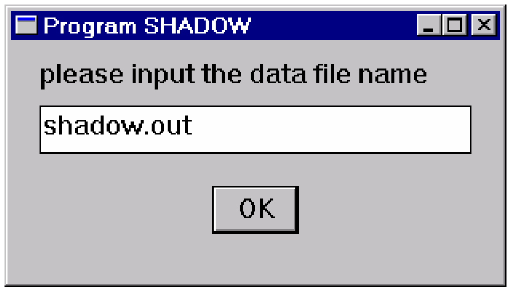
After clicking OK the computer program will display the following screen. This screen allows the user to input the initial calendar date, initial universal time and the simulation duration in days. It also permits the user to input the initial classical orbital elements of the satellite and the type of shadow calculations to use during the search.



After clicking the OK button the software will ask if you would like to create an ASCII data file of the shadow conditions with this next screen.



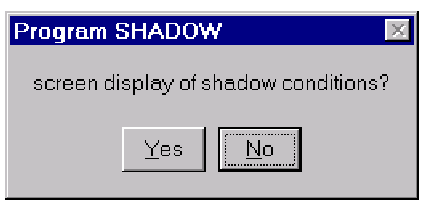
An input of Yes will cause the software to prompt you for the name of the output data file with this next input screen:



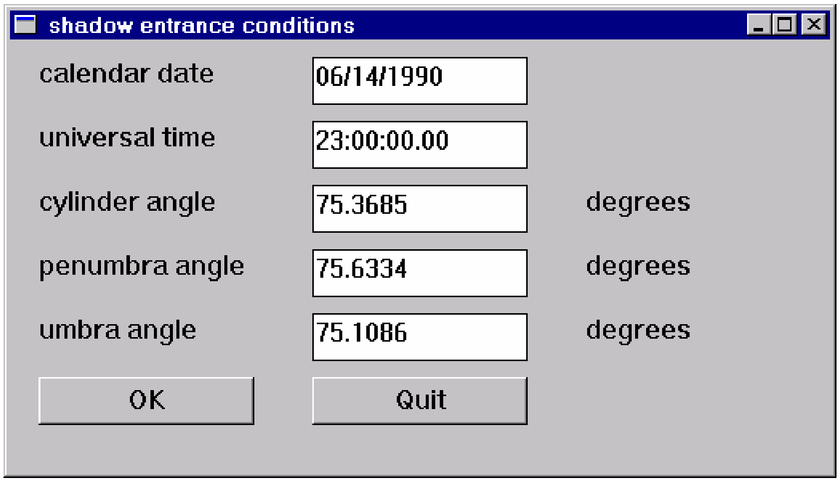
The following is part of a typical data file created with this program option. The Sim Time column is the elapsed time in days for the condition of *minimum* separation angle between the anti-Sun vector and the satellite.

```
 Entrance     Entrance        Exit         Exit        Sim Time      Duration
   Date         Time          Date         Time         (days)       (minutes)
06/14/1990, 23:00:00.00, 06/14/1990, 23:15:34.26, 0.20001E-02, 0.15571E+02
06/15/1990, 09:25:59.27, 06/15/1990, 09:49:35.12, 0.44227E+00, 0.23598E+02
06/15/1990, 19:59:58.15, 06/15/1990, 20:23:36.00, 0.88255E+00, 0.23631E+02
06/16/1990, 06:33:57.03, 06/16/1990, 06:57:36.88, 0.13228E+01, 0.23664E+02
06/16/1990, 17:07:55.91, 06/16/1990, 17:31:37.76, 0.17631E+01, 0.23698E+02
06/17/1990, 03:41:54.78, 06/17/1990, 04:05:38.65, 0.22034E+01, 0.23731E+02
06/17/1990, 14:15:53.65, 06/17/1990, 14:39:39.55, 0.26436E+01, 0.23765E+02
06/18/1990, 00:49:52.51, 06/18/1990, 01:13:40.45, 0.30839E+01, 0.23799E+02
```

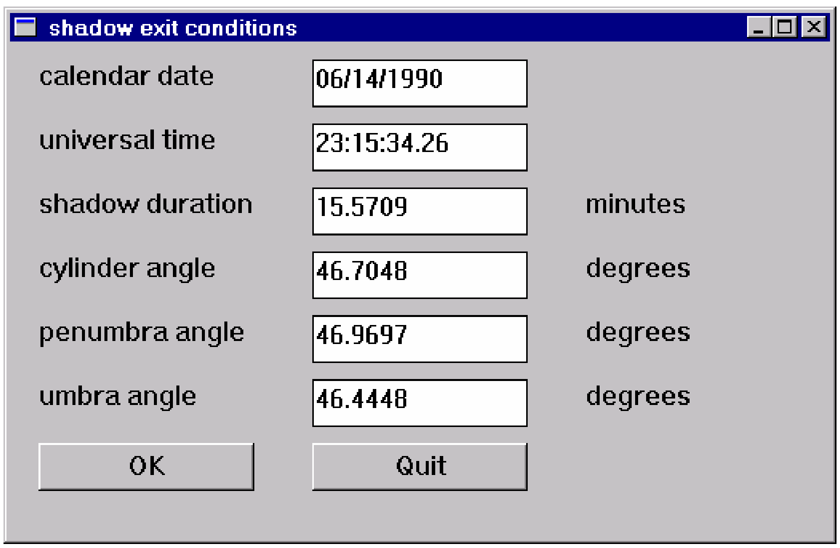
The software will then ask if you would like to display the shadow conditions on the screen. This request is as follows:



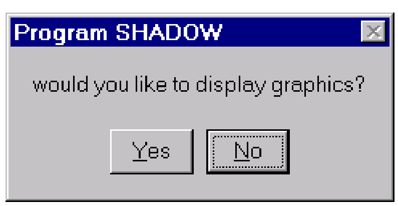
If you select Yes the software will display the shadow entrance and exit conditions screens which are similar to the following:



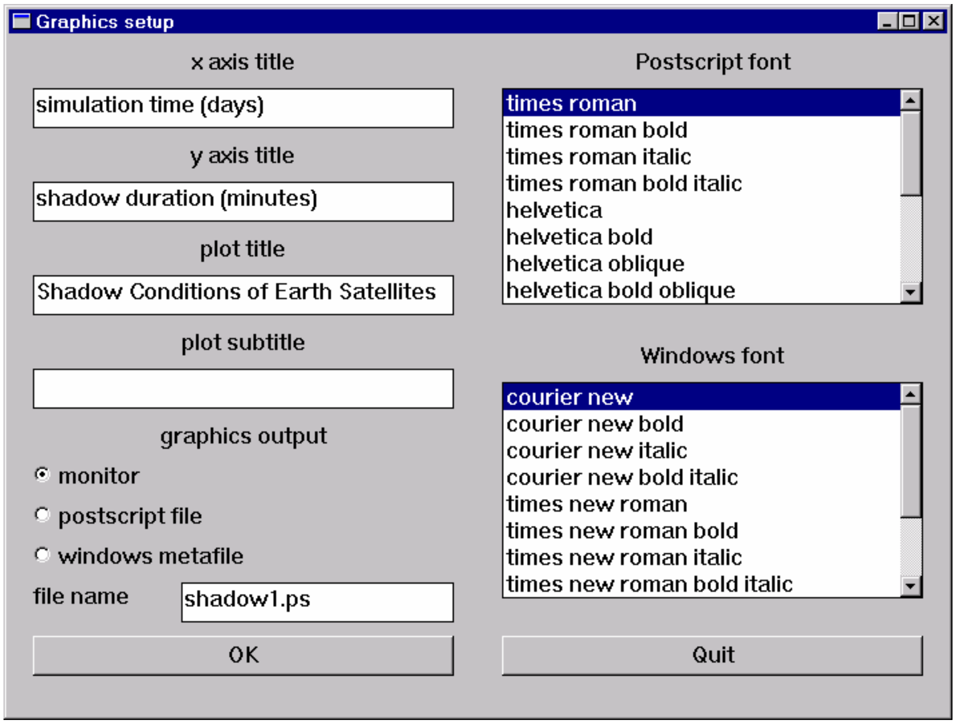
The second screen also displays the shadow duration.



After the simulation is complete the software will ask if you would like to display a graph of the shadow conditions. This interactive screen is as follows:



If you select Yes the software will display the following *graphics setup* screen:



This screen allows the user to define such things as the graphics output destination, the font to use and the titles for the graph. Please note that the Windows font selection defines the font for both screen (monitor) and Windows metafile graphics. The following is a typical graph of shadow duration as a function of simulation time. The simulation time corresponds to the condition of *minimum* angular separation between the anti-Sun vector and the satellite.

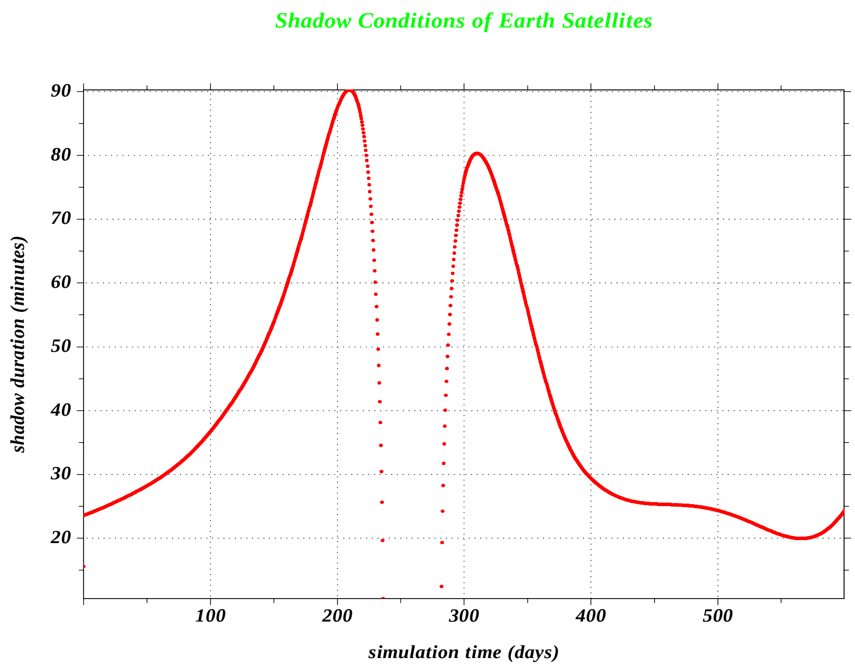


Figure 1. Graph of Shadow Duration versus Simulation Time

Technical Discussion

The *objective function* we wish to minimize is defined by

$$f_m = \cos^{-1}\left(-\mathbf{U}_{sun} \bullet \mathbf{U}_{sat}\right) \tag{1}$$

where $\mathbf{U}_{sun}$ and $\mathbf{U}_{sat}$ are the ECI unit pointing vectors of the Sun and satellite, respectively. This scalar value is equal to the angle between the satellite and the anti-Sun vector.

During the root-finding calculations the objective function is given by

$$f_r = \cos^{-1}\left(-\mathbf{U}_{sun} \bullet \mathbf{U}_{sat}\right) - \theta \tag{2}$$

where θ is either the penumbra, umbra or cylinder shadow angle. Before solving for the roots which define shadow entrance and exit, the solution is first bracketed using a *geometric acceleration* technique.

The following diagram illustrates the relationship between the Sun, Earth, satellite and shadow.

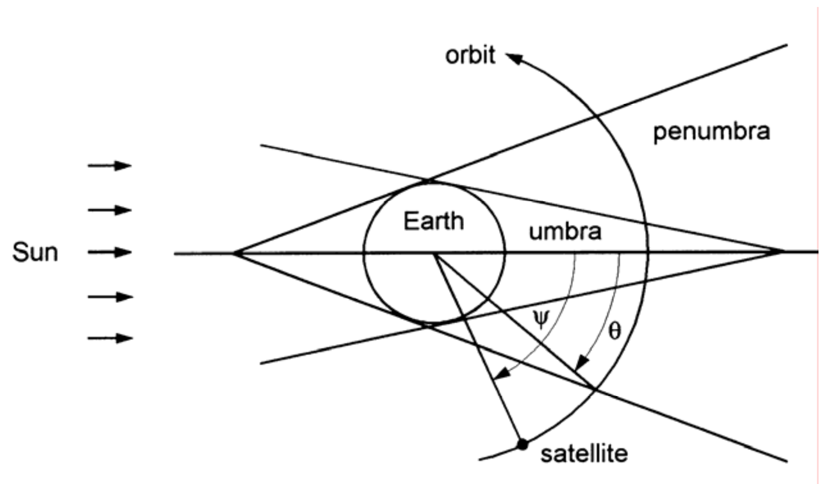


Figure 2. Shadow Geometry

The shadow angles are the angles between the anti-Sun ECI unit pointing vector and the shadow boundary at the satellite's geocentric distance. The *cylindrical* shadow angle at the point of shadow entrance or exit is given by:

$$\theta_c = \sin^{-1}\left(\frac{r_{eq}}{r_{sat}}\right) \tag{3}$$

The angle of the *umbra* portion of the shadow at the satellite's location is determined from

$$\theta_u = \sin^{-1}\left(\frac{d_{sun} - r_{eq}}{r_{sun}}\right) - \theta_c \tag{4}$$

The *penumbra* shadow angle can be calculated from the following expression:

$$\theta_p = \sin^{-1}\left(\frac{d_{sun} + r_{eq}}{r_{sun}}\right) - \theta_c \tag{5}$$

In these equations, r_{eq} is the equatorial radius of the Earth, d_{sun} is the radius of the Sun, and r_{sat} and r_{sun} are the geocentric distances of the satellite and Sun, respectively.

The *phase angle* at shadow entrance and exit is the angle between the ECI vectors to the Sun and satellite. The phase angle at the entrance and exit points can be calculated with the equation:

$$\psi = \cos^{-1}\left(\mathbf{U}_{sun} \bullet \mathbf{U}_{sat}\right) \tag{6}$$

where $\mathbf{U}_{sun}$ and $\mathbf{U}_{sat}$ are the ECI unit position vectors of the Sun and satellite, respectively. These unit vectors are evaluated at the points of shadow entrance and exit. The phase angle is an indication of the brightness or illumination of the satellite relative to an Earth observer. The actual brightness of a satellite is a function of its shape, reflective properties, and orientation or attitude in space. For a *spherical* satellite the illuminated fraction can be calculated from

$$I = \frac{1 + \cos\psi}{2} \tag{7}$$

The `shadow2` software uses a J_2 form of *Kozai's method* ("The Motion of a Close Earth Satellite", *The Astronomical Journal*, **64**, No. 1274, pp. 367-377) to propagate the orbit of an Earth satellite during the search for shadow. Therefore, all orbital elements input to the program are assumed to be Kozai "mean" elements. These orbital elements include the J_2 secular effects on mean anomaly, argument of perigee and right ascension of ascending node. All calculations in this program are performed in an Earth-centered inertial (ECI) coordinate system.

According to Kozai's method, the time evolution of the mean orbital elements is given by the following three equations:

$$\begin{aligned} M(t) &= M_0 + \tilde{n}(t - t_0) \\ \Omega(t) &= \Omega_0 + \dot{\Omega}(t - t_0) \\ \omega(t) &= \omega_0 + \dot{\omega}(t - t_0) \end{aligned} \tag{8}$$

where M_0 is the mean anomaly, Ω_0 is the right ascension of the ascending node (RAAN) and ω_0 is the argument of perigee, all at the initial time t_0. In the first expression $\tilde{n}$ is called the *perturbed mean motion* and is equal to the time rate of change of mean anomaly.

The perturbed mean motion can be calculated from:

$$\tilde{n} = \frac{dM}{dt} = n\left\{1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{p}\right)^2 \sqrt{1-e^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right\} \tag{9}$$

The time rate of change of the right ascension of the ascending node is determined from

$$\dot{\Omega} = \frac{d\Omega}{dt} = -\frac{3}{2} J_2 \tilde{n} \left(\frac{r_{eq}}{p}\right)^2 \cos i \tag{10}$$

The secular perturbation of the argument of perigee is given by:

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{3}{2} J_2 \tilde{n} \left(\frac{r_{eq}}{p}\right)^2 \left(2 - \frac{5}{2}\sin^2 i\right) \tag{11}$$

where

n = unperturbed or Keplerian mean motion

J_2 = Earth oblateness gravity term

r_{eq} = equatorial radius of the Earth

e = orbital eccentricity

i = orbital inclination

$p = a\left(1-e^2\right)$ = semiparameter

a = semimajor axis

The Earth radius used in the shadow calculations is increased by 2% to account for the atmosphere.