

A MATLAB Script for Earth-to-Mars Mission Design

This document is the user's manual for a MATLAB script named `e2m_matlab.m` that can be used to design and optimize ballistic interplanetary missions from Earth park orbit to B-plane encounter at Mars. The software assumes that interplanetary injection occurs *impulsively* from a circular Earth park orbit. The B-plane coordinates are expressed in a Mars-centered (areocentric) mean equator and IAU node of epoch coordinate system. These B-plane targets are enforced using either a combination of periapsis radius and orbital inclination, individual B-plane coordinates ($\mathbf{B}\cdot\mathbf{T}$ and $\mathbf{B}\cdot\mathbf{R}$) of the arrival hyperbola, or entry interface (EI) conditions at Mars. The type of targeting and the target values are defined by the user.

The first part of this MATLAB script solves for the minimum delta-v using a *patched-conic*, two-body Lambert solution for the transfer trajectory from Earth to Mars. The second part implements a simple *shooting* method that attempts to optimize the characteristics of the geocentric injection hyperbola while numerically integrating the spacecraft's geocentric and heliocentric equations of motion and targeting to components of the B-plane relative to Mars.

The spacecraft motion within the Earth's SOI includes the Earth's J_2 oblate gravity effect and the point-mass perturbations of the sun and moon. The heliocentric equations of motion include the point-mass gravity of the sun and the first seven planets of the solar system.

The user can select one of the following delta-v optimization options for the two-body solution of the interplanetary transfer trajectory:

- minimize departure delta-v
- minimize arrival delta-v
- minimize total delta-v
- no optimization

The major computational steps implemented in this script are as follows:

- solve the two-body, patched-conic interplanetary Lambert problem for the energy C_3 , declination (DLA) and asymptote (RLA) of the outgoing or departure hyperbola
- compute the orbital elements of the geocentric departure hyperbola and the components of the interplanetary injection delta-v vector
- perform geocentric orbit propagation from perigee of the geocentric departure hyperbola to the Earth's sphere-of-influence (SOI; default value = 925,000 kilometers)
- perform an n-body heliocentric orbit propagation from the Earth's SOI to closest approach at Mars
- target to the user-defined B-plane coordinates while minimizing the magnitude of the hyperbolic v-infinity at Earth departure (equivalent to minimizing the departure energy since $C_3 = V_\infty^2$)

This MATLAB script uses the SNOPT nonlinear programming algorithm to solve both the patched-conic and numerically integrated trajectory optimization problems. The planetary coordinates required by the software are computed using the JPL DE421 ephemeris.

Input file format and contents

This section describes a typical input data file for the `e2m_matlab` script. In the following discussion the actual input file contents are in *courier* font and all explanations are in *times italic* font.

Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input. The time scale for all calculations is Barycentric Dynamical Time (TDB).

The software allows the user to specify an initial guess for the departure and arrival calendar dates and a search interval. For any guess for departure time t_L and user-defined search interval Δt , the departure time t is constrained as follows:

$$t_L - \Delta t \leq t \leq t_L + \Delta t$$

Likewise, for any guess for arrival time t_A and user-defined search interval, the arrival time t is constrained as follows:

$$t_A - \Delta t \leq t \leq t_A + \Delta t$$

For fixed departure and/or arrival times, the search interval should be set to 0.

The first six lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with six and only six initial text lines.

```
*****
** interplanetary trajectory optimization
** script ==> e2m_matlab.m
** Mars '03 mars03.in
** June 30, 2011
*****
```

The first numerical input is an integer that defines the type of patched-conic trajectory optimization performed by this script. Please note that option 4 simply solves Lambert's two-point boundary value problem (TPBVP) using the inputs for departure and arrival calendar dates provided by the user.

```
*****
* simulation type *
*****
1 = minimize departure delta-v
2 = minimize arrival delta-v
3 = minimize total delta-v
4 = no optimization
-----
1
```

The next input defines an initial guess for the departure calendar date. Please be sure to include all digits of the calendar year.

```
departure calendar date initial guess (month, day, year)
6,1,2003
```

This next number defines the lower and upper search boundary for the departure calendar date.

```
departure date search boundary (days)
30
```

The next input defines an initial guess for the arrival calendar date. Please be sure to include all digits of the calendar year.

```
arrival calendar date initial guess (month, day, year)
12,1,2003
```

This number defines the lower and upper search boundary for the arrival calendar date.

```
arrival date search boundary (days)
30
```

The next set of inputs defines several characteristics of the departure hyperbola and initial flight conditions. The perigee altitude and launch site latitude are with respect to a spherical Earth. The launch azimuth is measured positive clockwise from north.

```
*****
* geocentric phase modeling
*****

perigee altitude of launch hyperbola (kilometers)
185.32

launch azimuth (degrees)
93.0

launch site latitude (degrees)
28.5
```

The next input specifies the type of targeting at Mars performed by the e2m_matlab script. Option 1 will target to user-defined components of the B-plane, option 2 will target to a Mars-centered hyperbola with a user specified radius of closest approach (periapsis) and orbital inclination, and program option 3 will target to a user-defined inertial flight path angle and altitude at entry interface (EI) at Mars.

```
*****
* encounter planet targeting
*****

type of targeting
(1 = B-plane, 2 = orbital elements, 3 = EI conditions)
2
```

The next two inputs are the user-defined B-plane components used with targeting option 1.

```
B dot T
4607.4

B dot R
-7888.0
```

These next two inputs define the radius of closest approach and the orbital inclination of the encounter hyperbola at Mars. These flight conditions are used by targeting option 2. The radius of closest approach is with respect to a spherical Mars model and the orbital inclination is with respect to the mean equator of Mars.

```
radius of closest approach (kilometers)
5000.0
```

```
orbital inclination (degrees)
60.0
```

The final two inputs define the inertial flight path angle and altitude of the entry interface at Mars. These flight conditions are used by targeting option 3. The altitude is with respect to a spherical Mars model. Targeting option 3 will target to the orbital inclination defined above.

```
EI flight path angle (degrees)
-2.0
```

```
EI altitude (kilometers)
100.0
```

Program example

The following is the solution created with this MATLAB script for this example. The output is organized by the following major sections.

- Two-body/Patched-conic Pass
 1. two body Lambert solution
 2. departure hyperbola orbital elements and state vector
 3. heliocentric coordinates of Earth at departure and Mars at arrival
 4. heliocentric coordinates of the spacecraft on the transfer trajectory
- Targeting/Optimization Pass
 1. optimized characteristics of the departure hyperbola
 2. heliocentric coordinates of the spacecraft and Mars at closest approach
 3. geocentric and heliocentric coordinates of the spacecraft at the Earth SOI

The first output section summarizes the two-body Lambert solution. The solution is provided in the heliocentric, Earth mean equator and equinox of J2000 (EME2000) coordinate system. The time scale is Barycentric Dynamical Time (TDB). Please see Appendix A for additional explanation.

```
Earth-to-Mars mission design
=====
two-body Lambert solution
=====

minimize departure delta-v

departure heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----

x-component of delta-v      2895.912618  meters/second
y-component of delta-v      -530.389044  meters/second
z-component of delta-v      -345.714310  meters/second

delta-v magnitude          2964.311187  meters/second

arrival heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----
```

x-component of delta-v -2063.021182 meters/second
y-component of delta-v 1164.270846 meters/second
z-component of delta-v 1311.949618 meters/second
delta-v magnitude 2707.913367 meters/second

heliocentric coordinates of the Earth at departure
(Earth mean equator and equinox of J2000)

TDB calendar date 05-Jun-2003
TDB time 14:46:46.546
TDB Julian Date 2452796.11581651

sma (km)	eccentricity	inclination (deg)	argper (deg)
1.4965147326e+008	1.6237346599e-002	2.3439054671e+001	1.0245240439e+002
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
7.2430845695e-004	1.5204742997e+002	2.5449983436e+002	3.6545322928e+002
rx (km)	ry (km)	rz (km)	rmag (km)
-4.05626079825043e+007	-1.34199491179377e+008	-5.81817199052164e+007	+1.51789133768775e+008
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+2.82279246211278e+001	-7.39786254931148e+000	-3.20748439166372e+000	+2.93569762550121e+001

spacecraft heliocentric coordinates after the first impulse
(Earth mean equator and equinox of J2000)

TDB calendar date 05-Jun-2003
TDB time 14:46:46.546
TDB Julian Date 2452796.11581651

sma (km)	eccentricity	inclination (deg)	argper (deg)
1.8838714746e+008	1.9427720614e-001	2.3490037881e+001	2.5349091882e+002
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
4.5596571320e-001	5.9131918849e-001	2.5408223801e+002	5.1616340902e+002
rx (km)	ry (km)	rz (km)	rmag (km)
-4.05626079825043e+007	-1.34199491179377e+008	-5.81817199052164e+007	+1.51789133768775e+008
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+3.11238372390479e+001	-7.92825159286771e+000	-3.55319870155481e+000	+3.23137066709359e+001

spacecraft heliocentric coordinates prior to the second impulse
(Earth mean equator and equinox of J2000)

TDB calendar date 24-Dec-2003
TDB time 15:23:10.886
TDB Julian Date 2452998.14109821

sma (km)	eccentricity	inclination (deg)	argper (deg)
1.8838714746e+008	1.9427720614e-001	2.3490037881e+001	2.5349091882e+002
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
4.5596571320e-001	1.5290995811e+002	4.6400876928e+001	5.1616340902e+002
rx (km)	ry (km)	rz (km)	rmag (km)
+1.49990801287589e+008	+1.46776341622975e+008	+6.32690486907151e+007	+2.19188292236025e+008
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-1.46793406853937e+001	+1.56263833449679e+001	+6.84186934966451e+000	+2.25050677759394e+001

spacecraft heliocentric coordinates after the second impulse
(Earth mean equator and equinox of J2000)

```
-----
TDB calendar date    24-Dec-2003
TDB time             15:23:10.886
TDB Julian Date      2452998.14109821

      sma (km)      eccentricity      inclination (deg)      argper (deg)
1.8838714746e+008  1.9427720614e-001  2.3490037881e+001  2.5349091882e+002

      raan (deg)    true anomaly (deg)    arglat (deg)      period (days)
4.5596571320e-001  1.5290995811e+002  4.6400876928e+001  5.1616340902e+002

      rx (km)      ry (km)      rz (km)      rmag (km)
+1.49990801287589e+008  +1.46776341622975e+008  +6.32690486907151e+007  +2.19188292236025e+008

      vx (kps)      vy (kps)      vz (kps)      vmag (kps)
-1.67423618678588e+001  +1.67906541904715e+001  +8.15381896779511e+000  +2.50742400247327e+001
```

heliocentric coordinates of Mars at arrival
(Earth mean equator and equinox of J2000)

```
-----
TDB calendar date    24-Dec-2003
TDB time             15:23:10.886
TDB Julian Date      2452998.14109821

      sma (km)      eccentricity      inclination (deg)      argper (deg)
2.2793930706e+008  9.3541889964e-002  2.4677224952e+001  3.3297923712e+002

      raan (deg)    true anomaly (deg)    arglat (deg)      period (days)
3.3716583265e+000  7.0759517454e+001  4.3738754577e+001  6.8697217107e+002

      rx (km)      ry (km)      rz (km)      rmag (km)
+1.49990801287589e+008  +1.46776341622975e+008  +6.32690486907151e+007  +2.19188292236025e+008

      vx (kps)      vy (kps)      vz (kps)      vmag (kps)
-1.67423618678588e+001  +1.67906541904715e+001  +8.15381896779511e+000  +2.50742400247327e+001
```

The following output summarizes the orbital characteristics of the initial circular park orbit and the departure hyperbola.

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)

```
-----
park orbit
-----

      sma (km)      eccentricity      inclination (deg)      argper (deg)
6.5634600000e+003  0.0000000000e+000  2.8644284856e+001  0.0000000000e+000

      raan (deg)    true anomaly (deg)    arglat (deg)      period (days)
2.0356395998e+000  1.9503978928e+002  1.9503978928e+002  6.1248647422e-002

      rx (km)      ry (km)      rz (km)      rmag (km)
-6.28154045279459e+003  -1.71891900967894e+003  -8.16439924103122e+002  +6.56346000000000e+003

      vx (kps)      vy (kps)      vz (kps)      vmag (kps)
+2.25553200554695e+000  -6.52893135110580e+000  -3.60775022895557e+000  +7.79296034444086e+000
```

departure hyperbola

```

c3                8.787141 km^2/sec^2
v-infinity        2964.311187 meters/second
asymptote right ascension  349.621254 degrees
asymptote declination    -6.697391 degrees
perigee altitude        185.320000 kilometers
launch azimuth         93.000000 degrees
launch site latitude    28.500000 degrees

TDB calendar date    05-Jun-2003
TDB time             14:46:46.546
TDB Julian Date      2452796.11581651

      sma (km)      eccentricity  inclination (deg)  argper (deg)
-4.5361790600e+004  1.1446913782e+000  2.8644284856e+001  1.9503978928e+002
      raan (deg)    true anomaly (deg)  arglat (deg)
2.0356395998e+000  0.0000000000e+000  1.9503978928e+002

      rx (km)              ry (km)              rz (km)              rmag (km)
-6.28154045279459e+003  -1.71891900967894e+003  -8.16439924103122e+002  +6.56346000000000e+003
      vx (kps)              vy (kps)              vz (kps)              vmag (kps)
+3.30317358566876e+000  -9.56146644276397e+000  -5.28346537786621e+000  +1.14126071811954e+001

```

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)

```

-----
x-component of delta-v    1047.641580 meters/sec
y-component of delta-v   -3032.535092 meters/sec
z-component of delta-v   -1675.715149 meters/sec

delta-v magnitude        3619.646837 meters/sec

```

The following program output summarizes the flight conditions determined by the n-body, numerically integrated optimized solution.

```

=====
optimal n-body solution
=====

```

orbital element targeting

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)

park orbit

```

      sma (km)      eccentricity  inclination (deg)  argper (deg)
6.5634600000e+003  0.0000000000e+000  2.8644284856e+001  0.0000000000e+000
      raan (deg)    true anomaly (deg)  arglat (deg)      period (days)
2.6752954460e+000  1.9474202498e+002  1.9474202498e+002  6.1248647422e-002

      rx (km)              ry (km)              rz (km)              rmag (km)
-6.27206557481663e+003  -1.76044876581034e+003  -8.00637708178284e+002  +6.56346000000000e+003
      vx (kps)              vy (kps)              vz (kps)              vmag (kps)
+2.28960895693892e+000  -6.51429477051137e+000  -3.61273932040786e+000  +7.79296034444086e+000

```

departure hyperbola

c3 8.789419 km^2/sec^2
v-infinity 2964.695455 meters/second
asymptote right ascension 349.992908 degrees
asymptote declination -6.838298 degrees

TDB calendar date 05-Jun-2003
TDB time 14:46:46.546
TDB Julian Date 2452796.11581651

sma (km) eccentricity inclination (deg) argper (deg)
-4.5350032242e+004 1.1447288938e+000 2.8644284856e+001 1.9474202498e+002

raan (deg) true anomaly (deg) arglat (deg)
2.6752954460e+000 3.6000000000e+002 1.9474202498e+002

rx (km) ry (km) rz (km) rmag (km)
-6.27206557481663e+003 -1.76044876581033e+003 -8.00637708178283e+002 +6.56346000000000e+003

vx (kps) vy (kps) vz (kps) vmag (kps)
+3.35310780609998e+000 -9.54011495283504e+000 -5.29081805866954e+000 +1.14127069971405e+001

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)

x-component of delta-v 1063.498849 meters/sec
y-component of delta-v -3025.820182 meters/sec
z-component of delta-v -1678.078738 meters/sec

delta-v magnitude 3619.746653 meters/sec

transfer time 201.332367 days

time and conditions at Mars closest approach
(Mars mean equator and IAU node of epoch)

TDB calendar date 23-Dec-2003
TDB time 22:45:23.059
TDB Julian Date 2452997.44818355

sma (km) eccentricity inclination (deg) argper (deg)
-5.8449948178e+003 1.8554325710e+000 5.9999935323e+001 1.1389611985e+002

raan (deg) true anomaly (deg) arglat (deg)
1.0573184793e+002 6.4522626225e-006 1.1389612631e+002

rx (km) ry (km) rz (km) rmag (km)
-1.65093000113875e+003 -2.56926586115266e+003 +3.95895093575761e+003 +4.99999894456795e+003

vx (kps) vy (kps) vz (kps) vmag (kps)
+2.19013550385939e+000 -4.08068117455877e+000 -1.73495269937855e+000 +4.94557511749085e+000

B-plane coordinates at Mars closest approach
(Mars mean equator and IAU node of epoch)

b-magnitude 9135.093064 kilometers
b dot r -7888.073792
b dot t 4607.408940
b-plane angle 300.289120 degrees

v-infinity 2706.909519 meters/second
r-periapsis 4999.998945 kilometers
decl-asymptote 7.541468 degrees
rasc-asymptote 281.348172 degrees

flight path angle +4.1926180998e-006 degrees

spacecraft heliocentric coordinates at closest approach
(Earth mean equator and equinox of J2000)

TDB calendar date 23-Dec-2003
TDB time 22:45:23.059
TDB Julian Date 2452997.44818355

 sma (km) eccentricity inclination (deg) argper (deg)
1.8440869773e+008 2.3701889822e-001 1.9041643538e+001 2.7135378370e+002

 raan (deg) true anomaly (deg) arglat (deg) period (days)
3.4391040122e+002 1.5010026197e+002 6.1454045668e+001 4.9989916374e+002

 rx (km) ry (km) rz (km) rmag (km)
+1.50991902536460e+008 +1.45763603665344e+008 +6.27810777022794e+007 +2.19059458829212e+008

 vx (kps) vy (kps) vz (kps) vmag (kps)
-1.35222504506597e+001 +1.70343808239226e+001 +4.35553051181676e+000 +2.21808934298002e+001

heliocentric coordinates of Mars at closest approach
(Earth mean equator and equinox of J2000)

TDB calendar date 23-Dec-2003
TDB time 22:45:23.059
TDB Julian Date 2452997.44818355

 sma (km) eccentricity inclination (deg) argper (deg)
2.2793935972e+008 9.3542132819e-002 2.4677224881e+001 3.3297930882e+002

 raan (deg) true anomaly (deg) arglat (deg) period (days)
3.3716581633e+000 7.0368251877e+001 4.3347560695e+001 6.8697240914e+002

 rx (km) ry (km) rz (km) rmag (km)
+1.50989733029159e+008 +1.45767812250595e+008 +6.27794711414507e+007 +2.19060303490482e+008

 vx (kps) vy (kps) vz (kps) vmag (kps)
-1.66287171231117e+001 +1.69011163747310e+001 +8.20141384887623e+000 +2.50883470358891e+001

The final program output summarizes both the geocentric and heliocentric spacecraft trajectory at the Earth's sphere-of-influence.

spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)

TDB calendar date 08-Jun-2003
TDB time 18:21:32.225
TDB Julian Date 2452799.26495631

 sma (km) eccentricity inclination (deg) argper (deg)
-4.5602517470e+004 1.1432735764e+000 2.8506935738e+001 1.9470041288e+002

 raan (deg) true anomaly (deg) arglat (deg)
2.6882352157e+000 1.4947861632e+002 3.4417902920e+002

 rx (km) ry (km) rz (km) rmag (km)
+8.99373827121205e+005 -1.79625739719469e+005 -1.20359099031243e+005 +9.25000000000012e+005

```

          vx (kps)          vy (kps)          vz (kps)          vmag (kps)
+3.03072560024046e+000  -5.32307292858440e-001  -3.65985341139533e-001  +3.09880525169172e+000

```

```

spacecraft heliocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)
-----

```

```

TDB calendar date      08-Jun-2003

```

```

TDB time              18:21:32.225

```

```

TDB Julian Date      2452799.26495631

```

```

          sma (km)          eccentricity          inclination (deg)          argper (deg)
1.9070346952e+008  2.0396240980e-001  2.3500006628e+001  2.5358986510e+002

          raan (deg)          true anomaly (deg)          arglat (deg)          period (days)
5.2629624554e-001  3.7899156831e+000  2.5737978078e+002  5.2571237522e+002

          rx (km)          ry (km)          rz (km)          rmag (km)
-3.19305372880214e+007  -1.36202139877065e+008  -5.90923669668953e+007  +1.51863392403645e+008

          vx (kps)          vy (kps)          vz (kps)          vmag (kps)
+3.16290575455184e+001  -6.53285231214628e+000  -2.96677124003074e+000  +3.24326559526152e+001

```

This script will also create a graphics display of the interplanetary and encounter trajectories. The program prompt for this option is

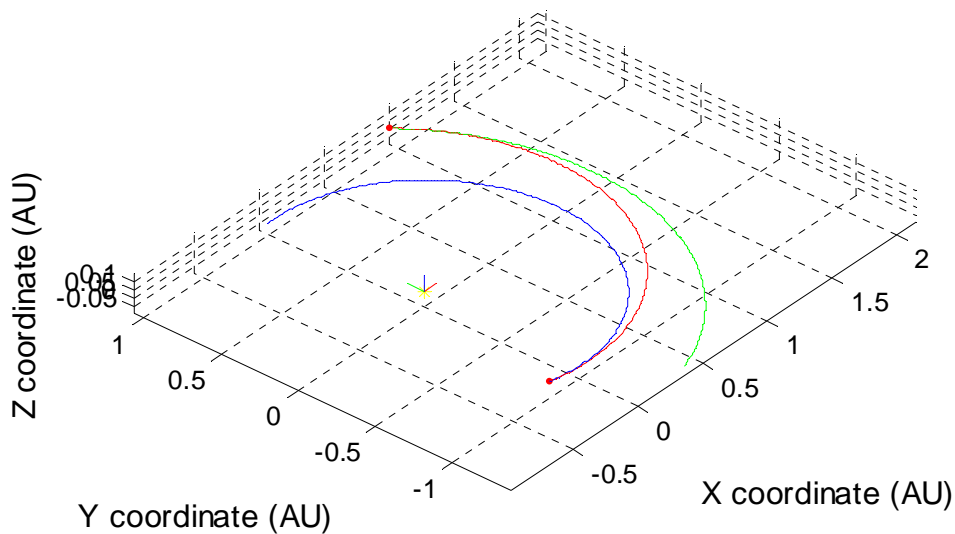
```

would you like to create trajectory graphics (y = yes, n = no)
?

```

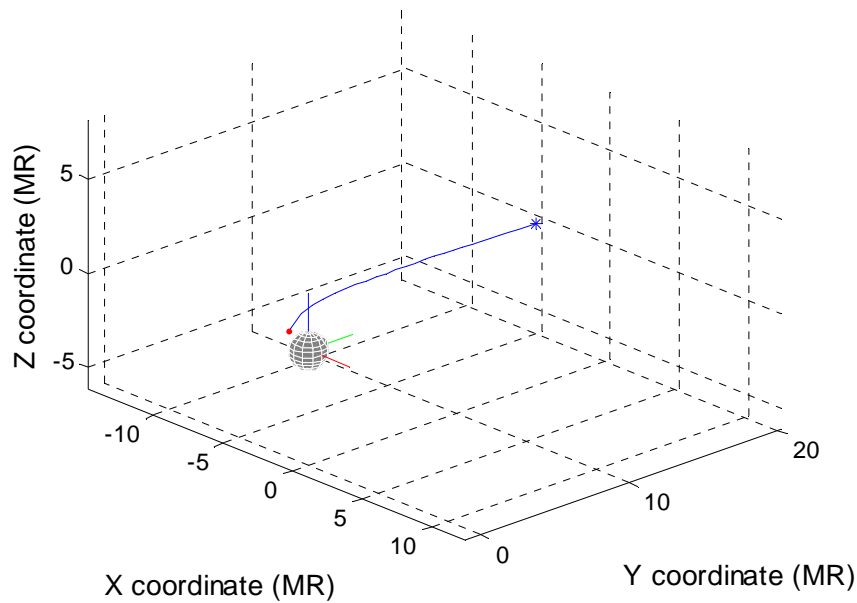
The interactive graphic features of MATLAB will allow the user to rotate and “zoom” the displays. These capabilities allow the user to interactively find the “best” viewpoint as well as verify orbital geometry of the heliocentric and areocentric trajectories. The following is a heliocentric, ecliptic view of the transfer and planetary orbits. The x-axis of this system is red, the y-axis green and the z-axis is blue.

Heliocentric Transfer Trajectory



This next plot is a view of the encounter trajectory in the Mars-centered mean equator and IAU node of epoch coordinate system. The small red dot is the periastron of the encounter hyperbola.

Mars-centered Trajectory



Technical discussion

Solving the two body Lambert problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamics problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis a of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length c of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof(r_i + r_f, c, a)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}} (E - e \sin E)$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}} [E - E_0 - e(\sin E - \sin E_0)]$$

where E is the eccentric anomaly associated with radius r , E_0 is the eccentric anomaly at r_0 , and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sum and difference identities:

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t = \sqrt{\frac{a^3}{\mu}} \left\{ E - E_0 - 2 \sin \frac{E - E_0}{2} \left(e \cos \frac{E + E_0}{2} \right) \right\}$$

With the two substitutions given by

$$e \cos \frac{E + E_0}{2} = \cos \frac{\alpha + \beta}{2}$$

$$\sin \frac{E - E_0}{2} = \sin \frac{\alpha - \beta}{2}$$

the time equation becomes

$$t = \sqrt{\frac{a^3}{\mu}} \left\{ (\alpha - \beta) - 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2} \right\}$$

From the elliptic relationships given by

$$r = a(1 - e \cos E)$$

$$x = a(\cos E - e)$$

$$y = a \sin E \sqrt{1 - e^2}$$

and some more manipulation, we have the following equations:

$$\cos \alpha = \left(1 - \frac{r + r_0}{2a} \right) - \frac{c}{2a} = 1 - \frac{r + r_0 + c}{2a} = 1 - \frac{s}{a}$$

$$\sin \beta = \left(1 - \frac{r + r_0}{2a} \right) + \frac{c}{2a} = 1 - \frac{r + r_0 - c}{2a} = 1 - \frac{s - c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2} = 1 - \frac{r + r_0}{2}$$

$$\sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2} = \sin \frac{E - E_0}{2} \sqrt{1 - \left(e \cos \frac{E + E_0}{2} \right)^2}$$

$$\left(\sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2} \right)^2 = \left(\frac{x - x_0}{2a} \right)^2 + \left(\frac{y - y_0}{2a} \right)^2 = \left(\frac{c}{2a} \right)^2$$

With the use of the half angle formulas given by

$$\sin \frac{\alpha}{2} = \sqrt{\frac{s}{2a}} \quad \sin \frac{\beta}{2} = \sqrt{\frac{s - c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert's theorem

$$t = \sqrt{\frac{a^3}{\mu}} [(\alpha - \beta) - (\sin \alpha - \sin \beta)]$$

A discussion about the angles α and β can be found in "Geometrical Interpretation of the Angles α and β in Lambert's Problem" by J. E. Prussing, *AIAA Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this MATLAB script is based on the method described in "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem" by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48**: 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

Designing the departure hyperbola

This section describes the algorithm used to determine the Earth-centered-inertial (ECI) state vector of a departure hyperbola for interplanetary missions. In the discussion that follows, interplanetary injection is assumed to occur *impulsively* at perigee of the departure hyperbola.

The departure trajectory for interplanetary missions can be defined using the specific (per unit mass) orbital energy C_3 , and the right ascension α_∞ (RLA) and declination δ_∞ (DLA) of the outgoing asymptote. The perigee radius of the departure hyperbola is calculated using the user's value for perigee altitude. The orbital inclination is computed from the user-defined launch azimuth Σ_L and launch site geocentric latitude ϕ_L using this equation

$$i = \cos^{-1}(\cos \phi_L \sin \Sigma_L)$$

The algorithm used to design the departure hyperbola is valid for geocentric orbit inclinations that satisfy the following constraint

$$|i| > |\delta_\infty|$$

If this inequality is not satisfied, the software will print the following error message

```
park orbit error!!
```

```
|inclination| must be > |asymptote declination|
```

The code will also print the inclination of the park orbit, the declination of the departure hyperbola and stop. The user can then change either the azimuth or launch site latitude to satisfy this constraint and restart the script.

A unit vector in the direction of the departure asymptote is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos \delta_\infty \cos \alpha_\infty \\ \cos \delta_\infty \sin \alpha_\infty \\ \sin \delta_\infty \end{Bmatrix}$$

where

α_∞ = right ascension of departure asymptote (RLA)

δ_∞ = declination of departure asymptote (DLA)

The T-axis direction of the B-plane coordinate system is determined from the following vector cross product:

$$\hat{\mathbf{T}} = \hat{\mathbf{S}} \times \hat{\mathbf{u}}_z$$

where $\hat{\mathbf{u}}_z = [0 \ 0 \ 1]^T$ is a unit vector perpendicular to the Earth's equator.

The following cross product operation completes the B-plane coordinate system.

$$\hat{\mathbf{R}} = \hat{\mathbf{S}} \times \hat{\mathbf{T}}$$

The B-plane angle is determined from the orbital inclination of the departure hyperbola i and the declination of the outgoing asymptote according to

$$\cos \theta = \frac{\cos i}{\cos \delta_\infty}$$

The unit angular momentum vector of the departure hyperbola is given by

$$\hat{\mathbf{h}} = \hat{\mathbf{T}} \sin \theta - \hat{\mathbf{R}} \cos \theta$$

The sine and cosine of the true anomaly at infinity are given by the next two equations

$$\cos \theta_\infty = -\frac{\mu}{r_p V_\infty^2 + \mu} \quad \sin \theta_\infty = \sqrt{1 - \cos^2 \theta_\infty}$$

where $V_\infty = \sqrt{C_3} = V_L - V_p$ is the spacecraft's velocity at infinity (v-infinity), V_L is the heliocentric departure velocity determined from the Lambert solution, V_p is the heliocentric velocity of the departure planet, and r_p is the user-specified perigee radius of the departure hyperbola.

A unit vector in the direction of perigee of the departure hyperbola is determined from

$$\hat{\mathbf{r}}_p = \hat{\mathbf{S}} \cos \theta_\infty - (\hat{\mathbf{h}} \times \hat{\mathbf{S}}) \sin \theta_\infty$$

The ECI position vector at perigee is

$$\mathbf{r}_p = r_p \hat{\mathbf{r}}_p$$

The scalar magnitude of the perigee velocity can be determined from

$$V_p = \sqrt{\frac{2\mu}{r_p} + V_\infty^2}$$

A unit vector aligned with the velocity vector at perigee is

$$\hat{\mathbf{v}}_p = \hat{\mathbf{h}} \times \hat{\mathbf{r}}_p$$

The ECI velocity vector at perigee of the departure hyperbola is given by

$$\mathbf{v}_p = V_p \hat{\mathbf{v}}_p$$

Finally, the classical orbital elements of the departure hyperbola can be determined from the position and velocity vectors at perigee. The injection delta-v vector and magnitude can be determined from the velocity difference between the circular park orbit and departure hyperbola at the orbital location of the impulsive maneuver.

Propagating the spacecraft's trajectory

The spacecraft's orbital motion is modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time.

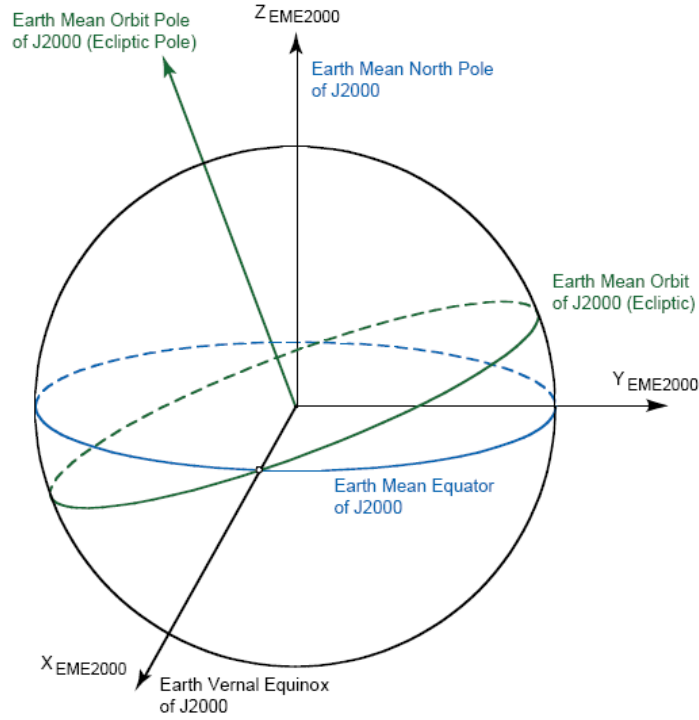


Figure 1. Earth mean equator and equinox of J2000 coordinate system

Geocentric trajectory propagation

This part of the trajectory analysis implements a *special perturbation* technique which numerically integrates the vector system of second-order, nonlinear differential equations of motion of a spacecraft given by

$$\vec{a}(\vec{r}, \vec{v}, t) = \vec{r}''(\vec{r}, \vec{v}, t) = \vec{a}_g(\vec{r}) + \vec{a}_m(\vec{r}, t)$$

where

- t = time
- \vec{r} = inertial position vector of the satellite
- \vec{v} = inertial velocity vector of the satellite
- \vec{a}_g = acceleration due to Earth gravity
- \vec{a}_m = acceleration due to the Moon

The system of six first-order differential equations subject to Earth gravity is defined by

$$\dot{y}_1 = v_x = y_4 \quad \dot{y}_2 = v_y = y_5 \quad \dot{y}_3 = v_z = y_6$$

$$\dot{y}_4 = -\mu \frac{r_x}{r^3} \left\{ 1 + \frac{3 J_2 r_{eq}^2}{2 r^2} \left(1 - \frac{5 r_z^2}{r^2} \right) \right\} \quad \dot{y}_5 = -\mu \frac{r_y}{r^3} \left\{ 1 + \frac{3 J_2 r_{eq}^2}{2 r^2} \left(1 - \frac{5 r_z^2}{r^2} \right) \right\} \quad \dot{y}_6 = -\mu \frac{r_z}{r^3} \left\{ 1 + \frac{3 J_2 r_{eq}^2}{2 r^2} \left(3 - \frac{5 r_z^2}{r^2} \right) \right\}$$

where $r = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{y_1^2 + y_2^2 + y_3^2}$. In these equations, μ and r_{eq} are the gravitational constant and equatorial radius of the Earth, respectively and J_2 is the oblateness gravity coefficient.

The acceleration contribution of the Moon represented by a *point mass* is given by

$$\bar{a}_m(\bar{r}, t) = -\mu_m \left(\frac{\bar{r}_{m-b}}{|\bar{r}_{m-b}|^3} + \frac{\bar{r}_{e-m}}{|\bar{r}_{e-m}|^3} \right)$$

where

$$\begin{aligned} \mu_m &= \text{gravitational constant of the Moon} \\ \bar{r}_{m-b} &= \text{position vector from the Moon to the satellite} \\ \bar{r}_{e-m} &= \text{position vector from the Earth to the Moon} \end{aligned}$$

The `e2m_matlab` script uses Battin's $F(q)$ function described in the next section to compute the point-mass gravitational effect of the Moon.

Heliocentric trajectory propagation

The general vector equation for *point-mass* perturbations such as the Moon or planets is given by

$$\ddot{\mathbf{r}} = -\sum_{j=1}^n \mu_j \left[\frac{\mathbf{d}_j}{d_j^3} + \frac{\mathbf{s}_j}{s_j^3} \right]$$

In this equation, \mathbf{s}_j is the vector from the primary body to the secondary body j , μ_j is the gravitational constant of the secondary body and $\mathbf{d}_j = \mathbf{r} - \mathbf{s}_j$, where \mathbf{r} is the position vector of the spacecraft relative to the primary body.

To avoid numerical problems, use is made of Battin's $F(q)$ function given by

$$F(q_k) = q_k \left[\frac{3 + 3q_k + q_k^2}{1 + (\sqrt{1 + q_k})^3} \right]$$

where

$$q_k = \frac{\mathbf{r}^T (\mathbf{r} - 2\mathbf{s}_k)}{\mathbf{s}_k^T \mathbf{s}_k}$$

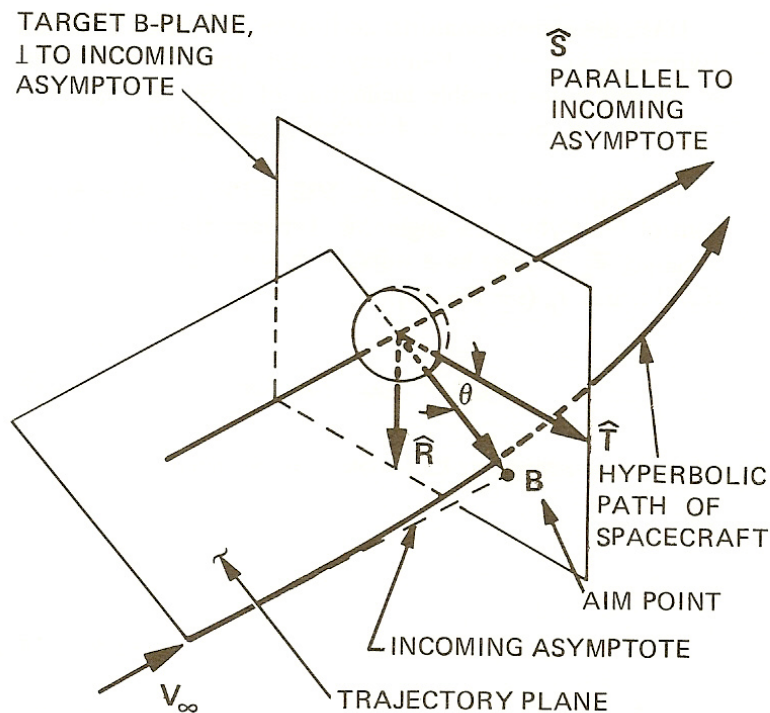
The third-body acceleration can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^n \frac{\mu_k}{d_k^3} \left[\mathbf{r} + F(q_k) \mathbf{s}_k \right]$$

In this MATLAB script the heliocentric coordinates of the Moon, planets and sun are based on the JPL Development Ephemeris DE421. These coordinates are evaluated in the Earth mean equator and equinox of J2000 coordinate system (EME2000).

B-plane targeting

The derivation of B-plane coordinates is described in the classic JPL reports, “A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories” and “Some Orbital Elements Useful in Space Trajectory Calculations”, both by William Kizner. The following diagram illustrates the fundamental geometry of the B-plane coordinate system.



The software solves the B-plane targeting problem by minimizing the delta-v vector at the SOI while satisfying two nonlinear *equality constraint* equations. These constraint equations are the differences between components of the *required* B-plane and the B-plane components *predicted* by the software.

Given the user-defined closest approach radius r_{ca} and orbital inclination i , the incoming v-infinity magnitude v_∞ , and the right ascension α_∞ (RLA) and declination δ_∞ (DLA) of the incoming asymptote vector at moment of closest approach, the following series of equations can be used to determine the required B-plane target vector:

$$\mathbf{B} \cdot \mathbf{T} = b_i \cos \theta \quad \mathbf{B} \cdot \mathbf{R} = b_i \sin \theta$$

where

$$b_i = \sqrt{\frac{2\mu r_{ca}}{v_\infty^2} + r_{ca}^2} = r_{ca} \sqrt{1 + \frac{2\mu}{r_{ca} v_\infty^2}}$$

and

$$\cos \theta = \frac{\cos i}{\cos \delta_\infty}$$
$$\sin \theta = -\sqrt{1 - \cos^2 \theta}$$
$$\sin \delta_\infty = |\hat{\mathbf{s}} \times \hat{\mathbf{z}}| = \sqrt{s_x^2 + s_y^2}$$
$$\hat{\mathbf{z}} = [0 \quad 0 \quad 1]^T$$

The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos \delta_\infty \cos \alpha_\infty \\ \cos \delta_\infty \sin \alpha_\infty \\ \sin \delta_\infty \end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola evaluated at closest approach to Mars.

Important note!!

This technique only works for aerocentric (Mars-centered) orbit inclinations that satisfy

$$|i| > |\delta_\infty|$$

If this inequality is not satisfied, the software will print the following error message

```
b-plane targeting error!!
```

```
|inclination| must be > |asymptote declination|
```

It will also display the actual declination of the asymptote and stop. The user should then edit the input file, include a valid orbital inclination and restart the simulation.

The following computational steps summarize the calculation of the *predicted* B-plane vector from a planet-centered position vector \mathbf{r} and velocity vector \mathbf{v} at closest approach.

angular momentum vectors

$$\mathbf{h} = \mathbf{r} \times \mathbf{v} \quad \hat{\mathbf{h}} = \frac{\mathbf{h}}{|\mathbf{h}|}$$

radius rate

$$\dot{r} = \frac{\mathbf{r} \cdot \mathbf{v}}{|\mathbf{r}|}$$

semiparameter

$$p = \frac{h^2}{\mu}$$

semimajor axis

$$a = \frac{r}{\left(2 - \frac{rv^2}{\mu}\right)}$$

orbital eccentricity

$$e = \sqrt{1 - p/a}$$

true anomaly

$$\cos \theta = \frac{p-r}{er} \quad \sin \theta = \frac{rh}{e\mu}$$

B-plane magnitude

$$B = \sqrt{p|a|}$$

fundamental vectors

$$\hat{\mathbf{z}} = \frac{r\mathbf{v} - \dot{r}\mathbf{r}}{h}$$

$$\hat{\mathbf{p}} = \cos \theta \hat{\mathbf{r}} - \sin \theta \hat{\mathbf{z}}$$

$$\hat{\mathbf{q}} = \sin \theta \hat{\mathbf{r}} + \cos \theta \hat{\mathbf{z}}$$

S vector

$$\mathbf{S} = -\frac{a}{\sqrt{a^2 + b^2}} \hat{\mathbf{p}} + \frac{b}{\sqrt{a^2 + b^2}} \hat{\mathbf{q}}$$

B vector

$$\mathbf{B} = \frac{b^2}{\sqrt{a^2 + b^2}} \hat{\mathbf{p}} + \frac{ab}{\sqrt{a^2 + b^2}} \hat{\mathbf{q}}$$

T vector

$$\mathbf{T} = \frac{(S_y^2, -S_x^2, 0)^T}{\sqrt{S_x^2 + S_y^2}}$$

R vector

$$\mathbf{R} = \mathbf{S} \times \mathbf{T} = (-S_z T_y, S_z T_x, S_x T_y - S_y T_x)^T$$

Targeting to the Mars-centered Periapsis Radius and Orbital Inclination

For this targeting option, the equality constraints enforced by the SNOPT nonlinear programming algorithm are

$$\begin{aligned}r_p - r_{ca} &= 0 \\ \cos i - \hat{\mathbf{h}}_z &= 0\end{aligned}$$

where r_p and i are the user-defined periapsis radius and orbital inclination, respectively, and $\hat{\mathbf{h}}_z$ is the z-component of the unit angular momentum vector at closest approach to Mars.

The mission elapsed time at which the spacecraft reaches closest approach to Mars is predicted using the event prediction capability of the MATLAB `ode45` algorithm. During the numerical integration of the spacecraft's heliocentric equations of motion, the `ode45` numerical method searches for the time at which the flight path angle *with respect to Mars* is nearly zero within a small tolerance. This constraint corresponds to closest approach to Mars.

Closest approach is predicted with the following *mission constraint*

$$\sin \gamma = \frac{\mathbf{r} \cdot \mathbf{v}}{|\mathbf{r} \cdot \mathbf{v}|} = 0$$

where \mathbf{r} and \mathbf{v} are the Mars-centered position and velocity vectors, respectively.

Targeting to user-defined B-plane coordinates

For this targeting option, the nonlinear equality constraints enforced by the SNOPT nonlinear programming algorithm are

$$\begin{aligned}(\mathbf{B} \cdot \mathbf{T})_p - (\mathbf{B} \cdot \mathbf{T})_u &= 0 \\ (\mathbf{B} \cdot \mathbf{R})_p - (\mathbf{B} \cdot \mathbf{R})_u &= 0\end{aligned}$$

where the p subscript refers to coordinates predicted by the software and the u subscript denotes coordinates provided by the user. The *predicted* B-plane coordinates are based on the Mars-centered flight conditions at closest approach.

Targeting to user-defined entry interface (EI) conditions

For this targeting option, the following equations can be used to determine the required B-plane components based on the user-defined EI targets which consist of the inertial flight path angle and altitude relative to a spherical Mars model.

$$\begin{aligned}\mathbf{B} \cdot \mathbf{T} &= b_t \cos \theta \\ \mathbf{B} \cdot \mathbf{R} &= b_t \sin \theta\end{aligned}$$

where

$$b_t = \cos \gamma_{ei} \sqrt{\frac{2\mu r_{ei}}{v_\infty^2} + r_{ei}^2}$$

In these equations, γ_{ei} is the user-defined flight path angle at the entry interface, and r_{ei} is the Mars-centered radius at the entry interface (sum of Mars equatorial radius plus user-defined EI altitude).

Entry interface at Mars is determined during the numerical integration of the spacecraft's heliocentric equations of motion by finding the time at which the difference between the *predicted* Mars-centered flight path angle and the user-defined *inertial* EI flight path angle is zero. This mission constraint is enforced as follows

$$\sin^{-1}\left(\frac{\mathbf{r} \cdot \mathbf{v}}{|\mathbf{r} \cdot \mathbf{v}|}\right) - \gamma_t = 0$$

where \mathbf{r} and \mathbf{v} are the Mars-centered position and velocity vectors, respectively, and γ_t is the user-defined EI flight path angle. The event prediction capability of the MATLAB ode45 algorithm is used to determine the time at which the spacecraft reaches the entry interface condition at Mars.

Geocentric-to-areocentric coordinate transformation

This section describes the transformation of coordinates between the Earth mean equator and equinox of J2000 and areocentric mean equator and IAU node of epoch coordinate systems. This transformation is used to compute the B-plane coordinates at encounter.

A unit vector in the direction of the pole of Mars can be determined from

$$\hat{\mathbf{p}}_{Mars} = \begin{bmatrix} \cos \alpha_p \cos \delta_p \\ \sin \alpha_p \cos \delta_p \\ \sin \delta_p \end{bmatrix}$$

The IAU 2000 right ascension and declination of the pole of Mars in the Earth mean equator and equinox of J2000 (EME2000) coordinate system are given by the following expressions

$$\alpha_p = 317.68143 - 0.1061T$$

$$\delta_p = 52.88650 - 0.0609T$$

where T is the time in Julian centuries given by $T = (JD - 2451545.0)/36525$ and JD is the TDB Julian Date.

The unit vector in the direction of the *IAU-defined* x-axis is computed from

$$\hat{\mathbf{x}} = \hat{\mathbf{p}}_{J2000} \times \hat{\mathbf{p}}_{Mars}$$

where $\hat{\mathbf{p}}_{J2000} = [0\ 0\ 1]^T$ is unit vector in the direction of the pole (z-axis) of the J2000 coordinate system.

The unit vector in the y-axis direction of this coordinate system is

$$\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Mars} \times \hat{\mathbf{x}}$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the Mars-centered mean equator and IAU node of epoch system are as follows:

$$\mathbf{M} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{p}}_{Mars} \end{bmatrix}$$

SNOPT algorithm implementation

This section provides details about the parts of the MATLAB script that solve these nonlinear programming (NLP) problems using the SNOPT algorithm. In the classic patched-conic trajectory optimization problem, the departure and arrival calendar dates are the *control variables* and the user-specified ΔV is the *objective function* or *performance index*.

MATLAB versions of SNOPT for several computer platforms can be found at Professor Philip Gill's web site which is located at <http://scicomp.ucsd.edu/~peg/>. Professor Gill's web site also includes a PDF version of the SNOPT software user's guide.

The SNOPT algorithm requires an initial guess for the control variables. For this problem they are given by

```
xg(1) = jdate1 - jdate0;
xg(2) = jdate2 - jdate0;
xg = xg';
```

where `jdate1` and `jdate2` are the initial user-provided departure and arrival date guesses, and `jdate0` is a reference Julian Date equal to 2451544.5 (January 1, 2000). This offset value is used to *scale* the control variables.

The algorithm also requires lower and upper bounds for the control variables. These are determined from the initial guesses and user-defined search boundaries as follows:

```
% bounds on control variables

xlwr(1) = xg(1) - ddays1;
xupr(1) = xg(1) + ddays1;

xlwr(2) = xg(2) - ddays2;
xupr(2) = xg(2) + ddays2;
```

```

xlwr = xlwr';
xupr = xupr';

xlwr = xlwr';
xupr = xupr';

```

where `d days1` and `d days2` are the user-defined departure and arrival search boundaries, respectively.

The algorithm also requires lower and upper bounds on the objective function. For this problem these bounds are given by

```

% bounds on objective function

flow(1) = 0.0d0;
fupp(1) = +Inf;

```

The actual call to the SNOPT MATLAB interface function is as follows

```

[x, f, inform, xmul, fmul] = snopt(xg, xlwr, xupr, flow, fupp, 'e2m_deltav');

```

where `e2m_deltav` is the name of the MATLAB function that solves Lambert's problem and computes the current value of the objective function.

The following is the MATLAB source code snippet for the departure hyperbola optimization algorithm. The control variables for this part of the computations are the v -infinity magnitude, and the right ascension and declination of the outgoing asymptote. The objective function for these calculations is the scalar magnitude of the departure v -infinity. The initial guess vector `xg` uses the values of v -infinity, RLA and DLA computed by the patched-conic Lambert solution.

The bounds for v -infinity are in units of kilometers per second and the bounds for RLA and DLA are in units of radians. The user can edit these bounds at this point in the source code.

```

% define lower and upper bounds for vinf, rla and dla

xlwr(1) = xg(1) - 0.05;
xupr(1) = xg(1) + 0.05;

xlwr(2) = xg(2) - 10.0 * dtr;
xupr(2) = xg(2) + 10.0 * dtr;

xlwr(3) = xg(3) - 1.0 * dtr;
xupr(3) = xg(3) + 1.0 * dtr;

if (otype == 4)

    % transpose bounds

    xlwr = xlwr';

    xupr = xupr';

end

% bounds on objective function

```

```
flow(1) = 0.0d0;
fupp(1) = +Inf;
% bounds on final b-plane/orbital element equality constraints
flow(2) = 0.0d0;
fupp(2) = 0.0d0;
flow(3) = 0.0d0;
fupp(3) = 0.0d0;
flow = flow';
fupp = fupp';
```

The actual call to the SNOPT MATLAB interface function for this part of the script is as follows

```
[x, f, inform, xmul, fmul] = snopt(xg, xlwr, xupr, flow, fupp, 'e2m_shoot');
```

where `e2m_shoot` is the name of the MATLAB function that implements the simple shooting method used to compute the time and flight characteristics at closest approach or entry interface relative to Mars.

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APPENDIX A

Contents of the Simulation Summary

This appendix is a brief summary of the information contained in the simulation summary screen displays produced by the e2m_matlab software.

The simulation summary screen display contains the following information:

TDB calendar date = TDB calendar date of trajectory event

TDB time = TDB time of trajectory event

TDB Julian Date = Julian Date of trajectory event on TDB time scale

sma (km) = semimajor axis in kilometers

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of periapsis in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (days) = orbital period in days

rx (km) = x-component of the spacecraft's position vector in kilometers

ry (km) = y-component of the spacecraft's position vector in kilometers

rz (km) = z-component of the spacecraft's position vector in kilometers

rmag (km) = scalar magnitude of the spacecraft's position vector in kilometers

vx (kps) = x-component of the spacecraft's velocity vector in kilometers per second

vy (kps) = y-component of the spacecraft's velocity vector in kilometers per second

vz (ksp) = z-component of the spacecraft's velocity vector in kilometers per second

vmag (kps) = scalar magnitude of the spacecraft's velocity vector in kilometers per second

b-magnitude = magnitude of the b-plane vector

b dot r = dot product of the b-vector and r-vector

b dot t = dot product of the b-vector and t-vector

theta = orientation of the b-plane vector in degrees

v-infinity = magnitude of outgoing or incoming v-infinity vector in kilometers/second

r-periapsis = periapsis radius of incoming hyperbola in kilometers

decl-asymptote = declination of incoming v-infinity vector in degrees

rasc-asymptote = right ascension of incoming v-infinity vector in degrees

fpa = flight path angle in degrees

APPENDIX B

Entry Interface Example

This appendix summarizes the trajectory characteristics of a typical entry interface simulation. The following is the input data file (mars_ei.in) for this example.

```
*****
** interplanetary trajectory optimization
** script ==> e2m_matlab.m
** Mars EI mars_ei.in
** July 18, 2011
*****

*****
* simulation type *
*****
 1 = minimize departure delta-v
 2 = minimize arrival delta-v
 3 = minimize total delta-v
 4 = no optimization
-----
1

departure calendar date initial guess (month, day, year)
6,1,2003

departure date search boundary (days)
30

arrival calendar date initial guess (month, day, year)
12,1,2003

arrival date search boundary (days)
30

*****
* geocentric phase modeling
*****

perigee altitude of launch hyperbola (kilometers)
185.32

launch azimuth (degrees)
93.0

launch site latitude (degrees)
28.5

*****
* encounter planet targeting
*****

type of targeting
(1 = B-plane, 2 = orbital elements, 3 = EI conditions)
3

B dot T
10965.197268

B dot R
-6109.036804

radius of closest approach (kilometers)
5000.0

orbital inclination (degrees)
60.0

EI flight path angle (degrees)
-2.0
```

EI altitude (kilometers)
100.0

The following is the two-body and optimal solution for this example.

Earth-to-Mars mission design

=====
two-body Lambert solution
=====

minimize departure delta-v

departure heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)

x-component of delta-v 2895.912618 meters/second
y-component of delta-v -530.389044 meters/second
z-component of delta-v -345.714310 meters/second

delta-v magnitude 2964.311187 meters/second

arrival heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)

x-component of delta-v -2063.021182 meters/second
y-component of delta-v 1164.270846 meters/second
z-component of delta-v 1311.949618 meters/second

delta-v magnitude 2707.913367 meters/second

heliocentric coordinates of the Earth at departure
(Earth mean equator and equinox of J2000)

TDB calendar date 05-Jun-2003

TDB time 14:46:46.546

TDB Julian Date 2452796.11581651

 sma (km) eccentricity inclination (deg) argper (deg)
1.4965147326e+008 1.6237346599e-002 2.3439054671e+001 1.0245240439e+002

 raan (deg) true anomaly (deg) arglat (deg) period (days)
7.2430845695e-004 1.5204742997e+002 2.5449983436e+002 3.6545322928e+002

 rx (km) ry (km) rz (km) rmag (km)
-4.05626079825043e+007 -1.34199491179377e+008 -5.81817199052164e+007 +1.51789133768775e+008

 vx (kps) vy (kps) vz (kps) vmag (kps)
+2.82279246211278e+001 -7.39786254931148e+000 -3.20748439166372e+000 +2.93569762550121e+001

spacecraft heliocentric coordinates after the first impulse
(Earth mean equator and equinox of J2000)

TDB calendar date 05-Jun-2003

TDB time 14:46:46.546

TDB Julian Date 2452796.11581651

 sma (km) eccentricity inclination (deg) argper (deg)
1.8838714746e+008 1.9427720614e-001 2.3490037881e+001 2.5349091882e+002

raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
4.5596571320e-001	5.9131918849e-001	2.5408223801e+002	5.1616340902e+002
rx (km)	ry (km)	rz (km)	rmag (km)
-4.05626079825043e+007	-1.34199491179377e+008	-5.81817199052164e+007	+1.51789133768775e+008
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+3.11238372390479e+001	-7.92825159286771e+000	-3.55319870155481e+000	+3.23137066709359e+001

spacecraft heliocentric coordinates prior to the second impulse
(Earth mean equator and equinox of J2000)

TDB calendar date	24-Dec-2003		
TDB time	15:23:10.886		
TDB Julian Date	2452998.14109821		
sma (km)	eccentricity	inclination (deg)	argper (deg)
1.8838714746e+008	1.9427720614e-001	2.3490037881e+001	2.5349091882e+002
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
4.5596571320e-001	1.5290995811e+002	4.6400876928e+001	5.1616340902e+002
rx (km)	ry (km)	rz (km)	rmag (km)
+1.49990801287589e+008	+1.46776341622975e+008	+6.32690486907151e+007	+2.19188292236025e+008
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-1.46793406853937e+001	+1.56263833449679e+001	+6.84186934966451e+000	+2.25050677759394e+001

spacecraft heliocentric coordinates after the second impulse
(Earth mean equator and equinox of J2000)

TDB calendar date	24-Dec-2003		
TDB time	15:23:10.886		
TDB Julian Date	2452998.14109821		
sma (km)	eccentricity	inclination (deg)	argper (deg)
1.8838714746e+008	1.9427720614e-001	2.3490037881e+001	2.5349091882e+002
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
4.5596571320e-001	1.5290995811e+002	4.6400876928e+001	5.1616340902e+002
rx (km)	ry (km)	rz (km)	rmag (km)
+1.49990801287589e+008	+1.46776341622975e+008	+6.32690486907151e+007	+2.19188292236025e+008
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-1.67423618678588e+001	+1.67906541904715e+001	+8.15381896779510e+000	+2.50742400247327e+001

heliocentric coordinates of Mars at arrival
(Earth mean equator and equinox of J2000)

TDB calendar date	24-Dec-2003		
TDB time	15:23:10.886		
TDB Julian Date	2452998.14109821		
sma (km)	eccentricity	inclination (deg)	argper (deg)
2.2793930706e+008	9.3541889964e-002	2.4677224952e+001	3.3297923712e+002
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
3.3716583265e+000	7.0759517454e+001	4.3738754577e+001	6.8697217107e+002
rx (km)	ry (km)	rz (km)	rmag (km)
+1.49990801287589e+008	+1.46776341622975e+008	+6.32690486907151e+007	+2.19188292236025e+008

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-1.67423618678588e+001	+1.67906541904715e+001	+8.15381896779511e+000	+2.50742400247327e+001

park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)

park orbit

sma (km)	eccentricity	inclination (deg)	argper (deg)
6.5634600000e+003	0.000000000e+000	2.8644284856e+001	0.000000000e+000
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
2.0356395998e+000	1.9503978928e+002	1.9503978928e+002	6.1248647422e-002
rx (km)	ry (km)	rz (km)	rmag (km)
-6.28154045279459e+003	-1.71891900967894e+003	-8.16439924103122e+002	+6.5634600000000e+003
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+2.25553200554695e+000	-6.52893135110580e+000	-3.60775022895557e+000	+7.79296034444086e+000

departure hyperbola

c3	8.787141	km^2/sec^2
v-infinity	2964.311187	meters/second
asymptote right ascension	349.621254	degrees
asymptote declination	-6.697391	degrees
perigee altitude	185.320000	kilometers
launch azimuth	93.000000	degrees
launch site latitude	28.500000	degrees

TDB calendar date	05-Jun-2003
TDB time	14:46:46.546
TDB Julian Date	2452796.11581651

sma (km)	eccentricity	inclination (deg)	argper (deg)
-4.5361790600e+004	1.1446913782e+000	2.8644284856e+001	1.9503978928e+002
raan (deg)	true anomaly (deg)	arglat (deg)	
2.0356395998e+000	0.000000000e+000	1.9503978928e+002	
rx (km)	ry (km)	rz (km)	rmag (km)
-6.28154045279459e+003	-1.71891900967894e+003	-8.16439924103122e+002	+6.5634600000000e+003
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+3.30317358566876e+000	-9.56146644276397e+000	-5.28346537786621e+000	+1.14126071811954e+001

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)

x-component of delta-v	1047.641580	meters/sec
y-component of delta-v	-3032.535092	meters/sec
z-component of delta-v	-1675.715149	meters/sec
delta-v magnitude	3619.646837	meters/sec

please wait, solving b-plane targeting problem ...

Nonlinear constraints	2	Linear constraints	1
Nonlinear variables	3	Linear variables	0
Jacobian variables	3	Objective variables	0
Total constraints	3	Total variables	3

The user has defined 0 out of 6 first derivatives

Major	Minors	Step	nCon	Feasible	Optimal	MeritFunction	nS	Penalty	
0	1		1	6.8E+00	6.8E-04	2.9643112E+00			r
1	0	1.9E-01	2	5.5E+00	6.2E-05	2.9642657E+00			n r
2	1	7.8E-03	3	5.4E+00	1.8E-03	2.9645833E+00	1	5.4E-07	s
3	1	2.2E-01	4	4.2E+00	9.0E-04	2.9646737E+00	1	7.1E-07	
4	1	2.0E-01	5	3.4E+00	1.2E-03	2.9647010E+00	1	8.0E-07	
5	1	3.1E-01	6	2.3E+00	1.1E-03	2.9647254E+00	1	9.3E-07	
6	1	4.6E-01	7	1.3E+00	8.4E-04	2.9647402E+00	1	1.1E-06	
7	1	1.0E+00	8	7.7E-03	4.7E-04	2.9647469E+00	1	1.3E-06	
8	1	1.0E+00	9	2.6E-03	2.5E-04	2.9647467E+00	1	1.3E-06	
9	1	1.0E+00	10	5.1E-04	(8.1E-07)	2.9647461E+00	1	1.3E-06	
9	2	1.0E+00	10	5.1E-04	2.1E-06	2.9647461E+00	1	1.3E-06	c
Major	Minors	Step	nCon	Feasible	Optimal	MeritFunction	nS	Penalty	
10	1	1.0E+00	11	(4.4E-08)	(3.8E-08)	2.9647461E+00	1	1.3E-06	c

SNOPTA EXIT 0 -- finished successfully
 SNOPTA INFO 1 -- optimality conditions satisfied

Problem name

No. of iterations	11	Objective value	2.9647460611E+00
No. of major iterations	10	Linear objective	2.9647460611E+00
Penalty parameter	1.327E-06	Nonlinear objective	0.0000000000E+00
No. of calls to funobj	74	No. of calls to funcon	74
Calls with modes 1,2 (known g)	11	Calls with modes 1,2 (known g)	11
Calls for forward differencing	30	Calls for forward differencing	30
Calls for central differencing	12	Calls for central differencing	12
No. of superbasics	1	No. of basic nonlinears	2
No. of degenerate steps	0	Percentage	.00
Max x	2 6.1E+00	Max pi	3 1.0E+00
Max Primal infeas	0 0.0E+00	Max Dual infeas	2 4.5E-07
Nonlinear constraint violn	2.8E-07		

Solution printed on file 9

=====
 optimal n-body solution
 =====

EI conditions targeting

park orbit and departure hyperbola characteristics
 (Earth mean equator and equinox of J2000)

 park orbit

sma (km)	eccentricity	inclination (deg)	argper (deg)
6.56346000000e+003	0.00000000000e+000	2.8644284856e+001	0.00000000000e+000
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
2.6903998800e+000	1.9473184832e+002	1.9473184832e+002	6.1248647422e-002
rx (km)	ry (km)	rz (km)	rmag (km)
-6.27194393271097e+003	-1.76112772480527e+003	-8.00097252496362e+002	+6.56346000000000e+003
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+2.29000354576512e+000	-6.51406245667044e+000	-3.61290810857509e+000	+7.79296034444086e+000

departure hyperbola

c3 8.789719 km^2/sec^2

flight path angle -2.0003978491e+000 degrees

spacecraft heliocentric coordinates at entry interface
(Earth mean equator and equinox of J2000)

TDB calendar date 23-Dec-2003
TDB time 22:53:30.730
TDB Julian Date 2452997.45382789
sma (km) eccentricity inclination (deg) argper (deg)
1.8526300488e+008 2.3907274334e-001 1.8140231257e+001 2.7906204419e+002
raan (deg) true anomaly (deg) arglat (deg) period (days)
3.3805649965e+002 1.4794454778e+002 6.7006591971e+001 5.0337699586e+002
rx (km) ry (km) rz (km) rmag (km)
+1.50983248982131e+008 +1.45773119835680e+008 +6.27844557902015e+007 +2.19060794823490e+008
vx (kps) vy (kps) vz (kps) vmag (kps)
-1.33074227929162e+001 +1.74558549659383e+001 +3.67544212479328e+000 +2.22554094276875e+001

heliocentric coordinates of Mars at entry interface
(Earth mean equator and equinox of J2000)

TDB calendar date 23-Dec-2003
TDB time 22:53:30.730
TDB Julian Date 2452997.45382789
sma (km) eccentricity inclination (deg) argper (deg)
2.2793935929e+008 9.3542130846e-002 2.4677224881e+001 3.3297930825e+002
raan (deg) true anomaly (deg) arglat (deg) period (days)
3.3716581647e+000 7.0371440879e+001 4.3350749125e+001 6.8697240722e+002
rx (km) ry (km) rz (km) rmag (km)
+1.50981623461024e+008 +1.45776054214411e+008 +6.27834706380841e+007 +2.19061344811477e+008
vx (kps) vy (kps) vz (kps) vmag (kps)
-1.66296466999596e+001 +1.69002188726838e+001 +8.20102730985110e+000 +2.50882322264247e+001

spacecraft geocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)

TDB calendar date 08-Jun-2003
TDB time 18:21:28.581
TDB Julian Date 2452799.26491414
sma (km) eccentricity inclination (deg) argper (deg)
-4.5600973344e+004 1.1432795939e+000 2.8507276552e+001 1.9469048862e+002
raan (deg) true anomaly (deg) arglat (deg)
2.7031848741e+000 1.4947808844e+002 3.4416857706e+002
rx (km) ry (km) rz (km) rmag (km)
+8.99381390808492e+005 -1.79535027410161e+005 -1.20437900181863e+005 +9.25000000000000e+005
vx (kps) vy (kps) vz (kps) vmag (kps)
+3.03079374009248e+000 -5.32010573301202e-001 -3.66256767289706e-001 +3.09885300791657e+000

spacecraft heliocentric coordinates at the Earth SOI
(Earth mean equator and equinox of J2000)

TDB calendar date	08-Jun-2003		
TDB time	18:21:28.581		
TDB Julian Date	2452799.26491414		
sma (km)	eccentricity	inclination (deg)	argper (deg)
1.9070386458e+008	2.0396404948e-001	2.3500188255e+001	2.5359027121e+002
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
5.2785929338e-001	3.7880346689e+000	2.5737830588e+002	5.2571400880e+002
rx (km)	ry (km)	rz (km)	rmag (km)
-3.19306339296161e+007	-1.36202027300211e+008	-5.90924362914067e+007	+1.51863338731534e+008
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+3.16291212795879e+001	-6.53257437667159e+000	-2.96705082114597e+000	+3.24326877005420e+001