A Computer Program for Earth-to-Mars Mission Design

This document is the user's manual for a Windows compatible Fortran computer program named `e2m_ftn` that can be used to design and optimize ballistic interplanetary missions from Earth park orbit to B-plane encounter at Mars. The software assumes that interplanetary injection occurs *impulsively* from a circular Earth park orbit. The B-plane coordinates are expressed in a Mars-centered (areocentric) mean equator and IAU node of epoch coordinate system. These B-plane targets are enforced using either a combination of flight path angle, periapsis radius and orbital inclination at a Mars-centered entry interface (EI) or individual B-plane coordinates ($\mathbf{B \bullet T}$ and $\mathbf{B \bullet R}$) of the arrival hyperbola. The type of targeting and the target values are defined by the user.

The first part of this computer program solves for the minimum delta-v using a patched-conic, two-body Lambert solution for the transfer trajectory from Earth to Mars. The second part implements a simple *shooting* method that attempts to optimize the characteristics of the geocentric injection hyperbola while numerically integrating the spacecraft's geocentric and heliocentric equations of motion and targeting to components of the B-plane relative to Mars. The spacecraft motion within the Earth's sphere-of-influence (SOI) includes non-spherical Earth gravity, the point-mass perturbation of the sun and moon, and optionally the point-mass perturbations of planets. The heliocentric equations of motion include the point-mass gravity of the sun, moon and the all planets of the solar system. The software also includes the option to include the effect of solar radiation pressure in both the geocentric and heliocentric phases of the spacecraft's motion.

The user can select one of the following delta-v optimization options for the two-body solution of the interplanetary transfer trajectory:

- minimize departure delta-v
- minimize arrival delta-v
- minimize total delta-v

The major computational steps implemented in this software are as follows:

- solve the two-body, patched-conic interplanetary Lambert problem for the energy C_3, declination (DLA) and asymptote (RLA) of the outgoing or departure hyperbola
- compute the orbital elements of the geocentric departure hyperbola and the components of the interplanetary injection delta-v vector
- perform geocentric orbit propagation from perigee of the geocentric departure hyperbola to the user-defined value of the Earth's sphere-of-influence (SOI)
- perform an n-body heliocentric orbit propagation from the Earth's SOI to either closest approach or entry interface (EI) at Mars
- target to the user-defined B-plane coordinates while minimizing the magnitude of the hyperbolic v-infinity at Earth departure (equivalent to minimizing the departure energy since $C_3 = V_\infty^2$)

This computer program uses a nonlinear programming (NLP) algorithm to solve both the patched-conic and numerically integrated trajectory optimization problems. The planetary coordinates required by the software are computed using the JPL DE421 ephemeris. This computer program was written using Intel Visual Fortran, version 11.1.

Program execution

An input file created by the user can be run from the command line or a simple batch file with a statement similar to the following:

```
e2m_ftn mars03.in
```

If the software is executed without an input file on the command line, the computer program will display the following prompt:

```
***********************************
*         program e2m_ftn         *
*                                 *
*    Earth-to-Mars trajectory     *
*     design & optimization       *
*                                 *
*        February 5, 2012         *
***********************************

please input the name of the simulation definition file
```

At this point the user should input the name of a valid input file, including the filename extension.

To create a DOS command window in Windows 7, select **start**, then **All Programs**, then **Accessories** and finally **Command Prompt**. The size, font and other characteristics of the screen can be controlled by the user with the **c:** icon in the upper left corner of the window. To log into the subdirectory created during the installation of the Fortran executable and support files, type **root:** and then **cd subdirectory** from the DOS command line where root is the name of the root directory, usually c:, and subdirectory is the name of the subdirectory created by the user.

The DOS command line prompt looks similar to **C:\e2m_ftn>_.**

Input file format and contents

The `e2m_ftn` software is “data-driven” by a user-created text file. The following is a typical input file used by this computer program. Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or change the number of lines reserved for each comment and data item. However, you may change them to reflect your own explanation or information. The annotation line also includes the correct units and when appropriate, the valid range of the input data items. ASCII text input is <u>not</u> case sensitive but must be spelled correctly. In the following discussion, the actual input file contents are in **`bold courier`** font and all explanations are in *times italic* font.

The first six lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with six and only six initial text lines.

```
*******************************
** Earth-to-Mars interplanetary
** trajectory optimization
** Mars '03 example - mars03.in
** February 5, 2012
*******************************
```

The first program input is the name of a "constants and models" data file. This ASCII data file contains user-defined astrodynamic constants and other information.

```
name of constants and models data file
--------------------------------------
e2m_cm.dat
```

The following is a typicaluser-defined constants and models data file named `e2m_cm.dat`. *Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment or data item. Also, please note the proper units for each data item.*

```
****************************************
* e2m_ftn constants and models data file
****************************************

astronomical unit (kilometers)
------------------------------
149597870.691d0

speed of light (meters/second)
------------------------------
299792458.0d0

solar flux at 1 AU (watts/meters**2)
------------------------------------
1366.1d0

Earth gravitational constant (km**3/sec**2)
-------------------------------------------
398600.4415d0

Earth equatorial radius (kilometers)
------------------------------------
6378.14d0

Earth sphere-of-influence value (kilometers)
--------------------------------------------
925000.0d0

Moon gravitational constant (km**3/sec**2)
------------------------------------------
4902.800238d0

Mars gravitational constant (km**3/sec**2)
------------------------------------------
42828.376212d0

Mars equatorial radius (kilometers)
-----------------------------------
3396.2d0
```

The second program input is the difference between ephemeris time (Terrestrial Time) and Universal Coordinated Time (UTC) in seconds.

```
ET-UTC (seconds)
64.132d0
```

The next data file input is an integer that defines the type of patched-conic trajectory optimization.

```
*******************
* simulation type *
*******************
 1 = minimize departure delta-v
```

```
 2 = minimize arrival delta-v
 3 = minimize total delta-v
--------------------------
1
```

The software allows the user to specify an initial guess for the departure and arrival calendar dates and a search interval. For any guess for departure time t_L and user-defined search interval Δt, the departure time t is constrained as follows:

$$t_L - \Delta t \le t \le t_L + \Delta t$$

Likewise, for any guess for arrival time t_A and user-defined search interval, the arrival time t is constrained as follows:

$$t_A - \Delta t \le t \le t_A + \Delta t$$

For fixed departure and/or arrival times, the search interval should be set to 0.

The next input defines an initial guess for the departure calendar date. Please be sure to include all digits of the calendar year.

```
departure calendar date initial guess (month, day, year)
6,1,2003
```

These two numbers define the lower and upper search interval for the departure calendar date.

```
departure date search boundary (days)
-30, +30
```

The next input defines an initial guess for the arrival calendar date.

```
arrival calendar date initial guess (month, day, year)
12,1,2003
```

These two numbers define the lower and upper search interval for the arrival calendar date.

```
arrival date search boundary (days)
-30, +30
```

The next set of inputs defines the perigee altitude of the departure hyperbola, the launch azimuth measured positive clockwise from north, and the geocentric latitude of the launch site. The azimuth and latitude are used to calculate the inclination of the park orbit and departure hyperbola.

```
***************************
* geocentric phase modeling
***************************

perigee altitude of departure hyperbola (kilometers)
185.32d0

launch azimuth (degrees)
93.0d0

launch site latitude (degrees)
28.5d0
```

The name of the ASCII data file containing the Earth gravity model data is specified in the next line. Please see the Technical Discussion section later in this document for a description and format of the data in this file.

```
name of Earth gravity model data file
egm96.dat
```

The order (zonals) of the Earth gravity model is an integer defined in the next line.

```
order of the gravity model (zonals)
8
```

The degree (tesserals) of the Earth gravity model is an integer defined in this next line.

```
degree of the gravity model (tesserals)
8
```

The next integer input "toggles" the option to include planetary point-mass perturbations during the geocentric phase of the targeting process.

```
include planetary point-mass perturbations (1 = yes, 0 = no)
1
```

The next input specifies the type of targeting at Mars performed by the `e2m_ftn` *computer program. Option 1 will target to user-defined components of the B-plane at closest approach to Mars, and option 2 will target to a Mars-centered hyperbola with a user-specified flight path angle, areocentric radius and orbital inclination.*

```
**************************
* Mars encounter targeting
**************************

type of targeting
(1 = B-plane, 2 = orbital elements)
2
```

The next two inputs are the user-defined B-plane components used with targeting option 1.

```
B dot T
4607.4

B dot R
-7888.0
```

The next three inputs define the flight path angle, radius and the orbital inclination of the encounter hyperbola at Mars. These flight conditions are used by targeting option 2. The radius is with respect to a spherical Mars model and the orbital inclination is with respect to the mean equator of Mars.

```
areocentric flight path angle (degrees)
0.0

areocentric radius (kilometers)
5000.0

areocentric orbital inclination (degrees)
60.0
```

The next series of inputs define the spacecraft characteristics used for solar radiation pressure perturbation calculations. These three items include the spacecraft's mass, reference cross-sectional area, and reflectivity coefficient. To exclude this perturbation, input a spacecraft mass of zero.

```
**********************************
spacecraft mass and SRP properties
**********************************
```

```
spacecraft mass (kilograms; input 0 to ignore SRP calculations)
3850.0d0

SRP reference area (square meters)
19.4

reflectivity coefficient (non-dimensional)
1.0d0
```

The last three inputs are algorithm control parameters that define the lower and upper bounds on the v-infinity, right ascension and declination of the geocentric departure hyperbola.

```
****************************
algorithm control parameters
****************************

v-infinity lower and upper bounds increment (meters/second)
50.0

asymptote right ascension lower and upper bounds increment (degrees)
10.0

asymptote declination lower and upper bounds increment (degrees)
1.0
```

Program example

The following is the solution created with this computer program for this example. The output is organized by the following major sections:

- Two-body/Patched-conic Pass
    1. two body Lambert solution
    2. departure hyperbola orbital elements and state vector
    3. heliocentric coordinates of Earth at departure and Mars at arrival
    4. heliocentric coordinates of the spacecraft on the transfer trajectory

- Targeting/Optimization Pass
    1. optimized characteristics of the departure hyperbola
    2. heliocentric coordinates of the spacecraft and Mars at encounter
    3. geocentric and heliocentric coordinates of the spacecraft at the Earth SOI

The first output section summarizes the two-body Lambert solution. The solution is provided in the heliocentric, Earth mean equator and equinox of J2000 (EME2000) coordinate system.

```
Earth-to-Mars mission design

========================
two-body Lambert solution
========================

minimize launch delta-v


departure heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
----------------------------------------
```

```
x-component of delta-v     2895.91191273315       meters/second
y-component of delta-v    -530.401772123313       meters/second
z-component of delta-v    -345.700686652980       meters/second

delta-v magnitude          2964.31118658849       meters/second
```

arrival heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)

```
x-component of delta-v    -2063.01128433645       meters/second
y-component of delta-v     1164.27006011528       meters/second
z-component of delta-v     1311.96071903865       meters/second

delta-v magnitude          2707.91086642097       meters/second
```

heliocentric coordinates of the Earth at departure
(Earth mean equator and equinox of J2000)

```
calendar date           June  5, 2003

UTC time                14:46:19.786

UTC Julian date         2452796.11550678

TDB time                14:47:23.918

TDB Julian date         2452796.11624905
```

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.100035823926D+01	0.162374015336D-01	0.234390546133D+02	0.102452224545D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.723788828213D-03	0.152048024408D+03	0.254500248953D+03	0.365453189944D+03
rx (km)	ry (km)	rz (km)	rmag (km)
-.405615530578D+08	-.134199767646D+09	-.581818397726D+08	0.151789142240D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.282279812575D+02	-.739767150582D+01	-.320740144564D+01	0.293569735091D+02

spacecraft heliocentric coordinates after the first impulse
(Earth mean equator and equinox of J2000)

```
calendar date           June  5, 2003

UTC time                14:46:19.786

UTC Julian date         2452796.11550678

TDB time                14:47:23.918

TDB Julian date         2452796.11624905
```

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.125929041710D+01	0.194277242259D+00	0.234900279793D+02	0.253491140553D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.455891057237D+00	0.591580038640D+00	0.254082720591D+03	0.516163481019D+03
rx (km)	ry (km)	rz (km)	rmag (km)
-.405615530578D+08	-.134199767646D+09	-.581818397726D+08	0.151789142240D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.311238931702D+02	-.792807327794D+01	-.355310213229D+01	0.323137061746D+02

spacecraft heliocentric coordinates prior to the second impulse
(Earth mean equator and equinox of J2000)
----

calendar date	December 24, 2003
UTC time	15:23:16.463
UTC Julian date	2452998.14116276
TDB time	15:24:20.595
TDB Julian date	2452998.14190503

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.125929041710D+01	0.194277242259D+00	0.234900279793D+02	0.253491140553D+03
raan (deg)	**true anomaly (deg)**	**arglat (deg)**	**period (days)**
0.455891057237D+00	0.152910259883D+03	0.464014004360D+02	0.516163481019D+03
rx (km)	**ry (km)**	**rz (km)**	**rmag (km)**
0.149989634184D+09	0.146777512084D+09	0.632696170860D+08	0.219188441443D+09
vx (kps)	**vy (kps)**	**vz (kps)**	**vmag (kps)**
-.146794823540D+02	0.156262551792D+02	0.684180266331D+01	0.225050509173D+02

spacecraft heliocentric coordinates after the second impulse
(Earth mean equator and equinox of J2000)
----

calendar date	December 24, 2003
UTC time	15:23:16.463
UTC Julian date	2452998.14116276
TDB time	15:24:20.595
TDB Julian date	2452998.14190503

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.152368015629D+01	0.935418896793D-01	0.246772249523D+02	0.332979237037D+03
raan (deg)	**true anomaly (deg)**	**arglat (deg)**	**period (days)**
0.337165832669D+01	0.707599727739D+02	0.437392098113D+02	0.686972170792D+03
rx (km)	**ry (km)**	**rz (km)**	**rmag (km)**
0.149989634184D+09	0.146777512084D+09	0.632696170860D+08	0.219188441443D+09
vx (kps)	**vy (kps)**	**vz (kps)**	**vmag (kps)**
-.167424936383D+02	0.167905252393D+02	0.815376338235D+01	0.250742235839D+02

heliocentric coordinates of Mars at arrival
(Earth mean equator and equinox of J2000)
----

calendar date	June 5, 2003
UTC time	14:46:19.786
UTC Julian date	2452796.11550678
TDB time	14:47:23.918
TDB Julian date	2452796.11624905

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.152368015629D+01	0.935418896802D-01	0.246772249523D+02	0.332979237037D+03

```
    raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 0.337165832666D+01    0.707599727737D+02    0.437392098108D+02    0.686972170792D+03

     rx (km)                ry (km)               rz (km)               rmag (km)
 0.149989634185D+09    0.146777512083D+09    0.632696170854D+08    0.219188441443D+09

     vx (kps)               vy (kps)              vz (kps)              vmag (kps)
 -.167424936382D+02    0.167905252395D+02    0.815376338242D+01    0.250742235840D+02
```

The following output summarizes the orbital characteristics of the initial circular park orbit and the departure hyperbola.

```
--------------------------------------------------
park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
--------------------------------------------------

park orbit
----------

calendar date           June  5, 2003

UTC time                14:46:19.786

UTC Julian date         2452796.11550678

TDB time                14:47:23.918

TDB Julian date         2452796.11624905

     sma (km)              eccentricity          inclination (deg)       argper (deg)
 0.656346000000D+04    0.257512306346D-15    0.286442848562D+02    0.000000000000D+00

    raan (deg)         true anomaly (deg)       arglat (deg)          period (hrs)
 0.203488961024D+01    0.195040355591D+03    0.195040355591D+03    0.146996753813D+01

     rx (km)                ry (km)               rz (km)               rmag (km)
 -.628154417661D+04    -.171889113623D+04    -.816469957040D+03    0.656346000000D+04

     vx (kps)               vy (kps)              vz (kps)              vmag (kps)
 0.225552026013D+01    -.652894070327D+01    -.360774064746D+01    0.779296034444D+01

departure hyperbola
-------------------

c3                   8.78714081093365       km**2/sec**2

v-infinity           2964.31118658849       meters/second

decl-asymptote      -6.69712585591636       degrees

rasc-asymptote       349.621008346580       degrees

launch azimuth       93.0000000000000       degrees

launch latitude      28.5000000000000       degrees

calendar date           June  5, 2003

UTC time                14:46:19.786

UTC Julian date         2452796.11550678

TDB time                14:47:23.918

TDB Julian date         2452796.11624905

     sma (km)              eccentricity          inclination (deg)       argper (deg)
 -.453617906070D+05    0.114469137819D+01    0.286442848562D+02    0.195040355591D+03
```

```
    raan (deg)         true anomaly (deg)       arglat (deg)
 0.203488961024D+01    0.000000000000D+00    0.195040355591D+03

     rx (km)                ry (km)                rz (km)              rmag (km)
 -.628154417661D+04    -.171889113623D+04    -.816469957040D+03    0.656346000000D+04

     vx (kps)               vy (kps)               vz (kps)             vmag (kps)
 0.330315638477D+01    -.956148013875D+01    -.528345134596D+01    0.114126071811D+02


hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

delta-vx             1047.63612463542       meters/seconds
delta-vx            -3032.53943547266       meters/seconds
delta-vx            -1675.71069850439       meters/seconds

delta-v magnitude    3619.64683669830       meters/seconds
```

The following program output summarizes the flight conditions determined by the n-body, numerically integrated optimized solution.

```
=======================
optimal n-body solution
=======================

orbital element targeting

--------------------------------------------------
park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
--------------------------------------------------

park orbit
----------

calendar date           June  5, 2003

UTC time                14:46:19.786

UTC Julian date         2452796.11550678

TDB time                14:47:23.918

TDB Julian date         2452796.11624905


     sma (km)            eccentricity       inclination (deg)       argper (deg)
 0.656346000000D+04    0.000000000000D+00    0.286442848562D+02    0.000000000000D+00

    raan (deg)         true anomaly (deg)       arglat (deg)          period (hrs)
 0.268721925840D+01    0.194746249218D+03    0.194746249218D+03    0.146996753813D+01

     rx (km)                ry (km)                rz (km)              rmag (km)
 -.627155679819D+04    -.176215847581D+04    -.800862038585D+03    0.656346000000D+04

     vx (kps)               vy (kps)               vz (kps)             vmag (kps)
 0.229151359953D+01    -.651366390188D+01    -.361266922463D+01    0.779296034444D+01

departure hyperbola
-------------------

c3                   8.79595859874293       km**2/sec**2

v-infinity           2965.79813856960       meters/second

decl-asymptote      -6.84084520107197       degrees

rasc-asymptote       349.999983002785       degrees
```

calendar date	June 5, 2003
UTC time	14:46:19.786
UTC Julian date	2452796.11550678
TDB time	14:47:23.918
TDB Julian date	2452796.11624905

sma (km)	eccentricity	inclination (deg)	argper (deg)
-.453163162406D+05	0.114483657421D+01	0.286442848562D+02	0.194746249218D+03

raan (deg)	true anomaly (deg)	arglat (deg)
0.268721925840D+01	0.000000000000D+00	0.194746249218D+03

rx (km)	ry (km)	rz (km)	rmag (km)
-.627155679819D+04	-.176215847581D+04	-.800862038585D+03	0.656346000000D+04

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.335598137849D+01	-.953943051657D+01	-.529084821797D+01	0.114129934925D+02

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)

---

delta-vx	1064.46777896427	meters/seconds
delta-vx	-3025.76661469037	meters/seconds
delta-vx	-1678.17899333961	meters/seconds
delta-v magnitude	3620.03314801319	meters/seconds
transfer time	201.320570961572	days

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)

---

calendar date	December 23, 2003
UTC time	22:27:57.117
UTC Julian date	2452997.43607774
TDB time	22:29:01.249
TDB Julian date	2452997.43682001

sma (km)	eccentricity	inclination (deg)	argper (deg)
-.584470525352D+04	0.185547513759D+01	0.600000000762D+02	0.113890120053D+03

raan (deg)	true anomaly (deg)	arglat (deg)
0.105737418369D+03	0.359999996513D+03	0.113890116566D+03

rx (km)	ry (km)	rz (km)	rmag (km)
-.165090853856D+04	-.256899297777D+04	0.395913833891D+04	0.500000003092D+04

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.219037534613D+01	-.408076499331D+01	-.173455626962D+01	0.494561144249D+01

B-plane coordinates at closest approach
(Mars mean equator and IAU node of epoch)

---

b-magnitude	9134.93586215633	kilometers
b dot r	-7887.91557903568	
b dot t	4607.36812331249	
theta	300.289399006638	degrees

```
v-infinity           2.70697657197526       km/sec
r-periapsis          5000.00003091632       kilometers
decl-asymptote       7.54592774962857       degrees
rasc-asymptote       281.351126770895       degrees

flight path angle  -2.265832235816536E-006  degrees


heliocentric coordinates of Mars at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date           December 23, 2003

UTC time                22:27:57.117

UTC Julian date         2452997.43607774

TDB time                22:29:01.249

TDB Julian date         2452997.43682001

     sma (au)            eccentricity       inclination (deg)       argper (deg)
 0.152368051445D+01    0.935421367896D-01    0.246772248793D+02    0.332979309970D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 0.337165816049D+01    0.703618314808D+02    0.433411414506D+02    0.686972413014D+03

     rx (km)               ry (km)               rz (km)              rmag (km)
 0.151006058357D+09    0.145751217673D+09    0.627714185266D+08    0.219058207099D+09

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 -.166268454477D+02    0.169029231683D+02    0.820219199604D+01    0.250885781725D+02


spacecraft heliocentric coordinates at closest approach
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date           December 23, 2003

UTC time                22:27:57.117

UTC Julian date         2452997.43607774

TDB time                22:29:01.249

TDB Julian date         2452997.43682001

     sma (au)            eccentricity       inclination (deg)       argper (deg)
 0.123269083106D+01    0.237029718878D+00    0.190411037964D+02    0.271348681572D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 0.343913878579D+03    0.150095738745D+03    0.614444203174D+02    0.499896015715D+03

     rx (km)               ry (km)               rz (km)              rmag (km)
 0.151008227801D+09    0.145747009174D+09    0.627730254016D+08    0.219057362955D+09

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 -.135199910005D+02    0.170361588795D+02    0.435657420053D+01    0.221810866458D+02
```

The final program output summarizes both the geocentric and heliocentric spacecraft trajectory characteristics at the Earth's sphere-of-influence.

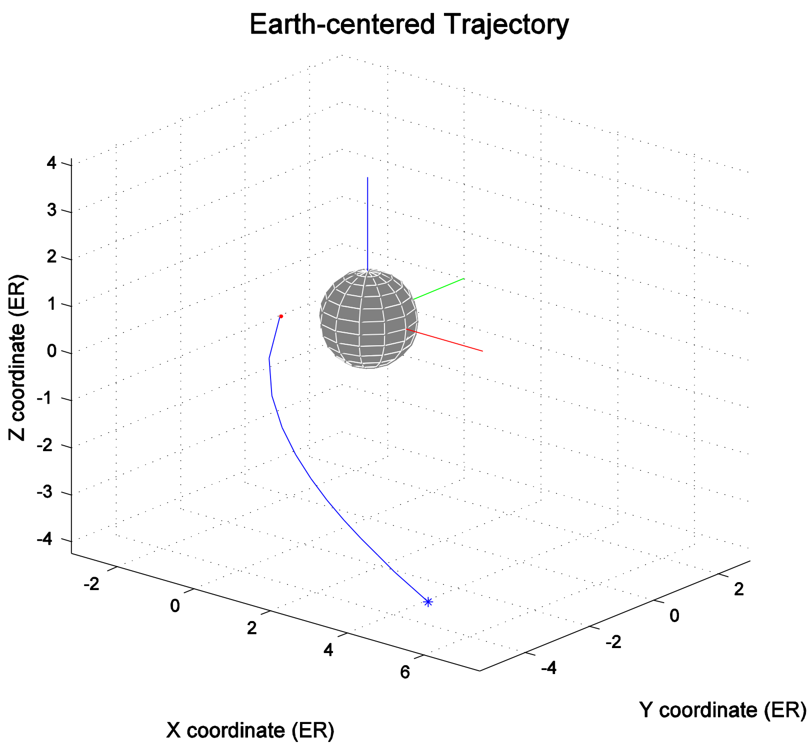
```
spacecraft geocentric coordinates at Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date           June  8, 2003
```

```
UTC time              18:19:27.718

UTC Julian date       2452799.26351525

TDB time              18:20:31.850

TDB Julian date       2452799.26425752

      sma (km)            eccentricity       inclination (deg)      argper (deg)
 -.455611424476D+05    0.114339388444D+01    0.285072368744D+02    0.194704225766D+03

    raan (deg)         true anomaly (deg)       arglat (deg)
 0.269901314894D+01    0.149468410190D+03    0.344172635957D+03

      rx (km)               ry (km)               rz (km)              rmag (km)
 0.899383566141D+06    -.179544414681D+06    -.120407658033D+06    0.925000000000D+06

      vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 0.303199931787D+01    -.532282408525D+00    -.366315743230D+00    0.310008574877D+01

spacecraft heliocentric coordinates at Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date         June  8, 2003

UTC time              18:19:27.718

UTC Julian date       2452799.26351525

TDB time              18:20:31.850

TDB Julian date       2452799.26425752

      sma (au)            eccentricity       inclination (deg)      argper (deg)
 0.127492335864D+01    0.204055659203D+00    0.235001563938D+02    0.253591594436D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 0.527611139104D+00    0.378631163570D+01    0.257377906072D+03    0.525804797231D+03

      rx (km)               ry (km)               rz (km)              rmag (km)
 -.319322541783D+08    -.136201696258D+09    -.590922584987D+08    0.151863313328D+09

      vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 0.316302582596D+02    -.653313867081D+01    -.296723676588D+01    0.324339271826D+02
```

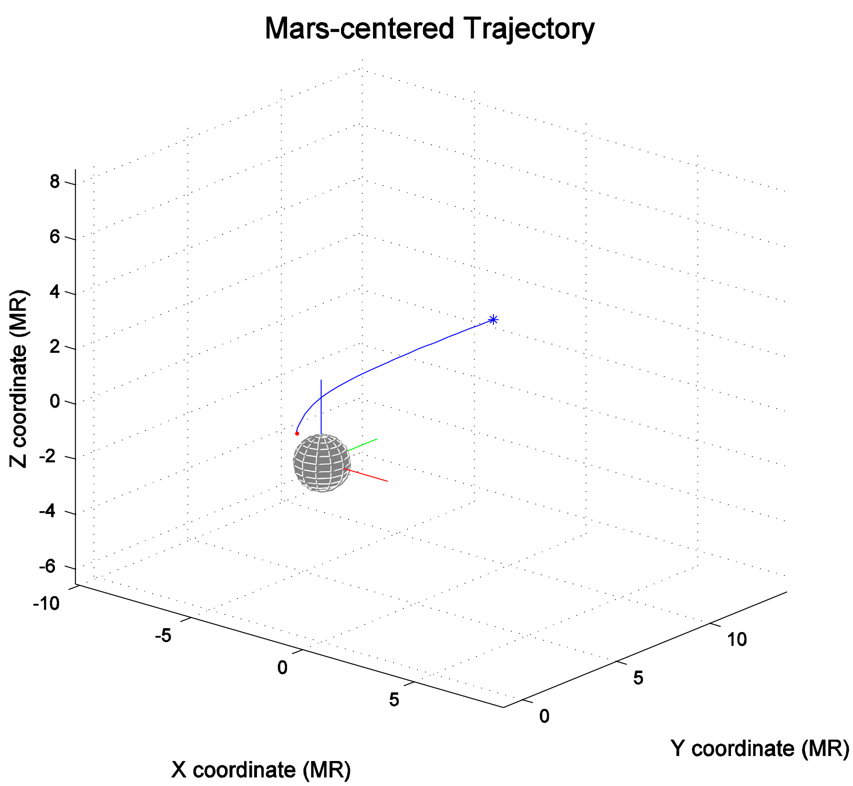
The `e2m_ftn` computer program will also create three comma-separated-variable (csv) data files. These files summarize the geocentric (`e2m_geo.csv`), heliocentric (`e2m_helio.csv`) and areocentric (`e2m_areo.csv`) trajectory characteristics of the spacecraft. Appendix B contains several plots created using these data files for this mission example.

This software suite also includes two MATLAB scripts that can be used to plot the geocentric (`marsplot2.m`) and areocentric (`marsplot1.m`) trajectories of the spacecraft. The following is a graphics display of the geocentric trajectory for this example for motion within 30,000 kilometers of the Earth. Injection occurs at perigee of the departure hyperbola which is denoted by the small red dot. The EME2000 x-axis (red), y-axis (green) and z-axis (blue) are also shown on the Earth. The coordinates are displayed in Earth radii.

Important Note: You must delete the first or "header" line of the solution file in order for the theses script to work. This script uses the MATLAB `csvread` function to read the data file which can only contain comma-separated-variable numerical data.



The following is a screen display of the areocentric or Mars-centered trajectory within 50,000 kilometers of Mars. Closest approach occurs at perigee of the approach hyperbola which is denoted by the small red dot. The x-axis (red), y-axis (green) and z-axis (blue) are also shown relative to the mean equator of Mars and the IAU node of epoch. The coordinates are displayed in Mars radii.



Technical discussion

Solving the two body Lambert problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamic problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis a of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length c of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof\left(r_i + r_f, c, a\right)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}}\left(E - e\sin E\right)$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}}\left[E - E_0 - e\left(\sin E - \sin E_0\right)\right]$$

where E is the eccentric anomaly associated with radius r, E_0 is the eccentric anomaly at r_0, and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sun and difference identities:

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha - \beta}{2}\cos\frac{a + \beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha - \beta}{2}\sin\frac{a + \beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha - \beta}{2}\cos\frac{a + \beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t = \sqrt{\frac{a^3}{\mu}}\left\{E - E_0 - 2\sin\frac{E - E_0}{2}\left(e\cos\frac{E + E_0}{2}\right)\right\}$$

With the two substitutions given by

$$e\cos\frac{E+E_0}{2}=\cos\frac{\alpha+\beta}{2} \qquad \sin\frac{E-E_0}{2}=\sin\frac{\alpha-\beta}{2}$$

the time equation becomes

$$t=\sqrt{\frac{a^3}{\mu}}\left\{(\alpha-\beta)-2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}\right\}$$

From the elliptic relationships given by

$$r=a(1-e\cos E)$$

$$x=a(\cos E-e)$$

$$y=a\sin E\sqrt{1-e^2}$$

and some more manipulation, we have the following equations:

$$\cos\alpha=\left(1-\frac{r+r_0}{2a}\right)-\frac{c}{2a}=1-\frac{r+r_0+c}{2a}=1-\frac{s}{a}$$

$$\sin\beta=\left(1-\frac{r+r_0}{2a}\right)+\frac{c}{2a}=1-\frac{r+r_0-c}{2a}=1-\frac{s-c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}=1-\frac{r+r_0}{2}$$

$$\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}=\sin\frac{E-E_0}{2}\sqrt{1-\left(e\cos\frac{E+E_0}{2}\right)^2}$$

$$\left(\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\right)^2=\left(\frac{x-x_0}{2a}\right)^2+\left(\frac{y-y_0}{2a}\right)^2=\left(\frac{c}{2a}\right)^2$$

With the use of the half angle formulas given by

$$\sin\frac{\alpha}{2}=\sqrt{\frac{s}{2a}} \qquad \sin\frac{\beta}{2}=\sqrt{\frac{s-c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert’s theorem

$$t=\sqrt{\frac{a^3}{\mu}}\left[(\alpha-\beta)-(\sin\alpha-\sin\beta)\right]$$

A discussion about the angles α and β can be found in "Geometrical Interpretation of the Angles α and β in Lambert's Problem" by J. E. Prussing, AIAA *Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this computer program is based on the method described in "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem" by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

Designing the departure hyperbola

This section describes the algorithm used to determine the Earth-centered-inertial (ECI) state vector of a departure hyperbola for interplanetary missions. In the discussion that follows, interplanetary injection is assumed to occur *impulsively* at perigee of the departure hyperbola.

The departure trajectory for interplanetary missions is specified by the orbital energy C_3, and the right ascension α_∞ and declination δ_∞ of the outgoing asymptote. The perigee radius of the departure hyperbola is specified and the orbital inclination is computed from the user-defined launch azimuth Σ_L and launch site geocentric latitude ϕ_L from the equation $i = \cos^{-1}\left(\cos\phi_L \sin\Sigma_L\right)$.

The algorithm used to design the departure hyperbola only works for geocentric orbit inclinations that satisfy the following constraint

$$|i| > |\delta_\infty|$$

If this inequality is not satisfied, the software will print the following error message

```
park orbit error!!

|inclination| must be > |asymptote declination|
```

The code will also print the inclination of the park orbit, the declination of the departure hyperbola and stop. The user can then change either the azimuth or launch site latitude to satisfy this constraint and restart the program.

A unit vector in the direction of the departure asymptote is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where

$\alpha_\infty =$ right ascension of departure asymptote

$\delta_\infty =$ declination of departure asymptote

The T-axis direction of the B-plane coordinate system is determined from the following vector cross product:

$$\hat{\mathbf{T}} = \hat{\mathbf{S}} \times \hat{\mathbf{u}}_z$$

where $\hat{\mathbf{u}}_z = \left[0\,0\,1\right]^T$ is a unit vector perpendicular to the Earth's equator.

The following cross product operation completes the B-plane coordinate system.

$$\hat{\mathbf{R}} = \hat{\mathbf{S}} \times \hat{\mathbf{T}}$$

The B-plane angle is determined from the orbital inclination of the departure hyperbola i and the declination of the outgoing asymptote according to

$$\cos\theta = \frac{\cos i}{\cos\delta_\infty}$$

The unit angular momentum vector of the departure hyperbola is given by

$$\hat{\mathbf{h}} = \hat{\mathbf{T}}\sin\theta - \hat{\mathbf{R}}\cos\theta$$

The sine and cosine of the true anomaly at infinity are given by the next two equations

$$\cos\theta_\infty = -\frac{\mu}{r_p V_\infty^2 + \mu} \qquad \sin\theta_\infty = \sqrt{1 - \cos^2\theta_\infty}$$

where $V_\infty = \sqrt{C_3} = V_L - V_p$ is the spacecraft's velocity at infinity, V_L is the heliocentric departure velocity determined from the Lambert solution, V_p is the heliocentric velocity of the departure planet, and r_p is the user-specified perigee radius of the departure hyperbola.

A unit vector in the direction of perigee of the departure hyperbola is determined from

$$\hat{\mathbf{r}}_p = \hat{\mathbf{S}}\cos\theta_\infty - \left(\hat{\mathbf{h}} \times \hat{\mathbf{S}}\right)\sin\theta_\infty$$

The ECI position vector at perigee is

$$\mathbf{r}_p = r_p\,\hat{\mathbf{r}}_p$$

The scalar magnitude of the perigee velocity can be determined from

$$V_p = \sqrt{\frac{2\mu}{r_p} + V_\infty^2}$$

A unit vector aligned with the velocity vector at perigee is

$$\hat{\mathbf{v}}_p = \hat{\mathbf{h}} \times \hat{\mathbf{r}}_p$$

The ECI velocity vector at perigee of the departure hyperbola is given by

$$\mathbf{v}_p = V_p \hat{\mathbf{v}}_p$$

Finally, the classical orbital elements of the departure hyperbola can be determined from the position and velocity vectors at perigee. The injection delta-v vector and magnitude can be determined from the velocity difference between the park orbit and departure hyperbola at the orbital location of the impulsive maneuver.

Propagating the spacecraft's trajectory

The spacecraft's orbital motion is modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time.

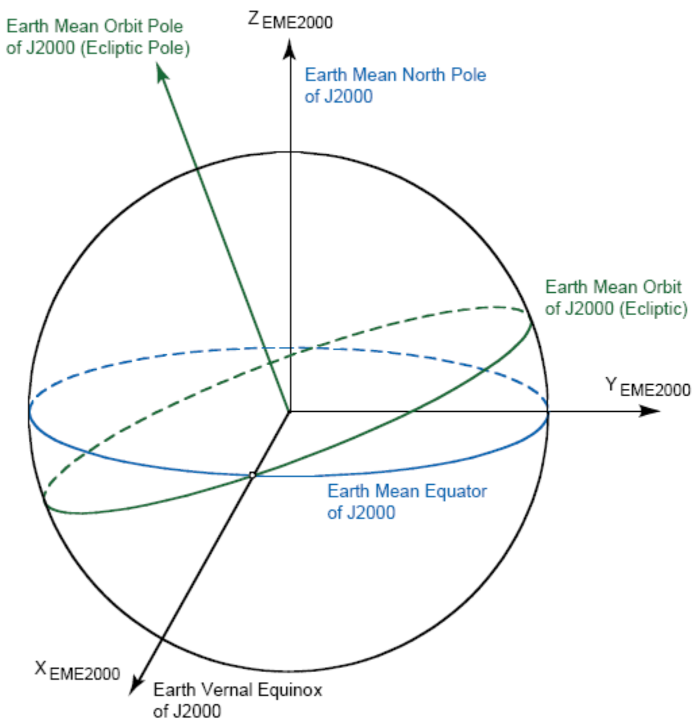


Figure 1. Earth mean equator and equinox of J2000 coordinate system

Program `e2m_ftn` implements a *special perturbation* technique which numerically integrates the vector system of second-order, nonlinear differential equations of motion of a spacecraft given by

$$\mathbf{a}(\mathbf{r},t) = \ddot{\mathbf{r}}(\mathbf{r},t) = \mathbf{a}_g(\mathbf{r},t) + \mathbf{a}_s(\mathbf{r},t) + \mathbf{a}_m(\mathbf{r},t) + \mathbf{a}_p(\mathbf{r},t) + \mathbf{a}_{srp}(\mathbf{r},t)$$

where

$t =$ dynamical time

$\mathbf{r} =$ inertial position vector of the spacecraft

$\mathbf{a}_g =$ acceleration due to Earth gravity

$\mathbf{a}_s =$ acceleration due to the sun

$\mathbf{a}_m =$ acceleration due to the moon

$\mathbf{a}_p =$ acceleration due to the planets

$\mathbf{a}_{srp} =$ acceleration due to solar radiation pressure

Geocentric acceleration due to non-spherical Earth gravity

The software uses a *spherical harmonic* representation of the Earth's geopotential function given by

$$\Phi(r,\phi,\lambda) = \frac{\mu}{r} + \frac{\mu}{r}\sum_{n=1}^{\infty} C_n^0 \left(\frac{R}{r}\right)^n P_n^0(u) + \frac{\mu}{r}\sum_{n=1}^{\infty}\sum_{m=1}^{n}\left(\frac{R}{r}\right)^n P_n^m(u)\left[S_n^m \sin m\lambda + C_n^m \cos m\lambda\right]$$

where ϕ is the geocentric latitude of the spacecraft, λ is the geocentric east longitude of the spacecraft and $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ is the geocentric distance of the spacecraft. In this expression the *S's* and *C's* are harmonic coefficients of the geopotential, and the *P's* are associated Legendre polynomials of degree *n* and order *m* with argument $u = \sin\phi$.

The software calculates the spacecraft's acceleration due to the Earth's gravity field with a vector equation derived from the gradient of the potential function expressed as

$$\mathbf{a}_g(\mathbf{r},t) = \nabla\Phi(\mathbf{r},t)$$

This acceleration vector is a combination of pure two-body or *point mass* gravity acceleration and the gravitational acceleration due to higher order nonspherical terms in the Earth's geopotential. In terms of the Earth's geopotential Φ, the inertial rectangular cartesian components of the spacecraft's acceleration vector are as follows:

$$\ddot{x} = \left(\frac{1}{r}\frac{\partial\Phi}{\partial r} - \frac{z}{r^2\sqrt{x^2+y^2}}\frac{\partial\Phi}{\partial\phi}\right)x - \left(\frac{1}{x^2+y^2}\frac{\partial\Phi}{\partial\lambda}\right)y$$

$$\ddot{y} = \left(\frac{1}{r}\frac{\partial\Phi}{\partial r} - \frac{z}{r^2\sqrt{x^2+y^2}}\frac{\partial\Phi}{\partial\phi}\right)y + \left(\frac{1}{x^2+y^2}\frac{\partial\Phi}{\partial\lambda}\right)x$$

$$\ddot{z} = \left(\frac{1}{r}\frac{\partial\Phi}{\partial r}\right)z + \left(\frac{\sqrt{x^2+y^2}}{r^2}\frac{\partial\Phi}{\partial\phi}\right)$$

The three partial derivatives of the geopotential with respect to r,ϕ,λ are given by

$$\frac{\partial\Phi}{\partial r} = -\frac{1}{r}\left(\frac{\mu}{r}\right)\sum_{n=2}^{N}\left(\frac{R}{r}\right)^n (n+1)\sum_{m=0}^{n}\left(C_n^m \cos m\lambda + S_n^m \sin m\lambda\right)P_n^m(\sin\phi)$$

$$\frac{\partial \Phi}{\partial \phi} = \left(\frac{\mu}{r}\right)\sum_{n=2}^{N}\left(\frac{R}{r}\right)^{n}\sum_{m=0}^{n}\left(C_n^m \cos m\lambda + S_n^m \sin m\lambda\right)\left[P_n^{m+1}\left(\sin\phi\right) - m\tan\phi\, P_n^m\left(\sin\phi\right)\right]$$

$$\frac{\partial \Phi}{\partial \lambda} = \left(\frac{\mu}{r}\right)\sum_{n=2}^{N}\left(\frac{R}{r}\right)^{n}\sum_{m=0}^{n} m\left(S_n^m \cos m\lambda - C_n^m \sin m\lambda\right)P_n^m\left(\sin\phi\right)$$

where

$R =$ radius of the Earth

$r =$ geocentric distance of the satellite

$S_n^m, C_n^m =$ harmonic coefficients

$\phi =$ geocentric declination of the satellite $= \sin^{-1}\left(\frac{z}{r}\right)$

$\lambda =$ longitude of the satellite $= \alpha - \alpha_g$

$\alpha =$ right ascension of the satellite $= \tan^{-1}\left(\frac{y}{x}\right)$

$\alpha_g =$ right ascension of Greenwich

The right ascension is measure positive east of the vernal equinox, longitude is measured positive east of Greenwich, and declination is positive above the Earth's equator and negative below.

For $m = 0$ the coefficients are called *zonal* terms, when $m = n$ the coefficients are *sectorial* terms, and for $n > m \neq 0$ the coefficients are called *tesseral* terms.

The Legendre polynomials with argument $\sin\phi$ are computed using recursion relationships given by:

$$\begin{aligned}
P_n^0\left(\sin\phi\right) &= \frac{1}{n}\left[\left(2n-1\right)\sin\phi\, P_{n-1}^0\left(\sin\phi\right) - \left(n-1\right)P_{n-2}^0\left(\sin\phi\right)\right] \\
P_n^n\left(\sin\phi\right) &= \left(2n-1\right)\cos\phi\, P_{n-1}^{n-1}\left(\sin\phi\right), \qquad m \neq 0,\ m < n \\
P_n^m\left(\sin\phi\right) &= P_{n-2}^m\left(\sin\phi\right) + \left(2n-1\right)\cos\phi\, P_{n-1}^{m-1}\left(\sin\phi\right), \qquad m \neq 0,\ m = n
\end{aligned}$$

where the first few associated Legendre functions are given by

$$P_0^0\left(\sin\phi\right) = 1, \quad P_1^0\left(\sin\phi\right) = \sin\phi, \quad P_1^1\left(\sin\phi\right) = \cos\phi$$

and $P_i^j = 0$ for $j > i$.

The trigonometric arguments are determined from expansions given by

$$\begin{aligned}
\sin m\lambda &= 2\cos\lambda \sin\left(m-1\right)\lambda - \sin\left(m-2\right)\lambda \\
\cos m\lambda &= 2\cos\lambda \cos\left(m-1\right)\lambda - \cos\left(m-2\right)\lambda \\
m\tan\phi &= \left(m-1\right)\tan\phi + \tan\phi
\end{aligned}$$

The gravity model data files used by the software are simple space delimited ASCII data files. The following is a portion of a typical gravity model data file. In this file, column one is the degree index, column two is the model order index, and columns three and four are the corresponding *un-normalized* gravity coefficients (zonals and tesserals, respectively).

```
  2   0  -0.10826300D-02   0.00000000D+00
  3   0   0.25321531D-05   0.00000000D+00
  4   0   0.16109876D-05   0.00000000D+00
  5   0   0.23578565D-06   0.00000000D+00
  6   0  -0.54316985D-06   0.00000000D+00
  7   0   0.33237640D-06   0.00000000D+00
  8   0   0.17721040D-06   0.00000000D+00
  9   0   0.14459876D-06   0.00000000D+00
 10   0   0.23339780D-06   0.00000000D+00
 11   0  -0.27870829D-06   0.00000000D+00
 12   0   0.17036617D-06   0.00000000D+00
 13   0   0.25024428D-06   0.00000000D+00
 14   0  -0.13764093D-06   0.00000000D+00
 15   0  -0.30920023D-07   0.00000000D+00
 16   0   0.55350560D-07   0.00000000D+00
```

Gravity model coefficients are often published in *normalized* form. The relationship between normalized $\bar{C}_{l,m}, \bar{S}_{l,m}$ and un-normalized gravity coefficients $C_{l,m}, S_{l,m}$ is given by the following expression:

$$\begin{Bmatrix} \bar{C}_{l,m} \\ \bar{S}_{l,m} \end{Bmatrix} = \left[\frac{1}{(2-\delta_{m0})(2l+1)} \frac{(l+m)!}{(l-m)!} \right]^{1/2} \begin{Bmatrix} C_{l,m} \\ S_{l,m} \end{Bmatrix}$$

where δ_{m0} is equal to 1 if m is zero and equal to zero if m is greater than zero.

Geocentric acceleration due to the sun, Moon and planets

The acceleration contribution of the Moon represented by a *point mass* is given by

$$\mathbf{a}_m(\mathbf{r},t) = -\mu_m \left(\frac{\mathbf{r}_{m-sc}}{|\mathbf{r}_{m-sc}|^3} + \frac{\mathbf{r}_{e-m}}{|\mathbf{r}_{e-m}|^3} \right)$$

where

$\mu_m =$ gravitational constant of the moon

$\mathbf{r}_{m-sc} =$ position vector from the moon to the spacecraft

$\mathbf{r}_{e-m} =$ position vector from the Earth to the moon

The acceleration contribution of the sun represented by a *point mass* is given by

$$\mathbf{a}_s(\mathbf{r},t) = -\mu_s \left(\frac{\mathbf{r}_{s-sc}}{|\mathbf{r}_{s-sc}|^3} + \frac{\mathbf{r}_{e-s}}{|\mathbf{r}_{e-s}|^3} \right)$$

where

$\mu_s =$ gravitational constant of the sun
$\mathbf{r}_{s-sc} =$ position vector from the sun to the spacecraft
$\mathbf{r}_{e-s} =$ position vector from the Earth to the sun

The acceleration contribution of a planet represented by a *point mass* is given by

$$\mathbf{a}_p(\mathbf{r},t) = -\mu_p \left(\frac{\mathbf{r}_{s-sc}}{\left|\mathbf{r}_{s-sc}\right|^3} + \frac{\mathbf{r}_{e-p}}{\left|\mathbf{r}_{e-p}\right|^3} \right)$$

where

$\mu_s =$ gravitational constant of the sun
$\mathbf{r}_{s-sc} =$ position vector from the sun to the spacecraft
$\mathbf{r}_{e-p} =$ position vector from the Earth to the planet

The first-order system of equations required by this computer program can be created from the second-order system by the method of *order reduction*. With the following definitions,

$$y_1 = r_x \quad y_2 = r_y \quad y_3 = r_z$$
$$y_4 = v_x \quad y_5 = v_y \quad y_6 = v_z$$

where v_x, v_y, v_z are the velocity vector components of the spacecraft, the first-order system of differential equations is given by

$$\dot{y}_1 = v_x \qquad \dot{y}_2 = v_y \qquad \dot{y}_3 = v_z$$
$$\dot{y}_4 = -\mu_s \frac{r_x}{r^3} + a_{x-m} + a_{x-p} + a_{x-srp}$$
$$\dot{y}_5 = -\mu_s \frac{r_y}{r^3} + a_{y-m} + a_{y-p} + a_{y-srp}$$
$$\dot{y}_6 = -\mu_s \frac{r_z}{r^3} + a_{z-m} + a_{z-p} + a_{z-srp}$$

In these equations, μ_s is the gravitational constant of the sun, a_{x-p}, a_{y-p} and a_{z-p} are the *x*, *y* and *z* gravitational contributions of the planets, a_{x-m}, a_{y-m} and a_{z-m} are the *x*, *y* and *z* gravitational contributions of the Moon, and a_{x-srp}, a_{y-srp} and a_{z-srp} are the *x*, *y* and *z* gravitational contributions due to solar radiation pressure.

To avoid numerical problems, use is made of Richard Battin's $f(q)$ function given by

$$f\left(q_k\right) = q_k\left[\frac{3+3q_k+q_k^2}{1+\left(\sqrt{1+q_k}\right)^3}\right]$$

where

$$q_k = \frac{\mathbf{r}^T\left(\mathbf{r}-2\mathbf{s}_k\right)}{\mathbf{s}_k^T\mathbf{s}_k}$$

The point-mass acceleration due to *n* gravitational bodies can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^{n}\frac{\mu_k}{d_k^3}\left[\mathbf{r}+f\left(q_k\right)\mathbf{s}_k\right]$$

In these equations, $\mathbf{s}_k$ is the vector from the primary body to the secondary body, μ_k is the gravitational constant of the secondary body and $\mathbf{d}_k = \mathbf{r}-\mathbf{s}_k$, where **r** is the position vector of the spacecraft relative to the primary body. The derivation of the $f\left(q\right)$ functions is described in Section 8.4 of "An *Introduction to the Mathematics and Methods of Astrodynamics*, Revised Edition", by Richard H. Battin, AIAA Education Series, 1999.

Geocentric acceleration due to solar radiation pressure

We can define a *solar radiation constant* for any spacecraft as a function of its size, mass and surface reflective properties according to the equation:

$$C_{srp} = \gamma\, P_s\, a^2\,\frac{A}{m}$$

where

$\gamma =$ reflectivity constant

$P_s =$ solar radiation pressure constant

$a =$ astronomical unit

$A =$ surface area normal to the incident radiation

$m =$ mass of the spacecraft

The reflectivity constant is a dimensionless number between 0 and 2. For a perfectly absorbent body $\gamma = 1$, for a perfectly reflective body $\gamma = 2$, and for a translucent body $\gamma < 1$. For example, the reflectivity constant for an aluminum surface is approximately 1.96.

The value of the solar radiation pressure on a perfectly absorbing spacecraft surface at a distance of one Astronomical Unit from the Sun is

$$P_s = \frac{G_1}{c} \Rightarrow \frac{Newton}{meters^2}$$

where G_1 is the solar flux at a distance of one Astronomical Unit in watts per square meter, and c is the speed of light in meters per second. The values of the solar flux and speed of light used during a simulation are defined by the user in the constants and models data file.

The acceleration vector of the spacecraft due to solar radiation pressure is given by:

$$\mathbf{a}_{srp} = c_{srp} \frac{\mathbf{r}_{sc-s}}{\left|\mathbf{r}_{sc-s}\right|^3}$$

where

$\mathbf{r}_{sc}$ = geocentric, inertial position vector of the spacecraft

$\mathbf{r}_{e-s}$ = geocentric, inertial position vector of the sun

$\mathbf{r}_{sc-s} = \mathbf{r}_{sc} - \mathbf{r}_{e-s}$

During the geocentric integration process, the software must determine if the spacecraft is in Earth shadow or sunlight. Obviously, there can be no solar radiation perturbation during Earth eclipse of the spacecraft orbit. The software makes use of a *shadow parameter* to determine eclipse conditions. This parameter is defined by the following expression:

$$\varphi = -\frac{\left|\mathbf{r}_{sc} \times \mathbf{r}_{e-s}\right|}{\mathbf{r}_{e-s}} sign(\mathbf{r}_{sc} \bullet \mathbf{r}_{e-s})$$

where $\mathbf{r}_{sc}$ is the geocentric, inertial position vector of the spacecraft and $\mathbf{r}_{e-s}$ is the geocentric, inertial position vector of the sun relative to the spacecraft.

The *critical* values of the shadow parameter for the penumbra (subscript p) and umbra part (subscript u) of the shadow are given by:

$$\varphi_p = \left|\mathbf{r}_{sc}\right| \sin \psi_p$$

$$\varphi_u = \left|\mathbf{r}_{sc}\right| \sin \psi_u$$

The penumbra and umbra shadow angles are found from:

$$\psi_p = \eta + \theta_p$$

$$\psi_u = \eta - \theta_u$$

They are the angles between the geocentric anti-sun vector and the vector to a spacecraft at the time of shadow entrance or exit.

If we represent the shadow as a cylinder, the shadow angle is given by:

$$\eta = \sin^{-1}\left(\frac{r_e}{r_{sc}}\right)$$

The corresponding penumbra and umbra *cone* angles are as follows:

$$\theta_p = \sin^{-1}\left(\frac{r_s + r_e}{r_{e-s}}\right) \qquad \theta_u = \sin^{-1}\left(\frac{r_s - r_e}{r_{e-s}}\right)$$

where

$r_e =$ radius of the Earth
$r_s =$ radius of the sun
$r_{e-s} =$ distance from the Earth to the sun

If the condition $\varphi_u < \varphi \le \varphi_p$ is true, the spacecraft is in the penumbra part of the Earth's shadow, and if the inequality $0 \le \varphi \le \varphi_u$ is true, the spacecraft is in the umbra part of the shadow. If the absolute value of the shadow parameter is larger than the penumbra value, the spacecraft is in full sunlight. The shadow calculations used in this computer program also assume the Earth's atmosphere increases the radius of the Earth by two percent.

Heliocentric trajectory propagation

The second-order, vector system of heliocentric equations of motion for *point-mass* gravity perturbations such as the Moon or planets are given by

$$\ddot{\mathbf{r}} = -\sum_{j=1}^{n} \mu_j \left[\frac{\mathbf{d}_j}{d_j^3} + \frac{\mathbf{s}_j}{s_j^3}\right]$$

In this equation, $\mathbf{s}_j$ is the vector from the primary body to the secondary body *j*, μ_j is the gravitational constant of the secondary body, and $\mathbf{d}_j = \mathbf{r} - \mathbf{s}_j$, where $\mathbf{r}$ is the position vector of the spacecraft relative to the primary body.

Following the geocentric formulation, use is again made of Battin's $F(q)$ function given by

$$F(q_k) = q_k \left[\frac{3 + 3q_k + q_k^2}{1 + \left(\sqrt{1 + q_k}\right)^3}\right]$$

where

$$q_k = \frac{\mathbf{r}^T(\mathbf{r} - 2\mathbf{s}_k)}{\mathbf{s}_k^T \mathbf{s}_k}$$

The third-body acceleration can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^{n} \frac{\mu_k}{d_k^3}\left[\mathbf{r} + F(q_k)\mathbf{s}_k\right]$$

Heliocentric acceleration due to solar radiation pressure

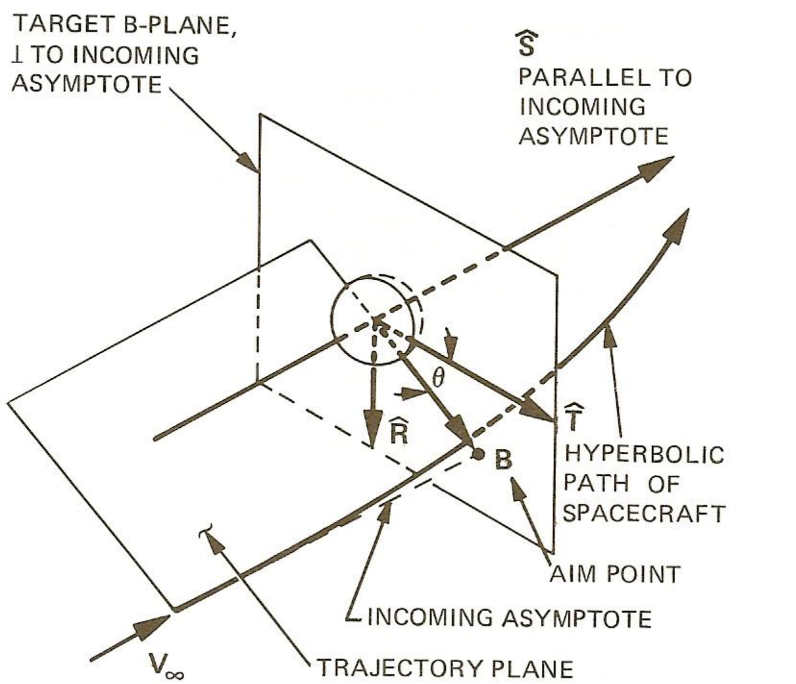
The heliocentric acceleration vector of the spacecraft due to solar radiation pressure is given by:

$$\mathbf{a}_{srp} = c_{srp} \frac{\mathbf{r}_{sc}}{\left|\mathbf{r}_{sc}\right|^3}$$

where $\mathbf{r}_{sc}$ is the heliocentric position vector of the spacecraft. The equation for c_{srp} is defined in the previous geocentric trajectory propagation discussion.

The B-plane

The derivation of B-plane coordinates is described in the classic JPL reports, “A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories” and “Some Orbital Elements Useful in Space Trajectory Calculations”, both by William Kizner. The following diagram illustrates the fundamental geometry of the B-plane coordinate system.



The following computational steps summarize the calculation of the *predicted* B-plane vector from a Mars-centered position vector **r** and velocity vector **v**.

angular momentum vector

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

radius rate

$$\dot{r} = \frac{\mathbf{r} \bullet \mathbf{v}}{\left|\mathbf{r}\right|}$$

semiparameter

$$p = \frac{h^2}{\mu}$$

semimajor axis

$$a = \frac{r}{\left(2 - \frac{r v^2}{\mu}\right)}$$

orbital eccentricity

$$e = \sqrt{1 - p/a}$$

true anomaly

$$\cos\theta = \frac{p - r}{e r} \qquad \sin\theta = \frac{\dot{r} h}{e\mu}$$

B-plane magnitude

$$B = \sqrt{p|a|}$$

fundamental vectors

$$\hat{\mathbf{z}} = \frac{r\,\mathbf{v} - \dot{r}\,\mathbf{r}}{h}$$

$$\hat{\mathbf{p}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\mathbf{z}} \qquad \hat{\mathbf{q}} = \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\mathbf{z}}$$

S vector

$$\mathbf{S} = -\frac{a}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{b}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

B vector

$$\mathbf{B} = \frac{b^2}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{ab}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

T vector

$$\mathbf{T} = \frac{\left(S_y^2, -S_x^2, 0\right)^T}{\sqrt{S_x^2 + S_y^2}}$$

R vector

$$\mathbf{R} = \mathbf{S} \times \mathbf{T} = \left(-S_z T_y,\ S_z T_x,\ S_x T_y - S_y T_x\right)^T$$

Predicting the conditions at the Earth's sphere of influence

The trajectory conditions at the boundary of the Earth's sphere of influence are determined during the numerical integration of the spacecraft's geocentric equations of motion by finding the time at which the difference between the geocentric distance and the user-defined value is essentially zero. This scalar mission constraint is computed as follows

$$\Delta r = \left|\mathbf{r}_{sc}\right|_p - r_{soi_u} \approx 0$$

where $\left|\mathbf{r}_{sc}\right|_p$ is the scalar magnitude of the *predicted* geocentric position vector of the spacecraft and r_{soi_u} is the user-defined value of the geocentric distance of the SOI boundary.

Targeting to the Mars-centered flight path angle, periapsis radius and orbital inclination

The software solves the B-plane targeting problem by minimizing the magnitude of the hyperbolic injection maneuver while satisfying two nonlinear *equality constraint* equations. These constraint equations are the differences between components of the *required* B-plane and the B-plane components *predicted* by the software.

The predicted flight path angle is computed as follows

$$\gamma_p = \sin^{-1}\left(\frac{\mathbf{r} \bullet \mathbf{v}}{\left|\mathbf{r} \bullet \mathbf{v}\right|}\right)$$

where **r** and **v** are the Mars-centered position and velocity vectors, respectively.

Both the geocentric SOI boundary and areocentric flight path angle are predicted using a Runge-Kutta-Fehlberg (RKF7(8)) integrator embedded with a one-dimensional form of Brent's root-finding method.

For this targeting option, the following series of equations can be used to determine the required B-plane target vector:

$$\mathbf{B} \bullet \mathbf{T} = b_t \cos\theta$$

$$\mathbf{B} \bullet \mathbf{R} = b_t \sin\theta$$

where

$$b_t = \cos\gamma_u \sqrt{\frac{2\mu r_u}{v_\infty^2} + r_u^2}$$

In these equations, γ_u is the user-defined flight path angle, and r_u is the user-defined Mars-centered radius at the entry interface. Note that a user-defined flight path angle equal to zero will predict closest approach at Mars. Furthermore, for this case *r* will be equal to the periapsis radius of the incoming areocentric hyperbola.

Also, from the user-defined areocentric inclination i_u ,

$$\cos\theta = \frac{\cos i_u}{\cos\delta_\infty}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta}$$

The areocentric declination of the incoming hyperbola can be determined from

$$\sin\delta_\infty = \left|\hat{\mathbf{s}}\times\hat{\mathbf{z}}\right| = \sqrt{s_x^2 + s_y^2}$$

$$\hat{\mathbf{z}} = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$$

The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix}\cos\delta_\infty\cos\alpha_\infty \\ \cos\delta_\infty\sin\alpha_\infty \\ \sin\delta_\infty\end{Bmatrix} = \begin{Bmatrix}s_x \\ s_y \\ s_z\end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola at Mars.

The nonlinear equality constraints enforced by the nonlinear programming algorithm are

$$(\mathbf{B}\bullet\mathbf{T})_p - (\mathbf{B}\bullet\mathbf{T})_r \approx 0$$

$$(\mathbf{B}\bullet\mathbf{R})_p - (\mathbf{B}\bullet\mathbf{R})_r \approx 0$$

where the *p* subscript refers to coordinates *predicted* by the software and the *r* subscript denotes the *required* B-plane coordinates defined previously.

Important note!!

This technique only works for aerocentric orbit inclinations that satisfy the following inequality

$$|i| > |\delta_\infty|$$

If this requirement is not satisfied, the software will print the following error message

```
b-plane targeting error!!

|inclination| must be > |asymptote declination|
```

It will also display the actual declination of the asymptote and stop. The user should then edit the input file, include a valid orbital inclination and restart the simulation.

Targeting to user-defined B-plane coordinates

For this targeting option, the nonlinear equality constraints enforced by the nonlinear programming algorithm are

$$(\mathbf{B \bullet T})_p - (\mathbf{B \bullet T})_u \approx 0$$

$$(\mathbf{B \bullet R})_p - (\mathbf{B \bullet R})_u \approx 0$$

where the *p* subscript refers to coordinates predicted by the software and the *u* subscript denotes coordinates provided by the user. The *predicted* B-plane coordinates are based on the Mars-centered flight conditions at closest approach.

Geocentric-to-areocentric coordinate transformation

This section describes the transformation of coordinates between the Earth mean equator and equinox of J2000 and areocentric (Mars-centered) mean equator and IAU node of epoch coordinate systems. This transformation is used to compute the areocentric state vector (position and velocity vectors) and B-plane coordinates at encounter.

A unit vector in the direction of the pole of Mars can be determined from

$$\hat{\mathbf{p}}_{Mars} = \begin{bmatrix} \cos\alpha_p \cos\delta_p \\ \sin\alpha_p \cos\delta_p \\ \sin\delta_p \end{bmatrix}$$

The IAU 2000 right ascension and declination of the pole of Mars in the EME2000 coordinate system are given by the following expressions

$$\alpha_p = 317.68143 - 0.1061T$$

$$\delta_p = 52.88650 - 0.0609T$$

where T is the time in Julian centuries given by $T = (JD - 2451545.0)/36525$ and JD is the TDB Julian Date.

The unit vector in the direction of the *IAU-defined* x-axis is computed from

$$\hat{\mathbf{x}} = \hat{\mathbf{p}}_{J2000} \times \hat{\mathbf{p}}_{Mars}$$

where $\hat{\mathbf{p}}_{J2000} = [0\,0\,1]^T$ is unit vector in the direction of the pole of the J2000 coordinate system.

The unit vector in the y-axis direction of this coordinate system is

$$\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Mars} \times \hat{\mathbf{x}}$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the Mars-centered mean equator and IAU node of epoch system are as follows:

$$\mathbf{M} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{p}}_{Mars} \end{bmatrix}$$

Terrestrial Time, TT

Terrestrial Time is the time scale that would be kept by an ideal clock on the geoid - approximately, sea level on the surface of the Earth. Since its unit of time is the SI (atomic) second, TT is independent of the variable rotation of the Earth. TT is meant to be a smooth and continuous "coordinate" time scale independent of Earth rotation. In practice TT is derived from International Atomic Time (TAI), a time scale kept by real clocks on the Earth's surface, by the relation **TT = TAI + $32^s.184$.** It is the time scale now used for the precise calculation of future astronomical events observable from Earth.

TT = TAI + 32.184 seconds

TT = UTC + (number of leap seconds) + 32.184 seconds

Barycentric Dynamical Time, TDB

Barycentric Dynamical Time is the time scale that would be kept by an ideal clock, free of gravitational fields, co-moving with the solar system barycenter. It is always within 2 milliseconds of TT, the difference caused by relativistic effects. TDB is the time scale now used for investigations of the dynamics of solar system bodies.

TDB = TT + periodic corrections

where typical periodic corrections (USNO Circular 179) are

$$\begin{aligned} TDB = TT &+ 0.001657\sin\left(628.3076T + 6.2401\right) \\ &+ 0.000022\sin\left(575.3385T + 4.2970\right) \\ &+ 0.000014\sin\left(1256.6152T + 6.1969\right) \\ &+ 0.000005\sin\left(606.9777T + 4.0212\right) \\ &+ 0.000005\sin\left(52.9691T + 0.4444\right) \\ &+ 0.000002\sin\left(21.3299T + 5.5431\right) \\ &+ 0.000010T\sin\left(628.3076T + 4.2490\right) + \cdots \end{aligned}$$

In this equation, the coefficients are in seconds, the angular arguments are in radians, and T is the number of Julian centuries of TT from J2000; T = (Julian Date(TT) – 2451545.0) / 36525.

Algorithm resources

(1) NOVAS (Naval Observatory Vector Astrometry Subroutines) software package, version 3.1, U.S. Naval Observatory, March 2011.

(2) *Explanatory Supplement to the Astronomical Almanac*, Edited by P. K. Seidelmann, University Science Books, 1992.

(3) “Update to Mars Coordinate Frame Definitions”, R. A. Mase, JPL IOM 312.B/015-99, 15 July 1999.

(4) “The Planetary and Lunar Ephemeris DE 421”, W. M. Folkner, J. G. Williams, D. H. Boggs, JPL IOM 343R-08-003, 31-March-2008.

(5) “Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 2009”, *Celestial Mechanics and Dynamical Astronomy*, **109**: 101-135, 2011.

(6) “IERS Conventions (2003)”, IERS Technical Note 32, November 2003.

(7) “Planetary Constants and Models”, R. Vaughan, JPL D-12947, December 1995.

(8) R. P. Brent, *Algorithms for Minimization Without Derivatives*, Prentice-Hall, 1972.

(9) W. Kizner, “A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories”, Publication 674, Jet Propulsion Laboratory, August 1, 1959.

(10) F. M. Sturms, Jr., “Error Analysis of Multiple Planet Trajectories”, JPL Space Programs Summary, No. 37-27, Vol. IV.

(11) R. H. Battin, *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA, 1987.

(12) “Interplanetary Mission Design Handbook, Volume 1, Part 2”, JPL Publication 82-43, September 15, 1983.

APPENDIX A

Contents of the Simulation Summary and CSV Files

This appendix is a brief summary of the information contained in the simulation summary screen displays and the CSV data files produced by the `e2m_ftn` software.

The simulation summary screen display contains the following information:

calendar date = UTC calendar date of trajectory event

UTC time = UTC time of trajectory event

UTC Julian Date = Julian Date of trajectory event on UTC time scale

TDB time = TDB time of trajectory event

TDB Julian Date = Julian Date of trajectory event on TDB time scale

sma (km) = semimajor axis in kilometers

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of periapsis in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (days) = orbital period in days

rx (km) = x-component of the spacecraft's position vector in kilometers

ry (km) = y-component of the spacecraft's position vector in kilometers

rz (km) = z-component of the spacecraft's position vector in kilometers

rmag (km) = scalar magnitude of the spacecraft's position vector in kilometers

vx (kps) = x-component of the spacecraft's velocity vector in kilometers per second

vy (kps) = y-component of the spacecraft's velocity vector in kilometers per second

vz (ksp) = z-component of the spacecraft's velocity vector in kilometers per second

vmag (kps) = scalar magnitude of the spacecraft's velocity vector in kilometers per second

b-magnitude = magnitude of the b-plane vector

b dot r = dot product of the B-plane b-vector and r-vector

b dot t = dot product of the B-plane b-vector and t-vector

theta = orientation of the b-plane vector in degrees

v-infinity = magnitude of outgoing or incoming v-infinity vector in kilometers/second

r-periapsis = periapsis radius of incoming or outgoing hyperbola in kilometers

decl-asymptote = declination of incoming v-infinity vector in degrees

rasc-asymptote = right ascension of incoming v-infinity vector in degrees

flight path angle = flight path angle in degrees

c3 = specific orbital energy in kilometers squared per seconds squared

launch azimuth = launch azimuth in degrees measure positive clockwise from north

launch latitude = geocentric latitude of the launch site in degrees

delta-vx = x-component of injection delta-v in meters per second

delta-vy = y-component of injection delta-v in meters per second

delta-vz = z-component of injection delta-v in meters per second

delta-v magnitude = scalar magnitude of the injection delta-v in meters per second

transfer time = Lambert two-body solution transfer time in days

The `e2m_geo.csv` disk file contains the following information:

time (hrs) = simulation time since interplanetary injection in hours

re2sc-x (km) = x-component of the spacecraft's geocentric position vector in kilometers

re2sc-y (km) = y-component of the spacecraft's geocentric position vector in kilometers

re2sc-z (km) = z-component of the spacecraft's geocentric position vector in kilometers

re2sc-mag (km) = the spacecraft's geocentric radius in kilometers

ve2sc-y (km/sec) = y-component of the spacecraft's geocentric velocity vector in kilometers per second

ve2sc-y (km/sec) = y-component of the spacecraft's geocentric velocity vector in kilometers per second

ve2sc-z (km/sec) = z-component of the spacecraft's geocentric position vector in kilometers per second

ve2sc-mag (km/sec) = the spacecraft's geocentric speed in kilometers per second

sma-geo (km) = geocentric semimajor axis of the spacecraft in kilometers

ecc-geo = geocentric orbital eccentricity of the spacecraft(non-dimensional)

inc-geo (deg) = geocentric orbital inclination of the spacecraft in degrees

argper-geo (deg) = geocentric argument of perigee of the spacecraft in degrees

raan-geo (deg) = geocentric right ascension of the ascending node of the spacecraft in degrees

tanom-geo (deg) = geocentric true anomaly of the spacecraft in degrees

The e2m_helio.csv disk file contains the following information:

time (days) = simulation time since TCM maneuver in days

rs2sc-x (au) = x-component of the spacecraft's heliocentric position vector in astronomical units

rs2sc-y (au) = y-component of the spacecraft's heliocentric position vector in astronomical units

rs2sc-z (au) = z-component of the spacecraft's heliocentric position vector in astronomical units

rs2sc-mag (au) = the spacecraft's heliocentric radius in astronomical units

vs2sc-y (km/sec) = y-component of the spacecraft's heliocentric velocity vector in kilometers per second

vs2sc-z (km/sec) = z-component of the spacecraft's heliocentric position vector in kilometers per second

vs2sc-mag (km/sec) = the spacecraft's heliocentric speed in kilometers per second

rs2e-x (au) = x-component of Earth's heliocentric position vector in astronomical units

rs2e-y (au) = y-component of Earth's heliocentric position vector in astronomical units

rs2e-z (au) = z-component of Earth's heliocentric position vector in astronomical units

rs2e-mag (au) = Earth heliocentric radius in astronomical units

rs2m-x (au) = x-component of Mars heliocentric position vector in astronomical units

rs2m-y (au) = y-component of Mars heliocentric position vector in astronomical units

rs2m-z (au) = z-component of Mars heliocentric position vector in astronomical units

rs2m-mag (au) = Mars heliocentric radius in astronomical units

sma-heo (au) = heliocentric semimajor axis of the spacecraft in astronomical units

ecc-heo = heliocentric orbital eccentricity of the spacecraft(non-dimensional)

inc-heo (deg) = heliocentric orbital inclination of the spacecraft in degrees

argper-heo (deg) = heliocentric argument of perigee of the spacecraft in degrees

raan-heo (deg) = heliocentric right ascension of the ascending node of the spacecraft in degrees

tanom-heo (deg) = heliocentric true anomaly of the spacecraft in degrees

The e2m_areo.csv disk file contains the following information:

time (days) = simulation time since exit from the Earth's SOI in days

rm2sc-x (km) = x-component of the spacecraft's areocentric position vector in kilometers

rm2sc-y (km) = y-component of the spacecraft's areocentric position vector in kilometers

rm2sc-z (km) = z-component of the spacecraft's areocentric position vector in kilometers

rm2sc-mag (km) = the spacecraft's areocentric radius in kilometers

vm2sc-y (km/sec) = y-component of the spacecraft's areocentric velocity vector in kilometers per second

vm2sc-y (km/sec) = y-component of the spacecraft's areocentric velocity vector in kilometers per second

vm2sc-z (km/sec) = z-component of the spacecraft's areocentric position vector in kilometers per second

vm2sc-mag (km/sec) = the spacecraft's areocentric speed in kilometers per second

sma-areo (km) = areocentric semimajor axis of the spacecraft in kilometers

ecc-areo = areocentric orbital eccentricity of the spacecraft(non-dimensional)

inc-areo (deg) = areocentric orbital inclination of the spacecraft in degrees

argper-areo (deg) = areocentric argument of perigee of the spacecraft in degrees

raan-areo (deg) = areocentric right ascension of the ascending node of the spacecraft in degrees

tanom-areo (deg) = areocentric true anomaly of the spacecraft in degrees

rp-aero (km) = areocentric periapsis radius in kilometers

b dot t = dot product of the B-plane b and t vectors

b dot r = dot product of the B-plane b and r vectors

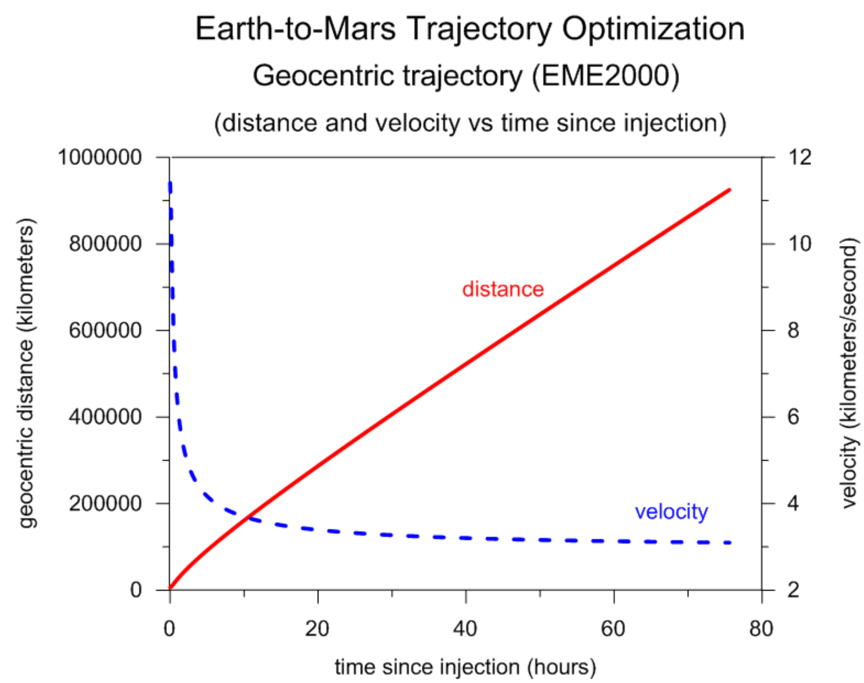
The geocentric and heliocentric coordinates of the spacecraft are with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The areocentric coordinates of the spacecraft are with respect to the areocentric (Mars-centered) mean equator and IAU node of epoch coordinate system.

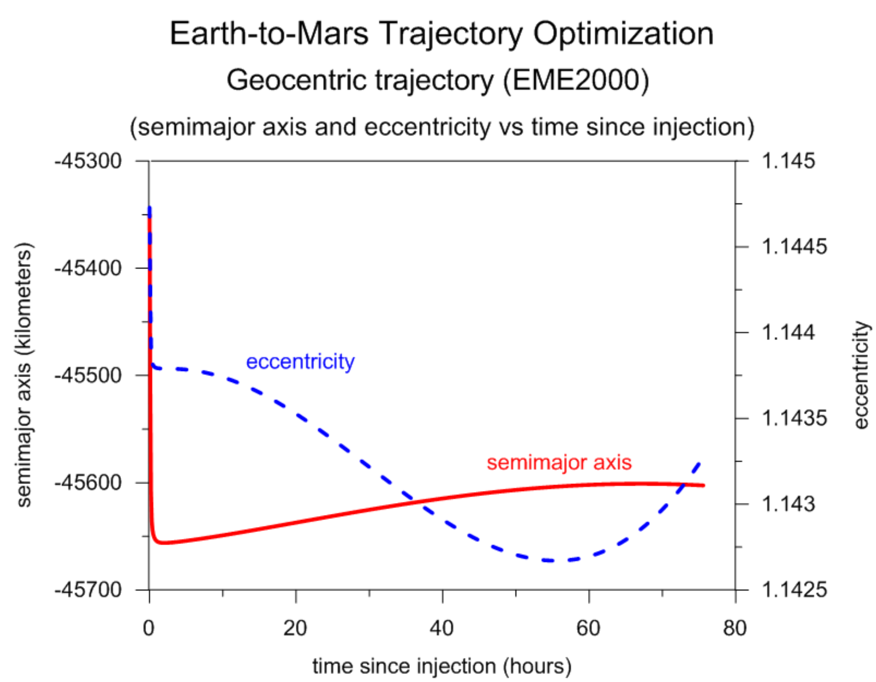
APPENDIX B

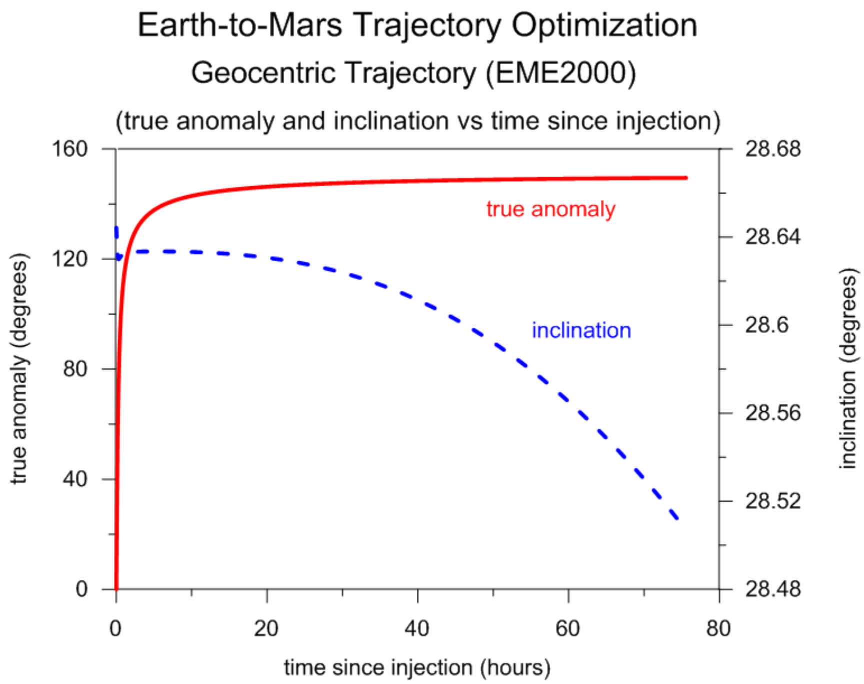
Graphic Displays of Flight Characteristics

This appendix contains typical graphic displays of important flight characteristics during the different phases of the mission. The geocentric and heliocentric plots are relative to the EME2000 coordinate system and the areocentric graphs are relative to the Mars mean equator and IAU node of epoch.

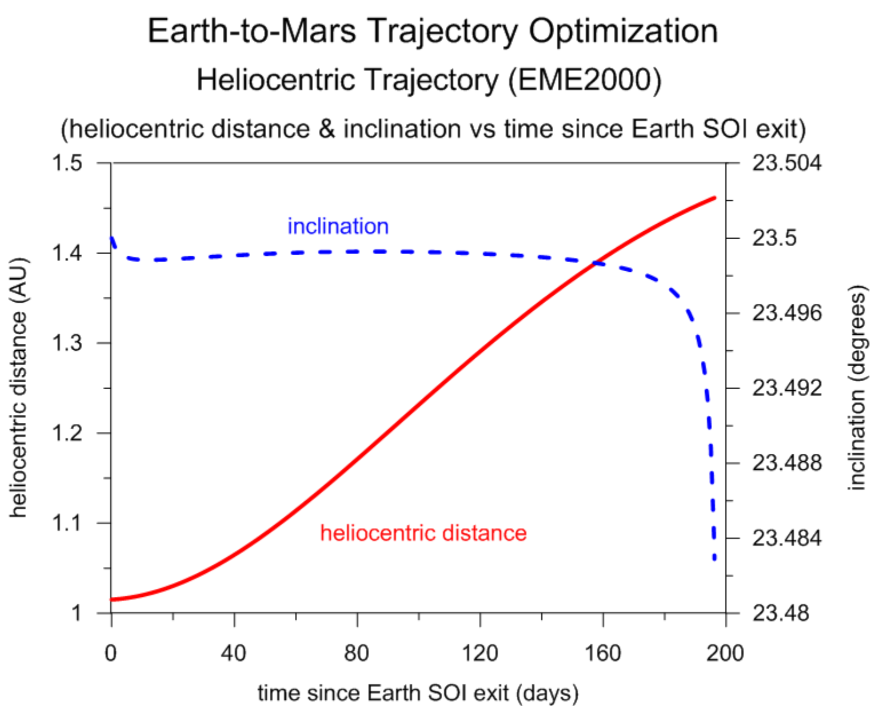
The first three plots are geocentric flight parameters as a function of time since the hyperbolic injection. The data displays terminate at exit from the Earth's sphere-of-influence (SOI).

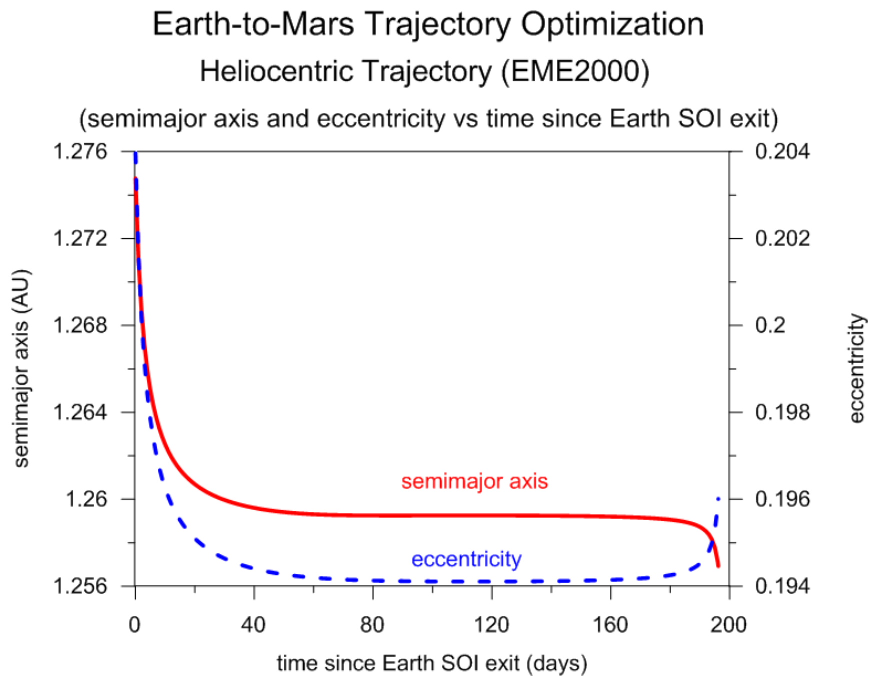




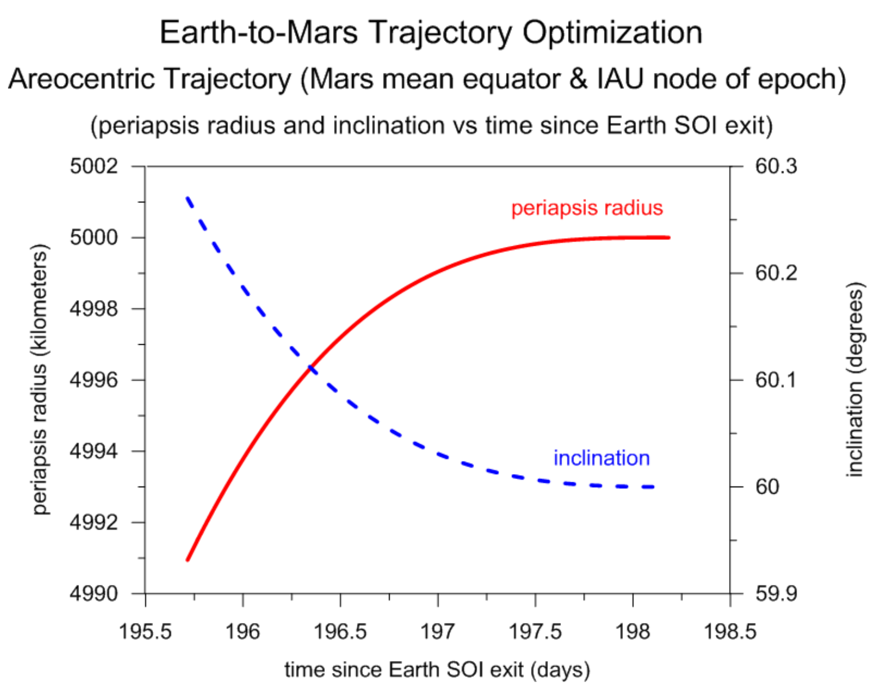


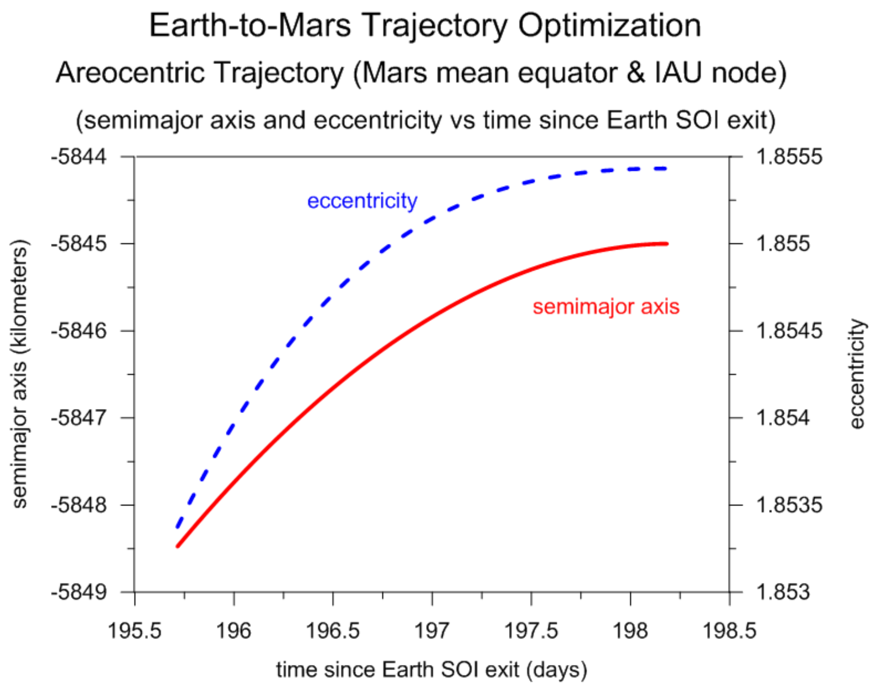
These next two plots are heliocentric trajectory characteristics as a function of time since exit from the Earth's sphere-of-influence (SOI). The data terminates when the spacecraft is within two days of closest approach to Mars.

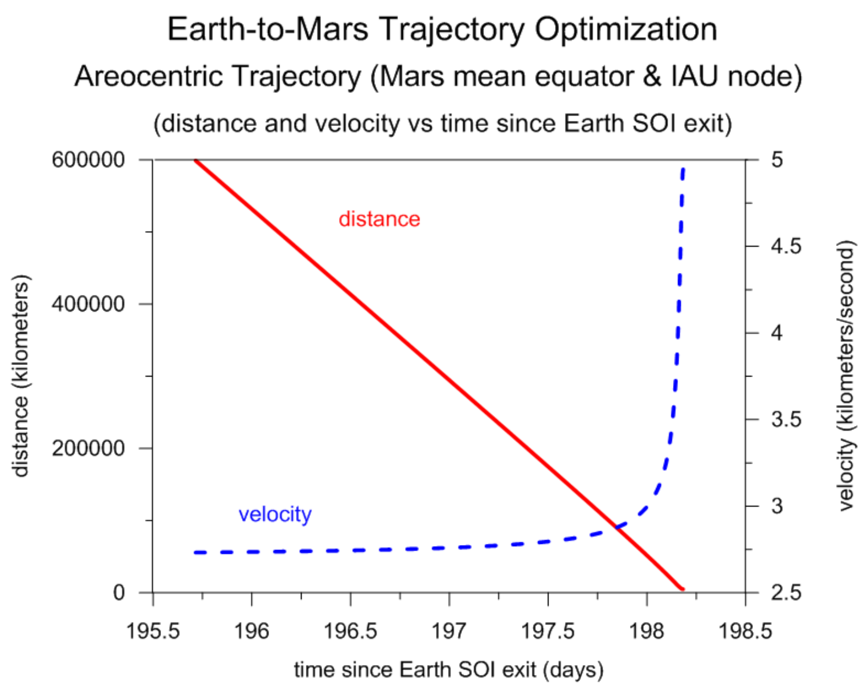




The final three plots are Mars-centered trajectory characteristics as a function of time since exit from the Earth's SOI. The data display starts when the spacecraft is 600,000 kilometers from Mars and ends at closest approach to Mars. The orbital inclination is relative to the mean equator of Mars.




Earth-to-Mars Trajectory Optimization
Areocentric Trajectory (Mars mean equator & IAU node)
(semimajor axis and eccentricity vs time since Earth SOI exit)
eccentricity
semimajor axis
semimajor axis (kilometers)
eccentricity
-5844
-5845
-5846
-5847
-5848
-5849
1.8555
1.855
1.8545
1.854
1.8535
1.853
195.5
196
196.5
197
197.5
198
198.5
time since Earth SOI exit (days)


Earth-to-Mars Trajectory Optimization
Areocentric Trajectory (Mars mean equator & IAU node)
(distance and velocity vs time since Earth SOI exit)
distance
velocity
distance (kilometers)
velocity (kilometers/second)
600000
400000
200000
0
5
4.5
4
3.5
3
2.5
195.5
196
196.5
197
197.5
198
198.5
time since Earth SOI exit (days)

APPENDIX C

Mars Entry Interface Example

The following is the `e2m_ftn` optimal n-body program output for a typical entry interface example. For this example, the areocentric flight path angle is -2 degrees, the areocentric radius is 3500 kilometers, and the areocentric orbital inclination target is 45 degrees.

```
=======================
optimal n-body solution
=======================

orbital element targeting

--------------------------------------------------
park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
--------------------------------------------------

park orbit
----------

calendar date           June  5, 2003

UTC time                14:46:19.786

UTC Julian date         2452796.11550678

TDB time                14:47:23.918

TDB Julian date         2452796.11624905

     sma (km)           eccentricity       inclination (deg)       argper (deg)
 0.656346000000D+04    0.110917091772D-15    0.286442848562D+02    0.000000000000D+00

    raan (deg)        true anomaly (deg)       arglat (deg)         period (hrs)
 0.270931886661D+01    0.194727110636D+03    0.194727110636D+03    0.146996753813D+01

     rx (km)               ry (km)               rz (km)              rmag (km)
 -.627152167852D+04    -.176274500778D+04    -.799845638258D+03    0.656346000000D+04

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 0.229153863954D+01    -.651347903009D+01    -.361298664793D+01    0.779296034444D+01

departure hyperbola
-------------------

c3                   8.79578987310781       km**2/sec**2

v-infinity           2965.76969320071       meters/second

decl-asymptote      -6.84967869961631       degrees

rasc-asymptote       350.005266398407       degrees

calendar date           June  5, 2003

UTC time                14:46:19.786

UTC Julian date         2452796.11550678

TDB time                14:47:23.918

TDB Julian date         2452796.11624905

     sma (km)           eccentricity       inclination (deg)       argper (deg)
 -.453171855229D+05    0.114483379593D+01    0.286442848562D+02    0.194727110636D+03
```

```
    raan (deg)         true anomaly (deg)       arglat (deg)
 0.270931886661D+01    0.000000000000D+00    0.194727110636D+03

     rx (km)               ry (km)               rz (km)             rmag (km)
 -.627152167852D+04    -.176274500778D+04    -.799845638258D+03    0.656346000000D+04

     vx (kps)              vy (kps)              vz (kps)            vmag (kps)
 0.335601587665D+01    -.953915358882D+01    -.529130966565D+01    0.114129861006D+02


hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

delta-vx             1064.47723711808       meters/seconds
delta-vx            -3025.67455872508       meters/seconds
delta-vx            -1678.32301772060       meters/seconds

delta-v magnitude    3620.02575618869       meters/seconds


transfer time        201.329916741233       days


time and conditions at Mars entry interface
(Mars mean equator and IAU node of epoch)
-----------------------------------------

calendar date           December 23, 2003

UTC time                22:41:24.592

UTC Julian date         2452997.44542352

TDB time                22:42:28.724

TDB Julian date         2452997.44616579


     sma (km)            eccentricity       inclination (deg)       argper (deg)
 -.584592533218D+04    0.159811483058D+01    0.450000001093D+02    0.118029275635D+03

    raan (deg)         true anomaly (deg)       arglat (deg)
 0.108956668867D+03    0.356748680746D+03    0.114777956381D+03

     rx (km)               ry (km)               rz (km)             rmag (km)
 -.164865109527D+04    -.211725961303D+04    0.224703384781D+04    0.349999997373D+04

     vx (kps)              vy (kps)              vz (kps)            vmag (kps)
 0.333447912072D+01    -.417773239851D+01    -.179648318722D+01    0.563910904683D+01


B-plane coordinates at Mars entry interface
(Mars mean equator and IAU node of epoch)
-----------------------------------------

b-magnitude          7287.43544300997       kilometers
b dot r             -5107.54726266006
b dot t              5198.04543032587
theta                315.503127761648       degrees
v-infinity           2.70669407634656       km/sec
r-periapsis          3496.53463966152       kilometers
decl-asymptote       7.54885673068842       degrees
rasc-asymptote       281.341426102540       degrees

flight path angle   -1.99999962370103       degrees


heliocentric coordinates of Mars at entry interface
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date           December 23, 2003
```

UTC time 22:41:24.592

UTC Julian date 2452997.44542352

TDB time 22:42:28.724

TDB Julian date 2452997.44616579

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.152368050974D+01	0.935421335240D-01	0.246772248803D+02	0.332979309023D+03

raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.337165816278D+01	0.703671118493D+02	0.433464208718D+02	0.686972409831D+03

rx (km)	ry (km)	rz (km)	rmag (km)
0.150992631967D+09	0.145764865767D+09	0.627780413362D+08	0.219059931239D+09

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.166283847981D+02	0.169014372089D+02	0.820155202609D+01	0.250883880781D+02

spacecraft heliocentric coordinates at entry interface
(Earth mean equator and equinox of J2000)
----

calendar date December 23, 2003

UTC time 22:41:24.592

UTC Julian date 2452997.44542352

TDB time 22:42:28.724

TDB Julian date 2452997.44616579

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.124418233994D+01	0.260193891025D+00	0.185282046360D+02	0.281382377649D+03

raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.340795030083D+03	0.143020395729D+03	0.644027733781D+02	0.506902549814D+03

rx (km)	ry (km)	rz (km)	rmag (km)
0.150993772736D+09	0.145761597092D+09	0.627785555405D+08	0.219058689916D+09

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.127216565242D+02	0.178536794502D+02	0.424803969529D+01	0.223302542727D+02