A Computer Program for Earth-to-Mars TCM Optimization

This document is the user's manual for a Windows-compatible Fortran computer program called `e2m_tcm_ftn` which can be used to solve the classic one impulse trajectory correction maneuver (TCM) optimization problem for Earth-to-Mars interplanetary missions. The software implements a simple shooting method that attempts to minimize the scalar magnitude of the TCM delta-v while numerically integrating a spacecraft's n-body heliocentric equations of motion and targeting to user-defined final orbit conditions at the entry interface (EI) at Mars.

The important features of this scientific simulation are as follows

- heliocentric, inertial cartesian equations of motion with point-mass planetary perturbations
- elliptical, non-coplanar planetary orbits
- JPL DE421 planetary ephemeris model
- B-plane coordinates of the encounter hyperbola

The final conditions at Mars can be determined using one of the following user-defined options

- flight path angle, radius and orbital inclination
- user-defined B-plane coordinates
- grazing flyby; user-defined B-plane angle

Running the software

An input file created by the user can be run from the command line or a simple batch file with a statement similar to the following:

```
e2m_tcm_ftn e2m1.in
```

If the software is executed without an input file on the command line, the computer program will display the following information screen and file name prompt:

```
**********************************
*       program e2m_tcm_ftn      *
*                                *
*        Earth-to-Mars TCM       *
*     trajectory optimization    *
*                                *
*         January 7, 2012        *
**********************************

please input the name of the simulation definition file
```

At this point the user should provide the name of a compatible input data file, including the filename extension.

To create a DOS command window in Windows 7, select **start**, then **All Programs**, then **Accessories** and finally **Command Prompt**. The size, font and other characteristics of the screen can be controlled by the user with the **c:** icon in the upper left corner of the window. To log into the subdirectory created during the installation of the Fortran executable and support files, type **root:** and then **cd subdirectory** from the DOS command line where root is the name of the root directory, usually c:, and subdirectory is the name of the subdirectory created by the user. The DOS command line prompt looks similar to **C:\e2m_tcm_ftn>_.**

Input file format and contents

The `e2m_tcm_ftn` computer program is "data-driven" by a simple user-created text file. The following is a typical input or "simulation definition" file used by the software. This example is an Earth-to-Mars trajectory that begins at the Earth's sphere-of-influence (SOI) and ends at hyperbolic encounter with Mars. It can be found as `e2m1.in` in the software distribution.

In the following discussion the actual input file contents are in `courier` font and all explanations are in *times italic* font. Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input. ASCII text input is not case sensitive but must be spelled correctly.

The fundamental time system in this scientific simulation is Barycentric Dynamical Time (TDB).

The first five lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with five and only five initial text lines.

```
********************************************
* Earth-to-Mars TCM trajectory optimization
* program e2m_tcm_ftn  January 7, 2012
* data file ==> e2m1.in
********************************************
```

The first program input is the Julian date, on the TDB time scale, at which to perform the TCM.

```
TDB julian date of TCM
2452799.264399034436792d0
```

The next six inputs are the components of the spacecraft's heliocentric orbital elements prior to the actual trajectory correction maneuver. These coordinates are defined relative to the Earth mean equator and equinox of J2000 system (EME2000) at the time specified in the previous input. Please note the proper units for each item.

```
***************************************************
heliocentric EME2000 orbital elements prior to TCM
***************************************************

semimajor axis (kilometers)
190725765.750D0

orbital eccentricity (non-dimensional)
0.204056802425D0

orbital inclination (degrees)
23.4926769446D0

argument of perihelion (degrees)
253.488459798D0
```

```
right ascension of the ascending node (degrees)
0.463121032996D0

true anomaly (degrees)
3.94861259377D0
```

The next three inputs are the user's initial guess for the Cartesian components of the TCM delta-v vector relative to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. These values should be provided in the units of meters per second. If an initial guess is not available, the user should input 0 for each component.

```
************************************************************
initial guess and bounds for heliocentric TCM delta-v vector
************************************************************

x-component of TCM velocity vector (meters/second)
0.0

y-component of TCM velocity vector (meters/second)
0.0

z-component of TCM velocity vector (meters/second)
0.0
```

Lower and upper bounds for each component of the TCM delta-v vector are defined by the next two user inputs. The units for these two numbers are also meters per second.

```
lower bound for TCM delta-v components (meters/second)
-100.0

upper bound for TCM delta-v components (meters/second)
+100.0
```

The next integer input allows the user to specify the type of final orbit targeting at Mars.

```
*****************************
final orbit targeting options
*****************************
 1 = user-defined flight path angle, radius and orbital inclination
 2 = user-defined B-plane coordinates
 3 = grazing flyby; user-defined B-plane angle
----------------------------------------------
1
```

The next set of inputs defines the desired characteristics of the entry interface targets and encounter hyperbola at Mars. The orbital inclination and B-plane coordinates are defined with respect to the Mars mean equator and IAU node of epoch coordinate system.

```
***************************
final Mars-centered targets
***************************

flight path angle (degrees)
-2.0d0

periapsis radius (kilometers)
5000.0d0

orbital inclination (degrees)
60.0d0
```

```
user-defined b dot r target (kilometers)
-7889.908599155647607d0

user-defined b dot t target (kilometers)
4607.242716469171683d0

user-defined b-plane angle (degrees)
-60.0d0
```

The next two inputs set the root-finding and truncation error tolerances used by the numerical methods that calculate the initial guess, predict the time of entrance into the Martian sphere-of-influence, and the time of closest approach to Mars. Smaller values will improve the solution of the corresponding problem at the expense of longer computation times.

```
*********************************************
root-finding and integration algorithm control
*********************************************

root-finding tolerance
1.0d-8

RKF7(8) truncation error tolerance
1.0d-12
```

Optimal Solution

The following is the program output created by the `e2m_tcm_ftn` simulation for this example. Please see Appendix A for additional details about this information. The software will also provide the heliocentric coordinates of Mars at the time of the entry interface. These coordinates are provided relative to both the Earth mean equator and ecliptic of J2000 system.

```
-------------------
program e2m_tcm_ftn
-------------------

input data file ==> e2m1.in

user-defined flight path angle, radius and orbital inclination


time and conditions prior to TCM
(heliocentric EME2000)
----------------------

calendar date           June       8, 2003

TDB time                18:20:44.077

TDB Julian date         2452799.26439903

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 0.190725765750D+09    0.204056802425D+00    0.234926769446D+02    0.253488459798D+03

    raan (deg)         true anomaly (deg)       arglat (deg)         period (min)
 0.463121032996D+00    0.394861259377D+01    0.257437072392D+03    0.757158586476D+06

     rx (km)                ry (km)              rz (km)              rmag (km)
 -.319331575699D+08    -.136207676243D+09    -.590899587841D+08    0.151867971768D+09

     vx (kps)               vy (kps)             vz (kps)             vmag (kps)
 0.316260605833D+02    -.655290816096D+01    -.295930903551D+01    0.324330976528D+02
```

```
TCM delta-v vector and magnitude
(heliocentric EME2000)
----------------------

deltav-x           2.87778574549904      meters/second
deltav-y           19.7131087133301      meters/second
deltav-z          -2.55023251822207      meters/second

delta-v            20.0846207790330      meters/second


time and conditions after TCM
(heliocentric EME2000)
----------------------

calendar date           June       8, 2003

TDB time                18:20:44.077

TDB Julian date         2452799.26439903

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 0.190709092933D+09    0.203958741332D+00    0.234965981051D+02    0.253626932661D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.507184670796D+00    0.376972908329D+01    0.257396661744D+03    0.757059305000D+06

     rx (km)                 ry (km)              rz (km)              rmag (km)
 -.319331575699D+08    -.136207676243D+09    -.590899587841D+08    0.151867971768D+09

     vx (kps)               vy (kps)             vz (kps)              vmag (kps)
 0.316289383691D+02    -.653319505224D+01    -.296185926803D+01    0.324321598150D+02


time and conditions at Mars entry interface
(areocentric mean equator and IAU node of epoch)
------------------------------------------------

calendar date           December  24, 2003

TDB time                02:15:11.510

TDB Julian date         2452997.59388322

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 -.584689495200D+04    0.185435309815D+01    0.599999995774D+02    0.114000694775D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.105658372486D+03    0.356921621400D+03    0.110922316175D+03    0.000000000000D+00

     rx (km)                 ry (km)              rz (km)              rmag (km)
 -.176659071253D+04    -.234950844595D+04    0.404462211937D+04    0.500000007716D+04

     vx (kps)               vy (kps)             vz (kps)              vmag (kps)
 0.215666996409D+01    -.412585075230D+01    -.166807024336D+01    0.494533397265D+01


b-plane coordinates at Mars entry interface
(areocentric mean equator and IAU node of epoch)
------------------------------------------------

b-magnitude          9130.56891560387      kilometers
b dot r             -7884.61688095692
b dot t              4604.35721497810
b-plane angle        300.283524650251      degrees
v-infinity           2.70646963456321      km/sec
```

```
r-periapsis          4995.31281682413      kilometers
decl-asymptote       7.46960142968940      degrees
rasc-asymptote       281.317043562790      degrees

flight path angle   -1.99999396152751      degrees


transfer time        198.329484181013      days


time and conditions of Mars at EI
(heliocentric EME2000)
----------------------

calendar date           December  24, 2003

TDB time                02:15:11.510

TDB Julian date         2452997.59388322

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.152368043519D+01    0.935420818716D-01    0.246772248961D+02    0.332979293981D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 0.337165819865D+01    0.704505612736D+02    0.434298552547D+02    0.686972359416D+03

     rx (km)                ry (km)                rz (km)               rmag (km)
 0.150780252293D+09    0.145980425249D+09    0.628826514074D+08    0.219087190145D+09

     vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 -.166526924348D+02    0.168779366412D+02    0.819142984095D+01    0.250853828640D+02


time and heliocentric conditions of Mars at EI
(Earth mean ecliptic and equinox J2000)
---------------------------------------

calendar date           December  24, 2003

TDB time                02:15:11.510

TDB Julian date         2452997.59388322

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.152368043520D+01    0.935420818720D-01    0.184937157887D+01    0.286517489686D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 0.495409238028D+02    0.704505612732D+02    0.356968050959D+03    0.686972359417D+03

     rx (km)                ry (km)                rz (km)               rmag (km)
 0.150780182227D+09    0.158947770282D+09    -.373972506352D+06    0.219087190145D+09

     vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 -.166527005357D+02    0.187435602938D+02    0.801839020836D+00    0.250853828640D+02
```

Technical Discussion

This section provides additional details about the numerical algorithms used in this computer program. The numerical methods discussed here include; propagating the spacecraft’s heliocentric trajectory, B-plane coordinates and the Mars-centered coordinate transformation. The software implements a simple *shooting* method that attempts to minimize the magnitude of the heliocentric TCM while numerically integrating the spacecraft’s heliocentric equations of motion and targeting to user-defined conditions at

Mars encounter. The scalar magnitude of the TCM maneuver is minimized using a nonlinear programming (NLP) numerical method.

The spacecraft's orbital motion is modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time.

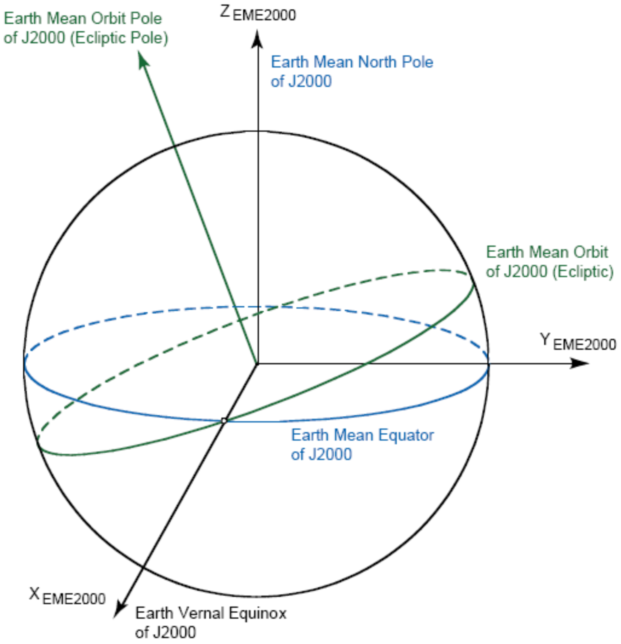


Figure 1. Earth mean equator and equinox of J2000 coordinate system

The objective function or performance index *J* for this simulation is the scalar magnitude of the TCM delta-v vector. For this classic trajectory optimization problem, this index is simply

$$J = \Delta V$$

The time and flight conditions at the Martian entry interface are determined during the numerical integration of the spacecraft's heliocentric equations of motion by finding the time at which the Mars-centered flight path angle matches the user-defined value. This mission constraint is computed as follows

$$\gamma = \sin^{-1}\left(\frac{\mathbf{r}\bullet\mathbf{v}}{\left|\mathbf{r}\bullet\mathbf{v}\right|}\right)$$

where **r** and **v** are the Mars-centered position and velocity vectors, respectively. The numerical method implemented in this computer program consists of a one-dimensional form of Brent's root-finding method imbedded within the RKF7(8) integrator.

Heliocentric equations of motion

The general, second-order vector equation of motion subject to *point-mass* perturbations such as the Moon or planets is given by

$$\ddot{\mathbf{r}} = -\sum_{j=1}^{n} \mu_j \left[\frac{\mathbf{d}_j}{d_j^3} + \frac{\mathbf{s}_j}{s_j^3} \right]$$

In this equation, $\mathbf{s}_j$ is the vector from the primary body to the secondary body *j*, μ_j is the gravitational constant of the secondary body and $\mathbf{d}_j = \mathbf{r} - \mathbf{s}_j$, where **r** is the position vector of the spacecraft relative to the primary body.

To avoid numerical problems, use is made of Professor Richard Battin's $F(q)$ function given by

$$F(q_k) = q_k \left[\frac{3 + 3q_k + q_k^2}{1 + \left(\sqrt{1 + q_k} \right)^3} \right]$$

where

$$q_k = \frac{\mathbf{r}^T (\mathbf{r} - 2\mathbf{s}_k)}{\mathbf{s}_k^T \mathbf{s}_k}$$

The third-body acceleration can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^{n} \frac{\mu_k}{d_k^3} \left[\mathbf{r} + F(q_k) \mathbf{s}_k \right]$$

The first-order system of equations required by this computer program can be created from the second-order system by the method of *order reduction*. With the following definitions,

$$\begin{aligned} y_1 &= r_x \quad & y_2 &= r_y \quad & y_3 &= r_z \\ y_4 &= v_x \quad & y_5 &= v_y \quad & y_6 &= v_z \end{aligned}$$

where v_x, v_y, v_z are the velocity vector components of the spacecraft, the first-order system of differential equations is given by

$$\dot{y}_1 = v_x \quad \dot{y}_2 = v_y \quad \dot{y}_3 = v_z$$

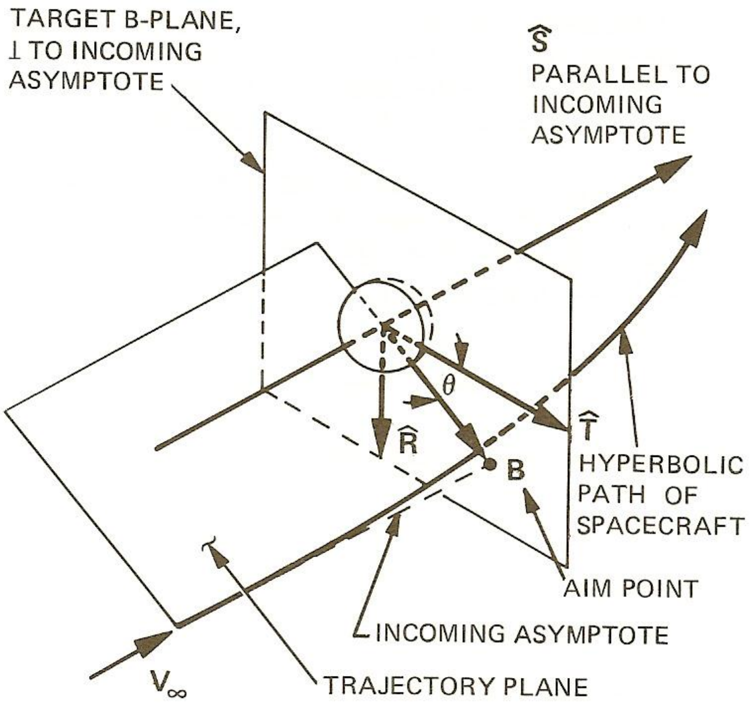
$$\dot{y}_4 = -\mu_s \frac{r_x}{r^3} + a_x \qquad \dot{y}_5 = -\mu_s \frac{r_y}{r^3} + a_y \qquad \dot{y}_6 = -\mu_s \frac{r_z}{r^3} + a_z$$

In these equations, μ_s is the gravitational constant of the sun, and a_x, a_y and a_z are the *x*, *y* and *z* gravitational contributions of the planets.

In this computer program the heliocentric coordinates of the planets are based on the JPL Development Ephemeris DE421. These coordinates are provided in the Earth mean equator and equinox of J2000 coordinate system (EME2000). The binary ephemeris file provided with this computer program was created for use on PC-compatible computers. For other platforms, you will need to create binary files specific to that system. Information and computer programs for creating these files can be found at the JPL solar system FTP site located at ftp://ssd.jpl.nasa.gov/pub/eph/planets/.

The B-plane

The derivation of B-plane coordinates is described in the classic JPL reports, “A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories” and “Some Orbital Elements Useful in Space Trajectory Calculations”, both by William Kizner. The following diagram illustrates the fundamental geometry of the B-plane coordinate system.



The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola.

The following computational steps summarize the calculation of the *predicted* B-plane vector from a Mars-centered position vector **r** and velocity vector **v** at the time of closest approach to Mars.

angular momentum vector

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

radius rate

$$\dot{r} = \mathbf{r} \bullet \mathbf{v} / |\mathbf{r}|$$

semiparameter

$$p = \frac{h^2}{\mu}$$

semimajor axis

$$a = \frac{r}{\left(2 - \frac{r v^2}{\mu}\right)}$$

orbital eccentricity

$$e = \sqrt{1 - p/a}$$

true anomaly

$$\cos\theta = \frac{p - r}{e r} \qquad \sin\theta = \frac{\dot{r} h}{e\mu}$$

B-plane magnitude

$$B = \sqrt{p|a|}$$

fundamental vectors

$$\hat{\mathbf{z}} = \frac{r\mathbf{v} - \dot{r}\mathbf{r}}{h}$$

$$\hat{\mathbf{p}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\mathbf{z}} \qquad \hat{\mathbf{q}} = \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\mathbf{z}}$$

S vector

$$\mathbf{S} = -\frac{a}{\sqrt{a^2 + b^2}}\hat{\mathbf{p}} + \frac{b}{\sqrt{a^2 + b^2}}\hat{\mathbf{q}}$$

B vector

$$\mathbf{B} = \frac{b^2}{\sqrt{a^2 + b^2}}\hat{\mathbf{p}} + \frac{ab}{\sqrt{a^2 + b^2}}\hat{\mathbf{q}}$$

T vector

$$\mathbf{T} = \frac{\left(S_y^2, -S_x^2, 0\right)^T}{\sqrt{S_x^2 + S_y^2}}$$

R vector

$$\mathbf{R} = \mathbf{S} \times \mathbf{T} = \left(-S_z T_y,\ S_z T_x,\ S_x T_y - S_y T_x\right)^T$$

Targeting to a Mars-centered periapsis radius and orbital inclination

For this targeting option, the following series of equations can be used to determine the required B-plane target vector:

$$\mathbf{B} \bullet \mathbf{T} = b_t \cos\theta$$

$$\mathbf{B} \bullet \mathbf{R} = b_t \sin\theta$$

where

$$b_t = \cos\gamma_{ei} \sqrt{\frac{2\mu r_{ei}}{v_\infty^2} + r_{ei}^2}$$

In these equations, γ_{ei} is the user-defined flight path angle at the entry interface, and r_{ei} is the user-defined Mars-centered radius at the entry interface.

Also,

$$\cos\theta = \frac{\cos i}{\cos\delta_\infty}$$

$$\sin\theta = -\sqrt{1 - \cos^2\theta}$$

$$\sin\delta_\infty = \left|\hat{\mathbf{s}} \times \hat{\mathbf{z}}\right| = \sqrt{s_x^2 + s_y^2}$$

$$\hat{\mathbf{z}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola.

Targeting to a Mars-centered grazing flyby

The general expression for the periapsis radius of an encounter hyperbola at Mars is given by

$$\tilde{r}_p = \frac{1}{\tilde{v}_\infty^2}\left(\sqrt{1 + \tilde{b}_\infty^2 \tilde{v}_\infty^4} - 1\right)$$

where the *normalized* quantities are

$\tilde{r}_p =$ normalized periapsis radius $= r_p / r_m$

$\tilde{b}_\infty =$ normalized b-plane magnitude $= b_\infty / r_m$

$\tilde{v}_\infty =$ normalized v-infinity speed $= v_\infty / v_{lc}$

$v_{lc} =$ local circular speed at Mars $= \sqrt{\mu_m / r_m}$

$r_m =$ radius of Mars

$\mu_m =$ gravitational constant of Mars

For a grazing flyby, $\tilde{r}_p = 1$ and the normalized B-plane distance is equal to

$$\tilde{b}_\infty = \sqrt{1 + \frac{2}{\tilde{v}_\infty^2}}$$

The required B-plane equality constraints are computed from

$$\mathbf{B \bullet T} = b_\infty \cos\theta$$

$$\mathbf{B \bullet R} = b_\infty \sin\theta$$

where θ is the user-defined B-plane angle of the grazing trajectory. Please note that the B-plane angle is measured positive clockwise from the **T** axis of the B-plane coordinate system. The two equality constraints for this program option are simply the difference between the predicted and required $\mathbf{B \bullet T}$ and $\mathbf{B \bullet R}$ components.

Targeting to user-defined B-plane coordinates

For this program option, the two equality constraints are simply the difference between the predicted and the user-defined $\mathbf{B \bullet T}$ and $\mathbf{B \bullet R}$ components. The B-plane constraints for this and the previous option are scaled by dividing by the equatorial radius of Mars.

Geocentric-to-areocentric coordinate transformation

This section describes the transformation of coordinates between the Earth mean equator and equinox of J2000 and areocentric mean equator and IAU node of epoch coordinate systems. This transformation is used to compute the B-plane coordinates at encounter.

A unit vector in the direction of the pole of Mars can be determined from

$$\hat{\mathbf{p}}_{Mars} = \begin{bmatrix} \cos\alpha_p \cos\delta_p \\ \sin\alpha_p \cos\delta_p \\ \sin\delta_p \end{bmatrix}$$

The IAU 2000 right ascension and declination of the pole of Mars in the EME2000 coordinate system are given by the following expressions

$$\alpha_p = 317.68143 - 0.1061T$$

$$\delta_p = 52.88650 - 0.0609T$$

where T is the time in Julian centuries given by $T = (JD - 2451545.0)/36525$ and JD is the TDB Julian Date.

The unit vector in the direction of the *IAU-defined* x-axis is computed from

$$\hat{\mathbf{x}} = \hat{\mathbf{p}}_{J2000} \times \hat{\mathbf{p}}_{Mars}$$

where $\hat{\mathbf{p}}_{J2000} = [0\;0\;1]^T$ is unit vector in the direction of the pole of the J2000 coordinate system.

The unit vector in the y-axis direction of this coordinate system is

$$\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Mars} \times \hat{\mathbf{x}}$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the Mars-centered mean equator and IAU node of epoch system are as follows:

$$\mathbf{M} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{p}}_{Mars} \end{bmatrix}$$

Algorithm resources

“Update to Mars Coordinate Frame Definitions”, R. A. Mase, JPL IOM 312.B/015-99, 15 July 1999.

“The Planetary and Lunar Ephemeris DE 421”, W. M. Folkner, J. G. Williams, and D. H. Boggs, JPL IOM 343R-08-003, 31 March 2008.

“Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 2000”, *Celestial Mechanics and Dynamical Astronomy*, **82**: 83-110, 2002.

“IERS Conventions (2003)”, IERS Technical Note 32, November 2003.

“Planetary Constants and Models”, R. Vaughan, JPL D-12947, December 1995.

“Preliminary Mars Planetary Constants and Models for Mars Sample Return”, D. Lyons, JPL IOM 312/99.DTL-1, 20 January 1999.

“Interplanetary Mission Design Handbook, Volume 1, Part 2”, JPL Publication 82-43, September 15, 1983.

“Interplanetary Mission Analysis and Design”, Stephen Kemble, Praxis Publishing, 2006.

“Optimal Interplanetary Orbit Transfers by Direct Transcription”, John T. Betts, *The Journal of the Astronautical Sciences*, Vol. 42, No. 3, July-September 1994, pp. 247-268.

“User’s Guide for SNOPT Version 6, A Fortran Package for Large-Scale Nonlinear Programming”, Philip E. Gill, Walter Murray and Michael A. Saunders, December 2002.

APPENDIX A

Contents of the Simulation Summary

This appendix is a brief summary of the information contained in the simulation summary screen displays produced by the `e2m_tcm_ftn` software.

The simulation summary screen display contains the following information:

calendar date = calendar date of trajectory event

TDB time = TDB time of trajectory event

TDB Julian date = julian date of trajectory event on TDB time scale

sma (au) = semimajor axis in astronomical unit

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of periapsis in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (days) = orbital period in days

rx (km) = x-component of the spacecraft’s position vector in kilometers

ry (km) = y-component of the spacecraft’s position vector in kilometers

rz (km) = z-component of the spacecraft’s position vector in kilometers

rmag (km) = scalar magnitude of the spacecraft’s position vector in kilometers

vx (kps) = x-component of the spacecraft’s velocity vector in kilometers per second

vy (kps) = y-component of the spacecraft’s velocity vector in kilometers per second

vz (ksp) = z-component of the spacecraft’s velocity vector in kilometers per second

vmag (kps) = scalar magnitude of the spacecraft’s velocity vector in kilometers per second

deltav-x = x-component of the impulsive TCM velocity vector in meters/second

deltav-y = y-component of the impulsive TCM velocity vector in meters/second

deltav-z = z-component of the impulsive TCM velocity vector in meters/second

delta-v = scalar magnitude of the impulsive TCM delta-v in meters/seconds

b-magnitude = magnitude of the b-plane vector

b dot r = dot product of the b-vector and r-vector

b dot t = dot product of the b-vector and t-vector

theta = orientation of the b-plane vector in degrees

v-infinity = magnitude of incoming v-infinity vector in kilometers/second

r-periapsis = periapsis radius of incoming hyperbola in kilometers

decl-asy = declination of incoming v-infinity vector in degrees

rasc-asy = right ascension of incoming v-infinity vector in degrees

fpa = flight path angle in degrees

The heliocentric coordinates of the spacecraft are with respect to the Earth mean equator and equinox of J2000 coordinate system. The areocentric coordinates of the spacecraft are with respect to the Mars-centered mean equator and IAU node of epoch coordinate system.

APPENDIX B

Additional Program Examples

This appendix provides typical output from the `e2m_tcm_ftn` computer program for the other types of user-defined final orbit targeting options.

The first example illustrates the solution for the “user-defined B-plane coordinates” program option.

```
-------------------
program e2m_tcm_ftn
-------------------

input data file ==> e2m1.in

user-defined B-plane coordinates


time and conditions prior to TCM
(heliocentric EME2000)
----------------------

calendar date           June       8, 2003

TDB time                18:20:44.077

TDB Julian date         2452799.26439903

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 0.190725765750D+09    0.204056802425D+00    0.234926769446D+02    0.253488459798D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.463121032996D+00    0.394861259377D+01    0.257437072392D+03    0.757158586476D+06

     rx (km)                ry (km)              rz (km)              rmag (km)
 -.319331575699D+08    -.136207676243D+09    -.590899587841D+08    0.151867971768D+09

     vx (kps)               vy (kps)             vz (kps)              vmag (kps)
 0.316260605833D+02    -.655290816096D+01    -.295930903551D+01    0.324330976528D+02


TCM delta-v vector and magnitude
(heliocentric EME2000)
----------------------

deltav-x           2.87583041705154      meters/second
deltav-y           19.7105953931028      meters/second
deltav-z          -2.56326184040979      meters/second

delta-v            20.0835326225429      meters/second


time and conditions after TCM
(heliocentric EME2000)
----------------------

calendar date           June       8, 2003

TDB time                18:20:44.077

TDB Julian date         2452799.26439903

     sma (km)             eccentricity       inclination (deg)       argper (deg)
```

```
 0.190709089188D+09    0.203958738207D+00    0.234966023279D+02    0.253626808383D+03

    raan (deg)        true anomaly (deg)       arglat (deg)         period (min)
 0.507232041307D+00    0.376980991863D+01    0.257396618301D+03    0.757059282696D+06

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.319331575699D+08    -.136207676243D+09    -.590899587841D+08    0.151867971768D+09

     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 0.316289364137D+02    -.653319756556D+01    -.296187229735D+01    0.324321596043D+02


time and conditions at Mars entry interface
(areocentric mean equator and IAU node of epoch)
------------------------------------------------

calendar date           December  24, 2003

TDB time                02:14:13.280

TDB Julian date         2452997.59320926

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 -.584688372340D+04    0.185522345656D+01    0.600011116817D+02    0.113983346377D+03

    raan (deg)        true anomaly (deg)       arglat (deg)         period (min)
 0.105658514716D+03    0.356922111757D+03    0.110905458134D+03    0.000000000000D+00

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.176892959872D+04    -.235062666665D+04    0.404923477893D+04    0.500508341050D+04

     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 0.215539050136D+01    -.412511308029D+01    -.166633635452D+01    0.494357593094D+01


b-plane coordinates at Mars entry interface
(areocentric mean equator and IAU node of epoch)
------------------------------------------------

b-magnitude          9136.59365892181      kilometers
b dot r             -7889.90869587079
b dot t              4607.24260909633
b-plane angle        300.282415319264      degrees
v-infinity           2.70647223337079      km/sec
r-periapsis          5000.39210803575      kilometers
decl-asymptote       7.46980412143243      degrees
rasc-asymptote       281.317261391677      degrees

flight path angle   -2.00000410940689      degrees


transfer time        198.328810221516      days


time and conditions of Mars at EI
(heliocentric EME2000)
----------------------

calendar date           December  24, 2003

TDB time                02:14:13.280

TDB Julian date         2452997.59320926

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.152368043554D+01    0.935420821074D-01    0.246772248961D+02    0.332979294050D+03
```

```
    raan (deg)        true anomaly (deg)      arglat (deg)         period (days)
 0.337165819848D+01    0.704501805831D+02    0.434294746332D+02    0.686972359647D+03

     rx (km)                ry (km)              rz (km)               rmag (km)
 0.150781221977D+09    0.145979442442D+09    0.628821744182D+08    0.219087065744D+09

     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 -.166525816297D+02    0.168780439213D+02    0.819147605301D+01    0.250853965779D+02


time and heliocentric conditions of Mars at EI
(Earth mean ecliptic and equinox J2000)
---------------------------------------

calendar date           December  24, 2003

TDB time                02:14:13.280

TDB Julian date         2452997.59320926

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.152368043554D+01    0.935420821078D-01    0.184937157877D+01    0.286517489754D+03

    raan (deg)        true anomaly (deg)      arglat (deg)         period (days)
 0.495409238030D+02    0.704501805828D+02    0.356967670337D+03    0.686972359648D+03

     rx (km)                ry (km)              rz (km)               rmag (km)
 0.150781151912D+09    0.158946678839D+09    -.374019197511D+06    0.219087065744D+09

     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 -.166525897306D+02    0.187436771035D+02    0.801838746021D+00    0.250853965779D+02
```

This next example illustrates the solution for the “grazing flyby; user-defined B-plane angle” program option.

```
-------------------
program e2m_tcm_ftn
-------------------

input data file ==> e2m1.in

grazing flyby; user-defined b-plane angle


time and conditions prior to TCM
(heliocentric EME2000)
----------------------

calendar date           June       8, 2003

TDB time                18:20:44.077

TDB Julian date         2452799.26439903

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 0.190725765750D+09    0.204056802425D+00    0.234926769446D+02    0.253488459798D+03

    raan (deg)        true anomaly (deg)      arglat (deg)         period (min)
 0.463121032996D+00    0.394861259377D+01    0.257437072392D+03    0.757158586476D+06

     rx (km)                ry (km)              rz (km)               rmag (km)
 -.319331575699D+08    -.136207676243D+09    -.590899587841D+08    0.151867971768D+09
```

```
     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 0.316260605833D+02    -.655290816096D+01    -.295930903551D+01    0.324330976528D+02


TCM delta-v vector and magnitude
(heliocentric EME2000)
----------------------

deltav-x           2.97191183280518      meters/second
deltav-y           20.0357404505105      meters/second
deltav-z          -2.59001233743857      meters/second

delta-v            20.4198755934083      meters/second


time and conditions after TCM
(heliocentric EME2000)
----------------------

calendar date           June       8, 2003

TDB time                18:20:44.077

TDB Julian date         2452799.26439903

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 0.190709634005D+09    0.203960522666D+00    0.234966616283D+02    0.253629376458D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.507897248378D+00    0.376663179329D+01    0.257396008251D+03    0.757062526848D+06

     rx (km)                ry (km)              rz (km)              rmag (km)
 -.319331575699D+08    -.136207676243D+09    -.590899587841D+08    0.151867971768D+09

     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 0.316290324952D+02    -.653287242051D+01    -.296189904785D+01    0.324321902531D+02


time and conditions at Mars entry interface
(areocentric mean equator and IAU node of epoch)
------------------------------------------------

calendar date           December  24, 2003

TDB time                02:38:45.832

TDB Julian date         2452997.61025269

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 -.584741511436D+04    0.158080193829D+01    0.602802905792D+02    0.120632976557D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.105601975502D+03    0.592118302894D-05    0.120632982478D+03    0.000000000000D+00

     rx (km)                ry (km)              rz (km)              rmag (km)
 -.929926444039D+03    -.205636051434D+04    0.253785834514D+04    0.339619003241D+04

     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 0.270822036299D+01    -.434030677910D+01    -.252448626011D+01    0.570488837302D+01


b-plane coordinates at Mars entry interface
(areocentric mean equator and IAU node of epoch)
------------------------------------------------

b-magnitude          7159.04830196203      kilometers
```

```
b dot r             -6199.91768685642
b dot t              3579.52416754385
b-plane angle        300.000000153063      degrees
v-infinity           2.70634925359746      km/sec
r-periapsis          3396.19003241347      kilometers
decl-asymptote       7.46920078062836      degrees
rasc-asymptote       281.309880916624      degrees

flight path angle   3.626863982708618E-006 degrees


transfer time        198.345853652339      days


time and conditions of Mars at EI
(heliocentric EME2000)
----------------------

calendar date           December  24, 2003

TDB time                02:38:45.832

TDB Julian date         2452997.61025269

     sma (au)           eccentricity       inclination (deg)       argper (deg)
 0.152368042692D+01    0.935420761435D-01    0.246772248979D+02    0.332979292306D+03

    raan (deg)        true anomaly (deg)       arglat (deg)         period (days)
 0.337165820259D+01    0.704598075446D+02    0.434390998509D+02    0.686972353818D+03

     rx (km)               ry (km)              rz (km)              rmag (km)
 0.150756698115D+09    0.146004294249D+09    0.628942359357D+08    0.219090211739D+09

     vx (kps)              vy (kps)             vz (kps)             vmag (kps)
 -.166553834480D+02    0.168753308030D+02    0.819030733538D+01    0.250850497659D+02


time and heliocentric conditions of Mars at EI
(Earth mean ecliptic and equinox J2000)
---------------------------------------

calendar date           December  24, 2003

TDB time                02:38:45.832

TDB Julian date         2452997.61025269

     sma (au)           eccentricity       inclination (deg)       argper (deg)
 0.152368042692D+01    0.935420761439D-01    0.184937158126D+01    0.286517488018D+03

    raan (deg)        true anomaly (deg)       arglat (deg)         period (days)
 0.495409237989D+02    0.704598075442D+02    0.356977295562D+03    0.686972353819D+03

     rx (km)               ry (km)              rz (km)              rmag (km)
 0.150756628038D+09    0.158974277711D+09    -.372838442811D+06    0.219090211739D+09

     vx (kps)              vy (kps)             vz (kps)             vmag (kps)
 -.166553915476D+02    0.187407229757D+02    0.801845685006D+00    0.250850497659D+02
```