A Computer Program for Parametric Analysis of Ballistic Earth-to-Mars Trajectories

This document is the user's manual for a Windows compatible Fortran computer program named `e2m_sweep` that can be used to perform a parametric "sweep" of ballistic interplanetary trajectories from Earth park orbit to encounter at Mars. The user defines the initial departure calendar date, the sweep duration and the step size to use during the analysis.

The software assumes that interplanetary injection occurs *impulsively* from a circular Earth park orbit.

The first part of this computer program computes the ballistic trajectory characteristics using a patched-conic, two-body Lambert solution for the transfer trajectory from Earth to Mars. The second part implements a simple *shooting* method that attempts to adjust the characteristics of the geocentric injection hyperbola while numerically integrating the spacecraft's geocentric and heliocentric equations of motion and "targeting" to the three components of the heliocentric position vector of Mars at a user-defined calendar date.

The spacecraft motion within the Earth's sphere-of-influence (SOI) includes the Earth's J_2 oblate gravity effect and the point-mass perturbation of the sun and moon. The heliocentric equations of motion include the point-mass gravity of the sun and the first seven planets of the solar system. The point-mass gravity of Mars is switched off when the spacecraft is within 25,000 kilometers of Mars.

The major computational steps implemented in this software are as follows:

- solve the two-body, patched-conic interplanetary Lambert problem for the energy C_3, declination (DLA) and asymptote (RLA) of the outgoing or departure hyperbola
- compute the orbital elements of the geocentric departure hyperbola and the components of the interplanetary injection delta-v vector
- using the hyperbola computed in step two as an initial guess, perform geocentric orbit propagation from perigee of the geocentric hyperbola to the Earth's sphere-of-influence
- perform an n-body heliocentric orbit propagation from the Earth's sphere-of-influence to the user-defined arrival calendar date at Mars
- target to all three components of the heliocentric position vector of Mars at the arrival time by adjusting the orbital energy, right ascension and declination of the departure hyperbola

This computer program uses a Powell numerical algorithm to solve the system of nonlinear equations which result from this two-point boundary value problem (TPBVP). The lunar, solar and planetary coordinates required by the software are computed using the JPL DE421 ephemeris. This computer program was written and compiled using Intel Visual Fortran, version 11.1.

Program execution

An input file created by the user can be run from the command line or a simple batch file with a statement similar to the following:

```
e2m_sweep mars09.in
```

If the software is executed without an input file on the command line, the computer program will display the following prompt:

```
****************************************
*           program e2m_sweep          *
*                                      *
* parametric analysis of Earth-to-Mars *
* ballistic interplanetary trajectories *
*                                      *
*           February 16, 2012          *
****************************************

please input the name of the simulation definition file
```

At this point the user should input the name of a valid input file, including the filename extension.

The screen output created by the `e2m_sweep` computer program can be re-directed to a text file with a command line similar to

```
e2m_sweep mars09.in >mars09.txt
```

To create a DOS command window, select **start**, then All Programs, then Accessories and finally Command Prompt**.**

Input file format and contents

This section describes a typical input data file for the software. In the following discussion the actual input file contents are in `courier` font and all explanations are in *times italic* font. Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input.

The first six lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with six and only six initial text lines.

```
********************************************
** parametric sweep of Earth-to-Mars
** ballistic interplanetary trajectories
** Mars '09 example - data file => mars09.in
** program e2m_sweep.exe - February 16, 2012
********************************************
```

The first input is the Barycentric Dynamical Time (TDB) calendar date at which to begin the parametric sweep. Please note that the day value can be a floating point number. Please be sure to include all digits of the calendar year.

```
initial departure calendar date (TDB; month, day, year)
10, 1.0, 2009
```

The next input defines the departure date step size to use during the parametric sweep in days.

```
departure calendar date sweep step size (days)
0.125
```

The next number defines the total duration of the departure calendar date sweep in days.

```
total sweep duration (days)
30
```

The next input defines the TDB arrival calendar date at Mars. Please note that the day value can be a floating point number. Please be sure to include all digits of the calendar year.

```
arrival calendar date (TDB; month, day, year)
9, 3.0, 2010
```

The final set of inputs defines the characteristics of the departure park orbit, the value for the geocentric distance of the Earth's SOI, and which park orbit injection solution to use during the analysis. The orbital inclination of the departure park orbit is with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system.

```
********************************
* geocentric trajectory modeling
********************************

departure circular park orbit altitude (kilometers)
185.32

departure park orbit inclination (degrees)
28.5

geocentric sphere-of-influence distance (kilometers)
925000.0

departure hyperbola solution (1 = ascending, 2 = descending)
1
```

Program example

The following is the solution created with this computer program for this example. The solution is provided in the geocentric and heliocentric, Earth mean equator and equinox of J2000 (EME2000) coordinate system. The time scale is Barycentric Dynamical Time (TDB)

The initial screen output provided by the `e2m_sweep` software displays the heliocentric coordinates of Mars at the user-defined arrival date. The three components of the position vector are the targets used by the shooting method to solve this astrodyamics problem.

```
program e2m_sweep
=================

input data file ==> mars09.in

heliocentric coordinates of Mars at arrival
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date            September  3, 2010

TDB time                 00:00:00.000

TDB Julian date          2455442.50000000

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.152366688265D+01    0.933318668850D-01    0.246773131494D+02    0.333062924916D+03

    raan (deg)         true anomaly (deg)        arglat (deg)          period (days)
 0.336999178066D+01    0.251366265114D+03    0.224429190030D+03    0.686963193912D+03
```

```
     rx (km)                 ry (km)               rz (km)               rmag (km)
 -.157319457677D+09    -.157665380903D+09    -.680680045063D+08    0.232897053087D+09

     vx (kps)                vy (kps)              vz (kps)              vmag (kps)
 0.187756513088D+02    -.128123337554D+02    -.638380555352D+01    0.236100392138D+02
```

For each data point in the departure calendar date sweep, the software will also provide a screen display of the following information;

- the heliocentric coordinates of the Earth at departure
- the trajectory characteristics of the geocentric departure hyperbola
- the spacecraft's heliocentric coordinates at exit from the Earth's sphere-of-influence

The following is the program output for the first data point of this example.

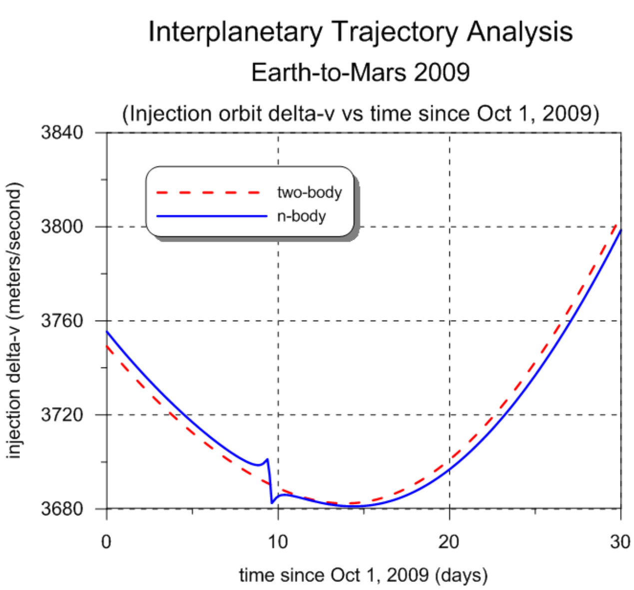
```
heliocentric coordinates of the Earth at departure
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date           October  1, 2009

TDB time                00:00:00.000

TDB Julian date         2455105.50000000

     sma (au)              eccentricity         inclination (deg)       argper (deg)
 0.999308262598D+00    0.169486903641D-01    0.234363347137D+02    0.105208899469D+03

    raan (deg)          true anomaly (deg)        arglat (deg)          period (days)
 0.820988007914D-04    0.262612001664D+03    0.782090113300D+01    0.364877971088D+03

     rx (km)                 ry (km)               rz (km)               rmag (km)
 0.148384649419D+09    0.187001268847D+08    0.810625882179D+07    0.149777870063D+09

     vx (kps)                vy (kps)              vz (kps)              vmag (kps)
 -.454240405752D+01    0.269650252118D+02    0.116891191673D+02    0.297385528600D+02

departure hyperbola trajectory characteristics
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date           October  1, 2009

TDB time                00:00:00.000

TDB Julian date         2455105.50000000

     sma (km)              eccentricity         inclination (deg)       argper (deg)
 -.334825616320D+05    0.119602622022D+01    0.285000000000D+02    0.349422806233D+03

    raan (deg)          true anomaly (deg)        arglat (deg)
 0.342227957360D+03    0.000000000000D+00    0.349422806233D+03

     rx (km)                 ry (km)               rz (km)               rmag (km)
 0.582086542341D+04    -.297759143592D+04    -.574875756024D+03    0.656346000000D+04

     vx (kps)                vy (kps)              vz (kps)              vmag (kps)
 0.506378658799D+01    0.885334468868D+01    0.541678250413D+01    0.115483842802D+02

 c3                   11.9047176242684      km**2/sec**2

 decl-asymptote       19.3016227912034      degrees

 rasc-asymptote       122.059466027731      degrees
```

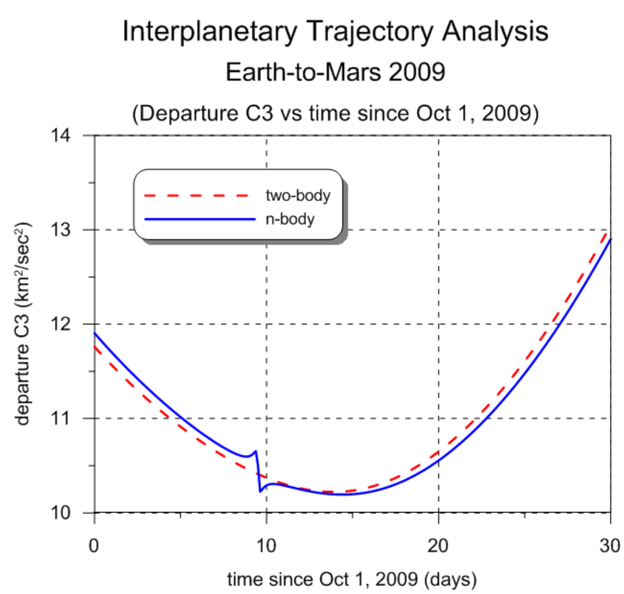
```
spacecraft heliocentric coordinates at Earth SOI
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date            October  3, 2009

TDB time                 18:48:35.355

TDB Julian date          2455108.28374253

     sma (au)              eccentricity        inclination (deg)       argper (deg)
 0.130882045509D+01     0.241187379558D+00     0.233418651063D+02     0.243801201101D+02

    raan (deg)           true anomaly (deg)       arglat (deg)          period (days)
 0.359964928539D+03     0.346547609248D+03     0.109277293580D+02     0.546913366475D+03

     rx (km)                 ry (km)                rz (km)               rmag (km)
 0.146676785502D+09     0.259088103833D+08     0.112192882846D+08     0.149369402122D+09

     vx (kps)                vy (kps)               vz (kps)              vmag (kps)
-.773828476070D+01     0.296001897094D+02     0.127714572148D+02     0.331536182228D+02
```

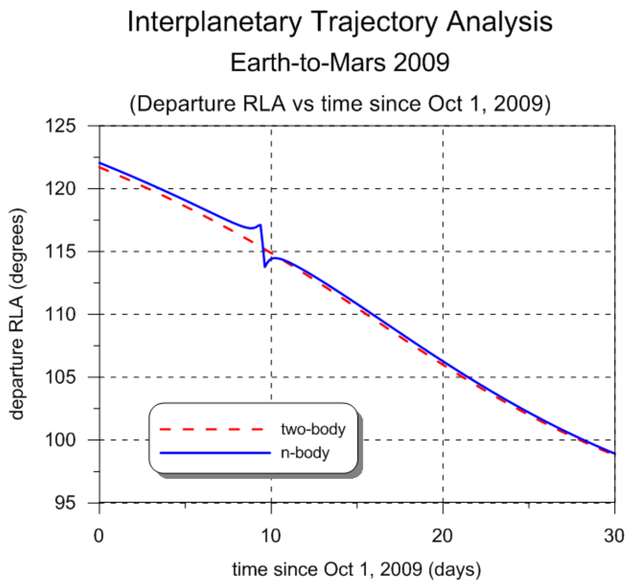
A brief explanation of these data items can be found in Appendix A.

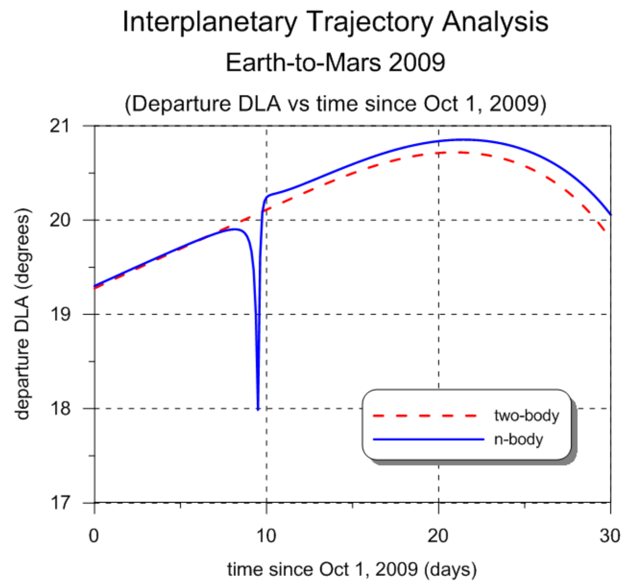
The following plots summarize the parametric sweep characteristics for this example. They illustrate the behavior of the geocentric injection delta-v, and the C3, RLA and DLA of the departure hyperbola for both the two-body and n-body simulations. Each trajectory parameter is displayed as a function of time since the initial departure calendar date of October 1, 2009.

The 'bump" in the n-body trajectory characteristics around nine days into the sweep is due to the effect of the point-mass gravity of the moon on the departure geocentric trajectory.

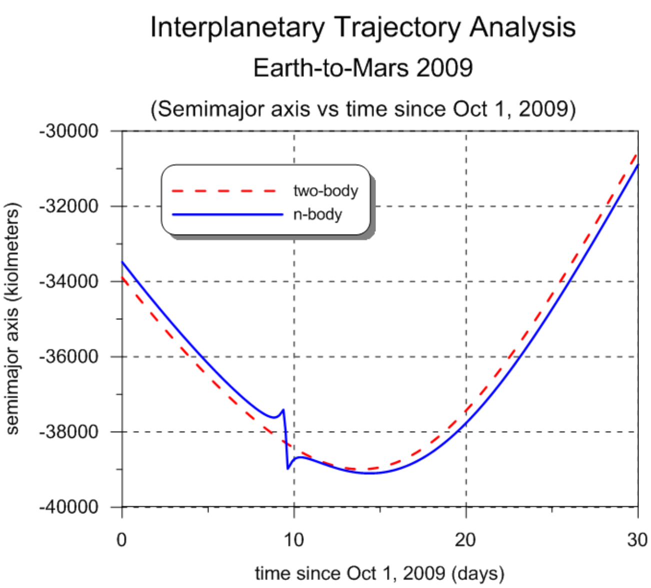


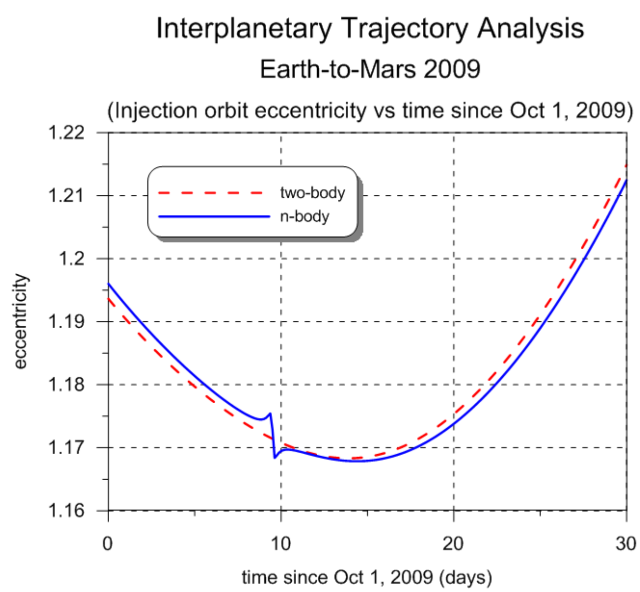


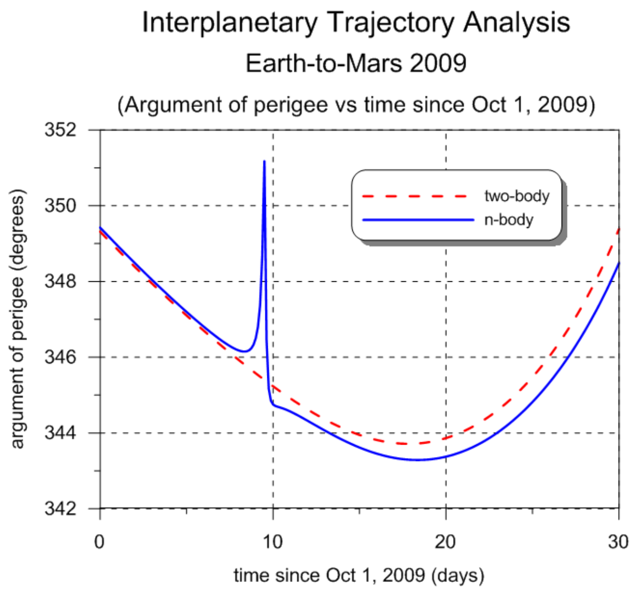


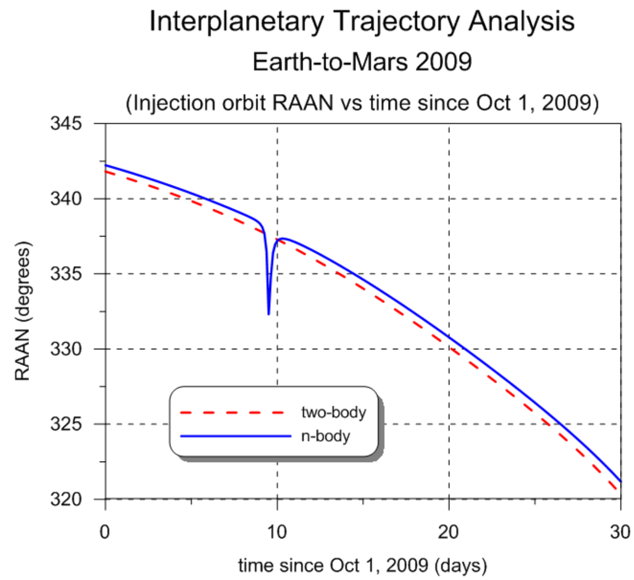


The next four plots illustrate the classical orbital elements of the departure hyperbola. The plots of orbital inclination and true anomaly are not shown since they have constant values for this example. The orbital inclination is constant because the declination of the outgoing asymptote throughout the sweep is always less than the user-defined park orbit inclination. Furthermore, the true anomaly is always zero since injection occurs at perigee of the departure hyperbola.









The `e2m_sweep` computer program will also create two comma-separated-variable (csv) data files. These files summarize the patched-conic, two-body (`e2m_2body.csv`) and integrated, n-body (`e2m_nbody.csv`) trajectory characteristics of the departure hyperbola. The columns of these two data files contain the following information;

`delta-t (days)`	=	elapsed time since initial departure date in days
`C3 launch (km^2/sec^2)`	=	twice the specific (per unit mass) orbital energy of the departure hyperbola in kilometers squared per second squared
`v-inf launch (km/sec)`	=	square root of departure C3 in kilometers per second
`RLA launch (deg)`	=	right ascension of the departure asymptote in degrees
`DLA launch (deg)`	=	declination of the departure asymptote in degrees
`C3 arrival (km^2/sec^2)`	=	twice the specific (per unit mass) orbital energy of the arrival hyperbola in kilometers squared per second squared
`v-inf arrival (km/sec)`	=	square root of arrival C3 in kilometers per second
`RLA arrival (deg)`	=	right ascension of the arrival asymptote in degrees
`DLA arrival (deg)`	=	declination of the arrival asymptote in degrees
`dv-inject (m/s)`	=	magnitude of impulsive injection delta-v in meters per second
`semimajor axis (km)`	=	semimajor axis of departure hyperbola in kilometers
`eccentricity`	=	orbital eccentricity of the departure hyperbola
`inclination (deg)`	=	orbital inclination of the departure hyperbola in degrees
`arg of perigee (deg)`	=	argument of perigee of the departure hyperbola in degrees
`raan (deg)`	=	right ascension of the ascending node of the departure hyperbola in degrees
`true anomaly (deg)`	=	true anomaly of the departure hyperbola in degrees

Please note that all angular elements are with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system.

Technical discussion

Solving the two body Lambert problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamic problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis a of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length c of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof\left(r_i + r_f, c, a\right)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}\left(E - e\sin E\right)}$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}}\left[E - E_0 - e\left(\sin E - \sin E_0\right)\right]$$

where E is the eccentric anomaly associated with radius r, E_0 is the eccentric anomaly at r_0, and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sun and difference identities:

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha - \beta}{2}\cos\frac{a + \beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha - \beta}{2}\sin\frac{a + \beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha - \beta}{2}\cos\frac{a + \beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t = \sqrt{\frac{a^3}{\mu}}\left\{E - E_0 - 2\sin\frac{E - E_0}{2}\left(e\cos\frac{E + E_0}{2}\right)\right\}$$

With the two substitutions given by

$$e\cos\frac{E+E_0}{2}=\cos\frac{\alpha+\beta}{2} \qquad \sin\frac{E-E_0}{2}=\sin\frac{\alpha-\beta}{2}$$

the time equation becomes

$$t=\sqrt{\frac{a^3}{\mu}\left\{(\alpha-\beta)-2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}\right\}}$$

From the elliptic relationships given by

$$r=a\left(1-e\cos E\right)$$

$$x=a\left(\cos E-e\right)$$

$$y=a\sin E\sqrt{1-e^2}$$

and some more manipulation, we have the following equations:

$$\cos\alpha=\left(1-\frac{r+r_0}{2a}\right)-\frac{c}{2a}=1-\frac{r+r_0+c}{2a}=1-\frac{s}{a}$$

$$\sin\beta=\left(1-\frac{r+r_0}{2a}\right)+\frac{c}{2a}=1-\frac{r+r_0-c}{2a}=1-\frac{s-c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}=1-\frac{r+r_0}{2}$$

$$\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}=\sin\frac{E-E_0}{2}\sqrt{1-\left(e\cos\frac{E+E_0}{2}\right)^2}$$

$$\left(\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\right)^2=\left(\frac{x-x_0}{2a}\right)^2+\left(\frac{y-y_0}{2a}\right)^2=\left(\frac{c}{2a}\right)^2$$

With the use of the half angle formulas given by

$$\sin\frac{\alpha}{2}=\sqrt{\frac{s}{2a}} \qquad \sin\frac{\beta}{2}=\sqrt{\frac{s-c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert’s theorem

$$t=\sqrt{\frac{a^3}{\mu}}\left[(\alpha-\beta)-(\sin\alpha-\sin\beta)\right]$$

A discussion about the angles α and β can be found in "Geometrical Interpretation of the Angles α and β in Lambert's Problem" by J. E. Prussing, AIAA *Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this computer program is based on the method described in "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem" by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

Modeling the launch hyperbola

The algorithm used to determine the trajectory characteristics of the launch hyperbola is described in Appendix B. This appendix also describes the situations which determine the number and orbital characteristics of possible departure hyperbolas.

Propagating the spacecraft's trajectory

The spacecraft's orbital motion is modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time.

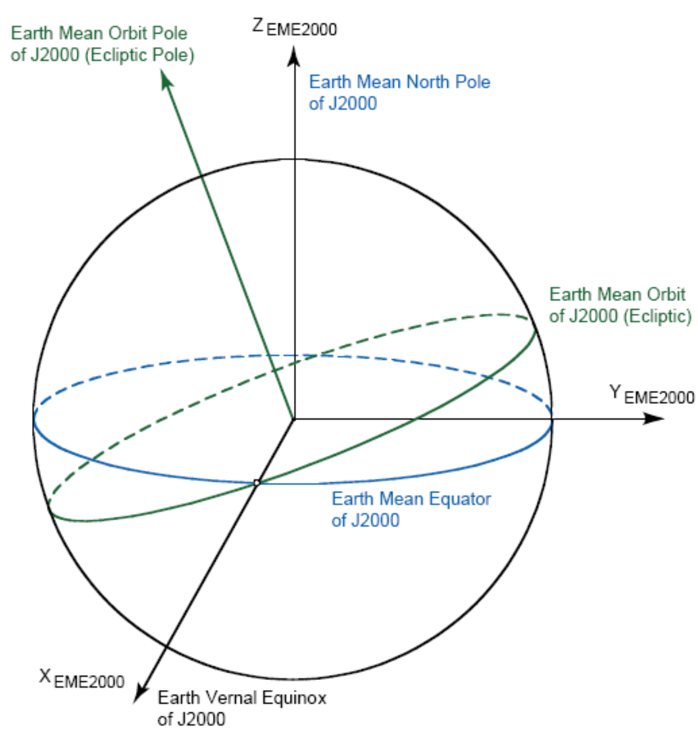


Figure 1. Earth mean equator and equinox of J2000 coordinate system

Geocentric trajectory propagation

This part of the trajectory analysis implements a *special perturbation* technique which numerically integrates the vector system of second-order, nonlinear differential equations of motion of a spacecraft given by

$$\vec{a}(\vec{r},\vec{v},t) = \ddot{\vec{r}}\left(\vec{r},\dot{\vec{r}},t\right) = \vec{a}_g(\vec{r}) + \vec{a}_m(\vec{r},t)$$

where

$t =$ time

$\vec{r} =$ inertial position vector of the satellite

$\vec{v} =$ inertial velocity vector of the satellite

$\vec{a}_g =$ acceleration due to Earth gravity

$\vec{a}_m =$ acceleration due to the Moon

The system of six first-order differential equations subject to Earth gravity is defined by

$$\dot{y}_1 = v_x = y_4 \qquad \dot{y}_2 = v_y = y_5 \qquad \dot{y}_2 = v_z = y_6$$

$$\dot{y}_4 = -\mu\frac{r_x}{r^3}\left\{1+\frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(1-\frac{5r_z^2}{r^2}\right)\right\}$$

$$\dot{y}_5 = -\mu\frac{r_y}{r^3}\left\{1+\frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(1-\frac{5r_z^2}{r^2}\right)\right\}$$

$$\dot{y}_6 = -\mu\frac{r_z}{r^3}\left\{1+\frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(3-\frac{5r_z^2}{r^2}\right)\right\}$$

where $r = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{y_1^2 + y_2^2 + y_3^2}$. In these equations μ and r_{eq} are the gravitational constant and equatorial radius of the Earth, respectively and J_2 is the oblateness gravity coefficient.

The acceleration contribution of the Moon represented by a *point mass* is given by

$$\vec{a}_m(\vec{r},t) = -\mu_m\left(\frac{\vec{r}_{m-b}}{\left|\vec{r}_{m-b}\right|^3} + \frac{\vec{r}_{e-m}}{\left|\vec{r}_{e-m}\right|^3}\right)$$

where

$\mu_m =$ gravitational constant of the Moon

$\vec{r}_{m-b} =$ position vector from the Moon to the satellite

$\vec{r}_{e-m} =$ position vector from the Earth to the Moon

The `e2m_sweep` computer program uses Battin's $F(q)$ function described in the next section to compute the point-mass gravitational effect of the Moon.

Heliocentric trajectory propagation

The general vector equation for *point-mass* perturbations such as the Moon or planets is given by

$$\ddot{\mathbf{r}} = -\sum_{j=1}^{n} \mu_j \left[\frac{\mathbf{d}_j}{d_j^3} + \frac{\mathbf{s}_j}{s_j^3} \right]$$

In this equation, $\mathbf{s}_j$ is the vector from the primary body to the secondary body *j*, μ_j is the gravitational constant of the secondary body, and $\mathbf{d}_j = \mathbf{r} - \mathbf{s}_j$, where **r** is the position vector of the spacecraft relative to the primary body.

To avoid numerical problems, use is made of Battin's $F(q)$ function given by

$$F(q_k) = q_k \left[\frac{3 + 3q_k + q_k^2}{1 + \left(\sqrt{1 + q_k}\right)^3} \right]$$

where

$$q_k = \frac{\mathbf{r}^T (\mathbf{r} - 2\mathbf{s}_k)}{\mathbf{s}_k^T \mathbf{s}_k}$$

The third-body acceleration can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^{n} \frac{\mu_k}{d_k^3} \left[\mathbf{r} + F(q_k) \mathbf{s}_k \right]$$

In this computer program the heliocentric coordinates of the moon, sun and planets are based on the JPL Development Ephemeris DE421. These coordinates are evaluated in the Earth mean equator and equinox of J2000 coordinate system (EME2000).

References and Bibliography

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"Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 2000", *Celestial Mechanics and Dynamical Astronomy*, **82**: 83-110, 2002.

"IERS Conventions (2003)", IERS Technical Note 32, November 2003.

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"Interplanetary Mission Design Handbook, Volume 1, Part 2", JPL Publication 82-43, September 15, 1983.

"A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem", R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990.

"A Computer Simulation of the Orbital Launch Window Problem", Archie C. Young and Pat R. Odom, AIAA 67-615, 1967.

"Launch Parameters for Interplanetary Flights", W. C. Riddell, *American Rocket Society Journal*, December 1960.

APPENDIX A

Contents of the Simulation Summary

This appendix is a brief summary of the information contained in the simulation summary screen displays produced by the `e2m_sweep` software.

The simulation summary screen display contains the following information:

calendar date = TDB calendar date of trajectory event

TDB time = TDB time of trajectory event

TDB Julian Date = Julian Date of trajectory event on TDB time scale

sma (km) = semimajor axis in kilometers

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of periapsis in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (days) = orbital period in days

rx (km) = x-component of the spacecraft's position vector in kilometers

ry (km) = y-component of the spacecraft's position vector in kilometers

rz (km) = z-component of the spacecraft's position vector in kilometers

rmag (km) = scalar magnitude of the spacecraft's position vector in kilometers

vx (kps) = x-component of the spacecraft's velocity vector in kilometers per second

vy (kps) = y-component of the spacecraft's velocity vector in kilometers per second

vz (ksp) = z-component of the spacecraft's velocity vector in kilometers per second

vmag (kps) = scalar magnitude of the spacecraft's velocity vector in kilometers per second

c3 = specific orbital energy in kilometers squared per seconds squared

decl-asymptote = declination of the hyperbolic asymptote in degrees

rasc-asymptote = right ascension of the hyperbolic asymptote in degrees

The geocentric and heliocentric coordinates of the spacecraft are with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system.

APPENDIX B

Interplanetary Injection from a Circular Park Orbit

The algorithm implemented in this scientific simulation assumes that the spacecraft is initially in a circular Earth park orbit. Furthermore, the orbital transfer maneuver is assumed to be impulsive which implies an instantaneous change in velocity but not change in position. In the following discussion, i is the orbital inclination of the initial circular Earth park orbit and δ_∞ is the declination of the outgoing or departure hyperbola.

Whenever $i > |\delta_\infty|$, there will be two coplanar opportunities to establish a departure hyperbola that will satisfy the energy and orientation of the outgoing asymptote. One injection opportunity will occur while the spacecraft is ascending and the other while the spacecraft is descending along the park orbit. For coplanar orbital transfer, the impulse is applied at the perigee of the departure hyperbola.

For the case where $|\delta_\infty| > i$, there will be a single *non-coplanar* injection opportunity. If the software encounters this situation, it will print the following warning and stop.

```
warning: non-coplanar transfer
```

This situation can be avoided by consulting an Earth-to-Mars porkchop plot for time spans during which the absolute value of the declination of the departure hyperbola is always less than the user-provide orbital inclination of the park orbit.

Coplanar Transfer - orientation of the park orbit and departure hyperbola

This section summarizes the equations used to determine the right ascension of the ascending node (RAAN) of the park orbit and the injection true anomaly on the park orbit.

A unit vector is the direction of the departure asymptote is given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where

$\alpha_\infty =$ right ascension of departure asymptote

$\delta_\infty =$ declination of departure asymptote

The angle between the outgoing asymptote and the spin axis of the Earth is given by

$$\beta = \cos^{-1}\left(\hat{\mathbf{s}} \bullet \hat{\mathbf{z}}\right)$$

where $\hat{\mathbf{z}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. Note that $\beta = 90^\circ - \delta_\infty$.

The park orbit right ascension of the ascending node for each opportunity can be determined from

$$\Omega_1 = 180^\circ + \alpha_\infty + \sin^{-1}\left(\frac{\cot\beta}{\tan i}\right)$$

$$\Omega_2 = 360^\circ + \alpha_\infty - \sin^{-1}\left(\frac{\cot\beta}{\tan i}\right)$$

The true anomaly on the park orbit for each injection opportunity can be determined from

$$\theta_1 = \cos^{-1}\left(\frac{\cos\beta}{\sin i}\right) - \eta$$

$$\theta_2 = -\cos^{-1}\left(\frac{\cos\beta}{\sin i}\right) - \eta$$

where

$$\eta = \sin^{-1}\left(\frac{1}{1 + r_p V_\infty^2 / \mu}\right)$$

In the last equation, r_p is the geocentric radius of the park orbit and μ is the gravitational constant of the Earth. The velocity vector at infinity V_∞ is determined from $V_\infty = \sqrt{C_3}$.

For a tangential impulsive injection maneuver that occurs at perigee of the hyperbola, the true anomaly on the hyperbola is zero. Furthermore, since the orbit transfer modeled by this software is coplanar, the right ascension of the ascending node computed above should be the same for both the park orbit and the departure hyperbola. This can be verified by examining the hyperbola's right ascension of the ascending node (RAAN) which is computed using the state vector at injection.

Coplanar Transfer - departure delta-V

The velocity vector at any geocentric position vector $\mathbf{r}$ required to achieve a departure hyperbola defined by V_∞, α_∞ and δ_∞ is given by

$$\mathbf{v}_h = \left(d + \frac{1}{2}V_\infty\right)\hat{\mathbf{s}} + \left(d - \frac{1}{2}V_\infty\right)\hat{\mathbf{r}}$$

where

$$d = \sqrt{\frac{\mu}{(1+\cos\psi)\, r_p} + \frac{V_\infty^2}{4}}$$

and ψ is the angle between the spacecraft's position vector and the departure asymptote unit vector which can be computed using

$$\cos\psi = \hat{\mathbf{s}} \bullet \hat{\mathbf{r}}$$

The injection $\Delta\mathbf{v}$ vector can be determined from the following expression

$$\Delta\mathbf{v} = \mathbf{v}_h - \mathbf{v}_p$$

where $\mathbf{v}_p$ is the inertial velocity vector in the park orbit prior to injection and $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$.

Finally, the scalar injection delta-v is $\Delta v = |\Delta\mathbf{v}|$. The injection delta-v is also given by

$$\Delta v = \sqrt{2\frac{\mu}{r_p} + V_\infty^2} - \sqrt{\frac{\mu}{r_p}}$$

The algorithm in this computer program is based on the equations derived in Chapter 4 of Richard Battin's classic text, *Astronautical Guidance*, and Chapter 11 of *An Introduction to the Mathematics and Methods of Astrodynamics*, also written by Professor Battin and published by the American Institute of Aeronautics and Astronautics (AIAA).