A MATLAB Script for Earth-to-Mars Interplanetary TCM Optimization

This document is the user's manual for a Matlab script called `e2m_tcm` which can be used to solve the classic one impulse, Earth-to-Mars interplanetary trajectory correction maneuver (TCM) optimization problem. The software implements a simple shooting method that attempts to minimize the TCM delta-v while numerically integrating the spacecraft's n-body heliocentric equations of motion and solving for user-defined final conditions at Mars. With the implementation of proper coordinate transformations, the software can be used for other planets.

The important features of this scientific simulation are as follows:

- heliocentric, inertial cartesian equations of motion with point-mass planetary perturbations
- elliptical, non-coplanar planetary orbits
- JPL DE421 planetary ephemeris model
- B-plane coordinates of the encounter hyperbola
- SNOPT non-linear programming algorithm

The final conditions at Mars can be determined using one of the following user-defined options;

- flight path angle, radius and orbital inclination
- user-defined B-plane coordinates
- grazing flyby; user-defined B-plane angle

Typical Input File

The `e2m_tcm` computer program is "data-driven" by a simple user-created text file. The following is a typical input or "simulation definition" file used by the software. This example is an Earth-to-Mars trajectory that begins at the Earth's sphere-of-influence (SOI) and ends at encounter with Mars. It can be found as `e2m1.in` in the software distribution.

In the following discussion the actual input file contents are in `courier` font and all explanations are in *times italic* font. Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input numerical values.

The first five lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with five and only five initial text lines.

```
*********************************
* interplanetary TCM optimization
* script e2m_tcm.m  June 17, 2008
* data file ==> e2m_tcm1.in
*********************************
```

The first program input is the julian date, on the TDB time scale, at which to perform the TCM.

```
TDB julian date of TCM
2452799.264399034436792
```

The next six inputs are the components of the spacecraft's heliocentric orbital elements prior to the actual trajectory correction maneuver. These coordinates are defined relative to the Earth mean equator and equinox of J2000 system (EME2000) at the time specified in the previous input. Please note the proper units for each item.

```
**************************************************
heliocentric EME2000 orbital elements prior to TCM
**************************************************

semimajor axis (kilometers)
190725765.750

orbital eccentricity (non-dimensional)
0.204056802425

orbital inclination (degrees)
23.4926769446

argument of perihelion (degrees)
253.488459798D0

right ascension of the ascending node (degrees)
0.463121032996

true anomaly (degrees)
3.94861259377
```

The next three inputs are the user's initial guess for the components of the TCM delta-v vector. These values should be provided in the units of meters per second. If an initial guess is not available, the user should input 0 for each component.

```
************************************************************
initial guess and bounds for heliocentric TCM delta-v vector
************************************************************

x-component of TCM velocity vector (meters/second)
0.0

y-component of TCM velocity vector (meters/second)
0.0

z-component of TCM velocity vector (meters/second)
0.0
```

Lower and upper bounds for each component of the TCM delta-v vector are defined by the next two user inputs. The units for these two numbers are also meters per second.

```
lower bound for TCM delta-v components (meters/second)
-100.0

upper bound for TCM delta-v components (meters/second)
+100.0
```

The next integer input allows the user to specify the type of final orbit targeting at Mars.

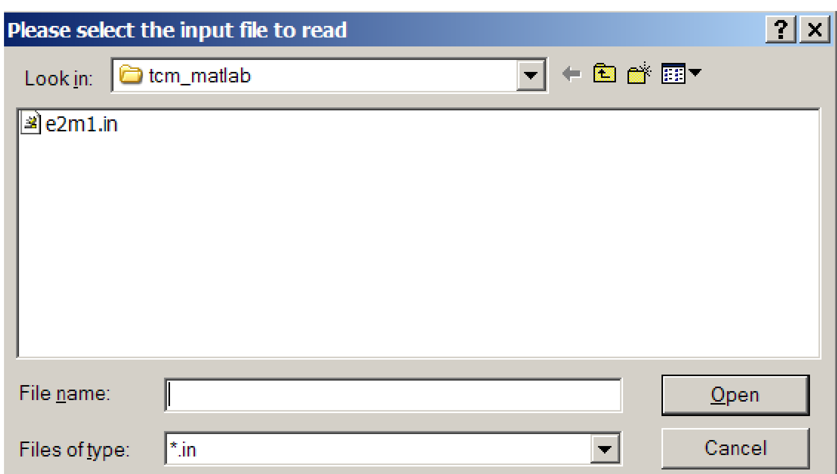
```
*****************************
final orbit targeting options
*****************************
 1 = user-defined flight path angle, radius and orbital inclination
 2 = user-defined B-plane coordinates
 3 = grazing flyby; user-defined b-plane angle
----------------------------------------------
1
```

The next set of inputs defines the desired characteristics of the encounter at Mars. The orbital inclination and B-plane coordinates are defined with respect to the Mars mean equator and IAU (International Astronomical Union) node of epoch coordinate system.

```
***************************
final Mars-centered targets
***************************

flight path angle (degrees)
0.0

periapsis radius (kilometers)
5000.0

orbital inclination (degrees)
60.0

user-defined b dot r target (kilometers)
-7889.908599155647607

user-defined b dot t target (kilometers)
4607.242716469171683

user-defined b-plane angle (degrees)
-60.0
```

Running the script

When the `e2m_tcm` script is started, the software will display the following screen which allows the user to select a data file for processing.



The file type defaults to names with a `*.in` filename extension. However, you can select any `e2m_tcm` compatible ASCII data file.

Optimal Solution

The following is the program output created by the `e2m_tcm` simulation for this example. Please see Appendix A for additional details about this information. The software will also provide the heliocentric coordinates of Mars at the encounter time. These coordinates are provided relative to both the mean Earth equator and ecliptic. The time system is Barycentric Dynamical Time (TDB).

```
e2m_tcm - Earth-to-Mars interplanetary TCM optimization
-------------------------------------------------------

user-defined flight path angle, radius and inclination


time and heliocentric conditions prior to TCM
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date      08-Jun-2003

TDB time           18:20:44.077

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.9072576575e+008  2.0405680243e-001  2.3492676945e+001  2.5348845980e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.6312103300e-001  3.9486125938e+000  2.5743707239e+002  5.2580457394e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -3.19331575698720e+007  -1.36207676243232e+008  -5.90899587841317e+007  +1.51867971768487e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.16260605833221e+001  -6.55290816095610e+000  -2.95930903550773e+000  +3.24330976527609e+001


TCM heliocentric delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

x-component of delta-v          2.865078  meters/second
y-component of delta-v         19.688192  meters/second
z-component of delta-v         -2.656675  meters/second

delta-v magnitude              20.072157  meters/second


time and heliocentric conditions after TCM
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date      08-Jun-2003

TDB time           18:20:44.077

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 1.9070913466e+008  2.0395902253e-001  2.3496631925e+001  2.5362589566e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 5.0756405084e-001  3.7704181608e+000  2.5739631382e+002  5.2573580103e+002

        rx (km)                ry (km)                rz (km)                rmag (km)
 -3.19331575698720e+007  -1.36207676243232e+008  -5.90899587841317e+007  +1.51867971768487e+008

        vx (kps)               vy (kps)               vz (kps)               vmag (kps)
 +3.16289256615583e+001  -6.53321996904229e+000  -2.96196571002918e+000  +3.24321621625149e+001


time and conditions at Mars encounter
(areocentric mean equator and IAU node of epoch)
```

```
------------------------------------------------

calendar date      24-Dec-2003

TDB time           02:09:11.496

TDB julian date    2452997.58971639

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 -5.8469185683e+003  1.8551518147e+000  6.0000063079e+001  1.1398230044e+002

     raan (deg)     true anomaly (deg)   arglat (deg)
 1.0566095703e+002  3.1848830856e-006  1.1398230362e+002

        rx (km)                ry (km)                rz (km)               rmag (km)
 -1.65077591497259e+003  -2.57342736535390e+003  +3.95631655883467e+003  +5.00000302399062e+003

        vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 +2.18743112374296e+000  -4.07936269756536e+000  -1.74075658226361e+000  +4.94532996048810e+000


b-plane coordinates of incoming hyperbola
(areocentric mean equator and IAU node of epoch)
------------------------------------------------

b-magnitude                 9136.150791  kilometers
b dot r                    -7889.428372
b dot t                     4607.186913
b-plane angle                300.283633  degrees
v-infinity                  2706.464169  meters/second
r-periapsis                 5000.003024  kilometers
decl-asymptote                 7.471851  degrees
rasc-asymptote               281.318314  degrees

flight path angle     2.0693966630e-006  degrees


Mars heliocentric coordinates at Mars encounter
(Earth mean equator and equinox of J2000)
-----------------------------------------

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 2.2793934903e+008  9.3542083330e-002  2.4677224896e+001  3.3297929441e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 3.3716581976e+000  7.0448207609e+001  4.3427502017e+001  6.8697236084e+002

        rx (km)                ry (km)                rz (km)               rmag (km)
 +1.50786247364147e+008  +1.45974348844320e+008  +6.28797023302901e+007  +2.19086421030220e+008

        vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 -1.66520073567141e+001  +1.68785999024438e+001  +8.19171554738645e+000  +2.50854676512943e+001


Mars heliocentric coordinates at Mars encounter
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

      sma (km)        eccentricity     inclination (deg)    argper (deg)
 2.2793934903e+008  9.3542083330e-002  1.8493715783e+000  2.8651749011e+002

     raan (deg)     true anomaly (deg)   arglat (deg)       period (days)
 4.9540923804e+001  7.0448207609e+001  3.5696569772e+002  6.8697236084e+002

        rx (km)                ry (km)                rz (km)               rmag (km)
 +1.50786177301423e+008  +1.58941022216520e+008  -3.74261178948078e+005  +2.19086421030238e+008

        vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 -1.66520154578682e+001  +1.87442824718834e+001  +8.01837321205413e-001  +2.50854676513019e+001
```

Technical Discussion

This section provides additional details about the numerical algorithms used in this MATLAB script. The numerical methods discussed here include; propagating the spacecraft's heliocentric trajectory, B-plane coordinates and the Mars-centered coordinate transformation. The software implements a simple *shooting* method that attempts to minimize the magnitude of the heliocentric TCM while numerically integrating the spacecraft's heliocentric equations of motion and targeting to user-defined conditions at Mars encounter.

The fundamental coordinate system used in this simulation is the Earth mean equator and equinox of J2000 (EME2000) and the fundamental time system is Barycentric Dynamical Time (TDB). All units are metric.

The objective function or performance index J for this simulation is the scalar magnitude of the TCM delta-v vector. For this classic trajectory optimization problem, this index is simply

$$J = \Delta V$$

The time and flight conditions at the Martian entry interface (EI) are determined during the numerical integration of the spacecraft's heliocentric equations of motion by finding the time at which the Mars-centered flight path angle matches the user-defined value. This mission constraint is computed as follows

$$\Delta = \sin^{-1}\left(\frac{\mathbf{r} \bullet \mathbf{v}}{\left|\mathbf{r} \bullet \mathbf{v}\right|}\right) - \gamma_t$$

where γ_t is the user-defined flight path angle at the entry interface, and $\mathbf{r}$ and $\mathbf{v}$ are the *predicted* Mars-centered position and velocity vectors, respectively. The numerical method implemented in this script uses the MATLAB `ode45` function to determine the time at which this mission constraint is within a small tolerance of zero.

Heliocentric equations of motion

The general, second-order vector equation of motion subject to *point-mass* perturbations such as the Moon or planets is given by

$$\ddot{\mathbf{r}} = -\sum_{j=1}^{n} \mu_j \left[\frac{\mathbf{d}_j}{d_j^3} + \frac{\mathbf{s}_j}{s_j^3}\right]$$

In this equation, $\mathbf{s}_j$ is the vector from the primary body to the secondary body j, μ_j is the gravitational constant of the secondary body and $\mathbf{d}_j = \mathbf{r} - \mathbf{s}_j$, where $\mathbf{r}$ is the position vector of the spacecraft relative to the primary body.

To avoid numerical problems, use is made of Professor Richard Battin's $F(q)$ function given by

$$F\left(q_k\right) = q_k \left[\frac{3 + 3q_k + q_k^2}{1 + \left(\sqrt{1+q_k}\right)^3} \right]$$

where

$$q_k = \frac{\mathbf{r}^T \left(\mathbf{r} - 2\mathbf{s}_k\right)}{\mathbf{s}_k^T \mathbf{s}_k}$$

The third-body acceleration can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^{n} \frac{\mu_k}{d_k^3} \left[\mathbf{r} + F\left(q_k\right) \mathbf{s}_k \right]$$

The first-order system of equations required by this computer program can be created from the second-order system by the method of *order reduction*. With the following definitions,

$$y_1 = r_x \quad y_2 = r_y \quad y_3 = r_z$$
$$y_4 = v_x \quad y_5 = v_y \quad y_6 = v_z$$

where v_x, v_y, v_z are the velocity vector components of the spacecraft, the first-order system of differential equations is given by

$$\dot{y}_1 = v_x \quad \dot{y}_2 = v_y \quad \dot{y}_3 = v_z$$

$$\dot{y}_4 = -\mu_s \frac{r_x}{r^3} + a_x$$

$$\dot{y}_5 = -\mu_s \frac{r_y}{r^3} + a_y$$

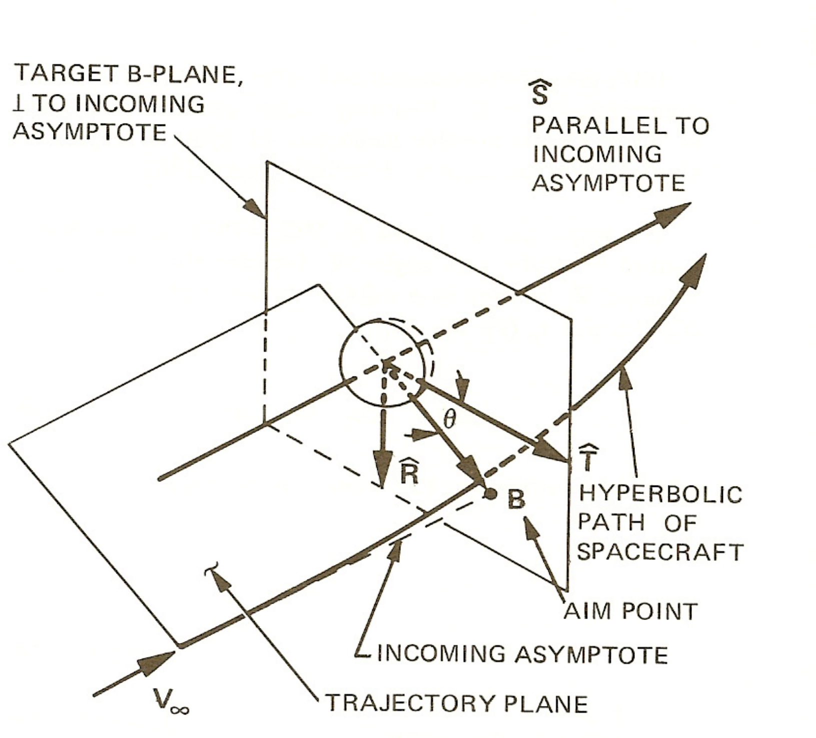
$$\dot{y}_6 = -\mu_s \frac{r_z}{r^3} + a_z$$

In these equations, μ_s is the gravitational constant of the sun, and a_x, a_y and a_z are the *x*, *y* and *z* gravitational contributions of the planets.

In this computer program the heliocentric coordinates of the planets are based on the JPL Development Ephemeris DE421. These coordinates are provided in the Earth mean equator and equinox of J2000 coordinate system (EME2000).

The B-plane

The derivation of B-plane coordinates is described in the classic JPL reports, “A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories” and “Some Orbital Elements Useful in Space Trajectory Calculations”, both by William Kizner. The following diagram illustrates the fundamental geometry of the B-plane coordinate system.



The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola.

The following computational steps summarize the calculation of the *predicted* B-plane vector from a Mars-centered position vector **r** and velocity vector **v** at the sphere-of-influence.

angular momentum vector

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

radius rate

$$\dot{r} = \frac{\mathbf{r} \bullet \mathbf{v}}{|\mathbf{r}|}$$

semiparameter

$$p = \frac{h^2}{\mu}$$

semimajor axis

$$a = \frac{r}{\left(2 - r\,v^2 / \mu\right)}$$

orbital eccentricity

$$e = \sqrt{1 - p/a}$$

true anomaly

$$\cos\theta = \frac{p - r}{e\,r}$$

$$\sin\theta = \frac{\dot{r}\,h}{e\mu}$$

B-plane magnitude

$$B = \sqrt{p\left|a\right|}$$

fundamental vectors

$$\hat{\mathbf{z}} = \frac{r\,\mathbf{v} - \dot{r}\,\mathbf{r}}{h}$$

$$\hat{\mathbf{p}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\mathbf{z}}$$

$$\hat{\mathbf{q}} = \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\mathbf{z}}$$

S vector

$$\mathbf{S} = -\frac{a}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{b}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

B vector

$$\mathbf{B} = \frac{b^2}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{ab}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

T vector

$$\mathbf{T} = \frac{\left(S_y^2, -S_x^2, 0\right)^T}{\sqrt{S_x^2 + S_y^2}}$$

R vector

$$\mathbf{R} = \mathbf{S} \times \mathbf{T} = \left(-S_z T_y,\; S_z T_x,\; S_x T_y - S_y T_x\right)^T$$

Targeting to a Mars-centered periapsis radius and orbital inclination

For this targeting option, the following series of equations can be used to determine the required B-plane target vector:

$$\mathbf{B}\bullet\mathbf{T} = b_t \cos\theta$$

$$\mathbf{B}\bullet\mathbf{R} = b_t \sin\theta$$

where

$$b_t = \cos\gamma_{ei}\sqrt{\frac{2\mu r_{ei}}{v_\infty^2} + r_{ei}^2}$$

In these equations, γ_{ei} is the user-defined flight path angle at the entry interface (EI), and r_{ei} is the user-defined Mars-centered radius at the entry interface.

Also,

$$\cos\theta = \frac{\cos i}{\cos\delta_\infty}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta}$$

$$\sin\delta_\infty = \left|\hat{\mathbf{s}}\times\hat{\mathbf{z}}\right| = \sqrt{s_x^2 + s_y^2}$$

$$\hat{\mathbf{z}} = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$$

The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix}\cos\delta_\infty\cos\alpha_\infty \\ \cos\delta_\infty\sin\alpha_\infty \\ \sin\delta_\infty\end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola.

Targeting to a Mars-centered grazing flyby

The general expression for the periapsis radius of an encounter hyperbola at Mars is given by

$$\tilde{r}_p = \frac{1}{\tilde{v}_\infty^2}\left(\sqrt{1+\tilde{b}_\infty^2\tilde{v}_\infty^4} - 1\right)$$

where the *normalized* quantities are

$$\tilde{r}_p = \text{ normalized periapsis radius} = r_p / r_m$$

$$\tilde{b}_\infty = \text{ normalized b-plane magnitude} = b_\infty / r_m$$

$$\tilde{v}_\infty = \text{ normalized v-infinity speed} = v_\infty / v_{lc}$$

$$v_{lc} = \text{ local circular speed at Mars} = \sqrt{\mu_m / r_m}$$

$$r_m = \text{radius of Mars}$$

$$\mu_m = \text{gravitational constant of Mars}$$

For a grazing flyby, $\tilde{r}_p = 1$ and the normalized B-plane distance is equal to

$$\tilde{b}_\infty = \sqrt{1 + \frac{2}{\tilde{v}_\infty^2}}$$

The required B-plane equality constraints are computed from

$$\mathbf{B \bullet T} = b_\infty \cos\theta$$

$$\mathbf{B \bullet R} = b_\infty \sin\theta$$

where θ is the user-defined B-plane angle of the grazing trajectory. Please note that the B-plane angle is measured positive clockwise from the **T** axis of the B-plane coordinate system. The two equality constraints for this program option are simply the difference between the predicted and required $\mathbf{B \bullet T}$ and $\mathbf{B \bullet R}$ components.

Targeting to user-defined B-plane coordinates

For this program option, the two equality constraints are simply the difference between the predicted and the user-defined $\mathbf{B \bullet T}$ and $\mathbf{B \bullet R}$ components. The B-plane constraints for this and the previous option are scaled by dividing by the equatorial radius of Mars.

Geocentric-to-areocentric coordinate transformation

This section describes the transformation of coordinates between the Earth mean equator and equinox of J2000 and areocentric mean equator and IAU node of epoch coordinate systems. This transformation is used to compute the B-plane coordinates at encounter.

A unit vector in the direction of the pole of Mars can be determined from

$$\hat{\mathbf{p}}_{Mars} = \begin{bmatrix} \cos\alpha_p \cos\delta_p \\ \sin\alpha_p \cos\delta_p \\ \sin\delta_p \end{bmatrix}$$

The IAU 2000 right ascension and declination of the pole of Mars in the EME2000 coordinate system are given by the following expressions

$$\alpha_p = 317.68143 - 0.1061T$$

$$\delta_p = 52.88650 - 0.0609T$$

where T is the time in Julian centuries given by $T = (JD - 2451545.0)/36525$ and JD is the TDB Julian Date.

The unit vector in the direction of the *IAU-defined* x-axis is computed from

$$\hat{\mathbf{x}} = \hat{\mathbf{p}}_{J2000} \times \hat{\mathbf{p}}_{Mars}$$

where $\hat{\mathbf{p}}_{J2000} = [0\ 0\ 1]^T$ is unit vector in the direction of the pole of the J2000 coordinate system.

The unit vector in the y-axis direction of this coordinate system is

$$\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Mars} \times \hat{\mathbf{x}}$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the Mars-centered mean equator and IAU node of epoch system are as follows:

$$\mathbf{M} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{p}}_{Mars} \end{bmatrix}$$

References and Bibliography

"Update to Mars Coordinate Frame Definitions", R. A. Mase, JPL IOM 312.B/015-99, 15 July 1999.

"JPL Planetary Ephemeris DE410", E. M. Standish, JPL IOM 312.N-03-009, 24 April 2003.

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"Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 2000", *Celestial Mechanics and Dynamical Astronomy*, **82**: 83-110, 2002.

"IERS Conventions (2003)", IERS Technical Note 32, November 2003.

"Planetary Constants and Models", R. Vaughan, JPL D-12947, December 1995.

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"Interplanetary Mission Design Handbook, Volume 1, Part 2", JPL Publication 82-43, September 15, 1983.

"Interplanetary Mission Analysis and Design", Stephen Kemble, Praxis Publishing, 2006.

"User's Guide for SNOPT Version 6, A Fortran Package for Large-Scale Nonlinear Programming", Philip E. Gill, Walter Murray and Michael A. Saunders, December 2002.

"User's Guide for SNOPT Version 7, A Fortran Package for Large-Scale Nonlinear Programming", Philip E. Gill, Walter Murray and Michael A. Saunders, March 20, 2006.

APPENDIX A

Contents of the Simulation Summary

This appendix is a brief summary of the information contained in the simulation summary screen displays produced by the `e2m_tcm` software.

The simulation summary screen display contains the following information:

calendar date = calendar date of trajectory event

TDB time = TDB time of trajectory event

TDB Julian date = julian date of trajectory event on TDB time scale

sma (au) = semimajor axis in astronomical unit

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of periapsis in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (days) = orbital period in days

rx (km) = x-component of the spacecraft's position vector in kilometers

ry (km) = y-component of the spacecraft's position vector in kilometers

rz (km) = z-component of the spacecraft's position vector in kilometers

rmag (km) = scalar magnitude of the spacecraft's position vector in kilometers

vx (kps) = x-component of the spacecraft's velocity vector in kilometers per second

vy (kps) = y-component of the spacecraft's velocity vector in kilometers per second

vz (ksp) = z-component of the spacecraft's velocity vector in kilometers per second

vmag (kps) = scalar magnitude of the spacecraft's velocity vector in kilometers per second

deltav-x = x-component of the impulsive TCM velocity vector in meters/second

deltav-y = y-component of the impulsive TCM velocity vector in meters/second

deltav-z = z-component of the impulsive TCM velocity vector in meters/second

delta-v = scalar magnitude of the impulsive TCM delta-v in meters/seconds

b-magnitude = magnitude of the b-plane vector

b dot r = dot product of the b-vector and r-vector

b dot t = dot product of the b-vector and t-vector

theta = orientation of the b-plane vector in degrees

v-infinity = magnitude of incoming v-infinity vector in kilometers/second

r-periapsis = periapsis radius of incoming hyperbola in kilometers

decl-asy = declination of incoming v-infinity vector in degrees

rasc-asy = right ascension of incoming v-infinity vector in degrees

fpa = flight path angle in degrees

The heliocentric coordinates of the spacecraft are with respect to the Earth mean equator and equinox of J2000 coordinate system. The areocentric coordinates of the spacecraft are with respect to the Mars-centered mean equator and IAU node of epoch coordinate system.