A Computer Program for Interplanetary Gravity-Assist Trajectory Design and Optimization

This document describes a Windows compatible Fortran computer program called `flyby_ftn` that can be used to design and optimize interplanetary trajectories that include a single gravity assist maneuver. The user specifies the departure and flyby planets, the arrival celestial body, and the desired flyby altitude. The algorithm also requires initial guesses for the departure, flyby and arrival calendar dates along with lower and upper search bounds about these dates. This scientific simulation searches for a *patched-conic*, gravity-assist trajectory that satisfies the flyby mission constraints (V_∞ matching and user-defined flyby altitude) and minimizes the departure, arrival or total *impulsive* delta-v for the mission. The type of delta-v optimization is specified by the user.

The planet positions and velocities are modeled using the JPL DE 421 ephemeris. The trajectory optimization is performed with a nonlinear programming (NLP) algorithm.

Program execution

An input file created by the user can be run from the command line or a simple batch file with a statement similar to the following:

```
flyby_ftn evm09.in
```

If the software is executed without an input file on the command line, the computer program will display the following prompt:

```
***********************************
*        program flyby_ftn        *
*                                 *
*    gravity-assist trajectory    *
*     design & optimization       *
*                                 *
*         December 7, 2011        *
***********************************

please input the name of the simulation definition file
```

At this point the user should input the name of a valid input file, including the filename extension.

To create a DOS command window in Windows 7, select **start**, then **All Programs**, then **Accessories** and finally **Command Prompt**. The size, font and other characteristics of the screen can be controlled by the user with the **c:** icon in the upper left corner of the window. To log into the subdirectory created during the installation of the Fortran executable and support files, type **root:** and then **cd subdirectory** from the DOS command line where root is the name of the root directory, usually c:, and subdirectory is the name of the subdirectory created by the user. The DOS command line prompt looks similar to **C:\flyby_ftn>_.**

Input file format and contents

This section describes a typical input data file for the software. In the following discussion the actual input file contents are in `courier` font and all explanations are in *times italic* font.

Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input.

The software allows the user to specify an initial guess for the departure, flyby and arrival calendar dates, and lower and upper bounds on the actual departure, flyby and arrival calendar dates found during the optimization process. For any guess for departure time t_L and user-defined departure time lower and upper bounds Δt_l and Δt_u, the departure time t is constrained as follows:

$$t_L - \Delta t_l \le t \le t_L + \Delta t_u$$

For any guess for flyby time t_{FB} and user-defined flyby time lower and upper bounds Δt_l and Δt_u, the flyby time t is constrained as follows:

$$t_{FB} - \Delta t_l \le t \le t_{FB} + \Delta t_u$$

Likewise, for any guess for arrival time t_A and user-defined arrival time bounds Δt_l and Δt_u, the arrival time t is constrained as follows:

$$t_A - \Delta t_l \le t \le t_A + \Delta t_u$$

The time scale for all calculations in this computer program is Barycentric Dynamical Time (TDB).

The first six lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with six and only six initial text lines.

```
********************************************
patched-conic, gravity-assist interplanetary
trajectory optimization
Earth-to-Venus-to-Mars 2009 - evm09.in
December 1, 2011
********************************************
```

The first numeric input for the program is an integer that specifies the type of optimization.

```
*******************
* simulation type *
*******************
 1 = minimize departure delta-v
 2 = minimize arrival delta-v
 3 = minimize total delta-v
---------------------------
3
```

The next input is the user's guess for the departure calendar date. Please be sure to include all four digits of the calendar year.

```
departure calendar date initial guess (month, day, year)
2,1,2009
```

The next two inputs are the lower and upper bounds, in days, for the departure date search.

```
departure calendar date search boundary (days)
-30, +30
```

The next input is the user's guess for the flyby calendar date. Please be sure to include all four digits of the calendar year.

```
flyby calendar date initial guess (month, day, year)
6,1,2009
```

The next two inputs are the lower and upper bounds, in days, for the flyby date search.

```
flyby calendar date search boundary (days)
-30, +30
```

The next input is the user's guess for the arrival calendar date. Please be sure to include all four digits of the calendar year.

```
arrival calendar date initial guess (month, day, year)
2,1,2010
```

The next two inputs are the lower and upper bounds, in days, for the arrival date search.

```
arrival calendar date search boundary (days)
-30, +30
```

The following input defines the altitude of the gravity-assist relative to a spherical flyby planet.

```
flyby altitude (kilometers)
500.0
```

The next two inputs specify the altitude and orbital inclination of the initial circular Earth park orbit. The inclination should be defined relative to the EME2000 equator.

```
**************************
park orbit characteristics
**************************

altitude of circular park orbit (kilometers)
185.32d0

inclination of circular park orbit (degrees)
28.5d0
```

The next input is an integer that specifies the departure planet.

```
********************
* departure planet *
********************
 1 = Mercury
 2 = Venus
 3 = Earth
 4 = Mars
 5 = Jupiter
 6 = Saturn
 7 = Uranus
 8 = Neptune
 9 = Pluto
----------
3
```

The next input is an integer that specifies the flyby planet.

```
****************
* flyby planet *
****************
```

```
 1 = Mercury
 2 = Venus
 3 = Earth
 4 = Mars
 5 = Jupiter
 6 = Saturn
 7 = Uranus
 8 = Neptune
 9 = Pluto
----------
2
```

The next input is an integer that specifies the arrival celestial body.

```
********************************
* arrival planet/asteroid/comet *
********************************
 1 = Mercury
 2 = Venus
 3 = Earth
 4 = Mars
 5 = Jupiter
 6 = Saturn
 7 = Uranus
 8 = Neptune
 9 = Pluto
10 = asteroid/comet
-------------------
4
```

The next series of inputs include the name and classical orbital elements of a comet or asteroid (arrival celestial body = 10). Please note that the angular orbital elements must be specified with respect to a heliocentric, Earth mean ecliptic and equinox of J2000 coordinate system. The calendar date of perihelion passage should be with respect to the TDB time system.

```
***********************************
* asteroid/comet orbital elements *
* (heliocentric, ecliptic J2000)  *
***********************************

asteroid/comet name
Tempel 1

calendar date of perihelion passage (month, day, year)
7, 5.3153, 2005

perihelion distance (au)
1.506167

orbital eccentricity (nd)
0.517491

orbital inclination (degrees)
10.5301
argument of perihelion (degrees)
178.8390

longitude of the ascending node (degrees)
68.9734
```

Please see the Technical discussion section for additional information about these orbital elements.

Optimal solution and trajectory graphics

This section summarizes the program output for this example. The software provides the heliocentric orbital elements of each leg of the transfer trajectory in the Earth mean ecliptic and equinox of J2000 coordinate system. These numerical results also include the characteristics of the hyperbolic flyby trajectory with respect to the flyby planet. The time scale is Barycentric Dynamical Time (TDB).

```
program flyby_ftn - gravity assist mission design
-------------------------------------------------

minimize total delta-v


departure heliocentric delta-v vector and magnitude
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

x-component of delta-v     3598.23961390701       meters/second
y-component of delta-v    -107.481272154242       meters/second
z-component of delta-v    -4086.59036661132       meters/second

delta-v magnitude          5446.01701865079       meters/second


arrival heliocentric delta-v vector and magnitude
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

x-component of delta-v     3467.15189037889       meters/second
y-component of delta-v     2289.33044772484       meters/second
z-component of delta-v    -755.160235415933       meters/second

delta-v magnitude          4222.84774897123       meters/second


heliocentric coordinates of the planet at departure
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

calendar date           January 26, 2009

TDB time                22:44:21.920

TDB Julian date         2454858.44747593

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.100089454405D+01    0.174899272470D-01    0.151803840927D-02    0.187661808577D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 0.276571390730D+03    0.227664668681D+02    0.210428275445D+03    0.365747115495D+03

     rx (km)                ry (km)               rz (km)               rmag (km)
 -.886527921296D+08    0.117647654908D+09    -.197668503679D+04    0.147310177047D+09

     vx (kps)               vy (kps)              vz (kps)              vmag (kps)
 -.242850147743D+02    -.180475089531D+02    -.693920244544D-03    0.302568095225D+02


spacecraft heliocentric coordinates after the first impulse
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

calendar date           January 26, 2009
```

```
TDB time               22:44:21.920

TDB Julian date        2454858.44747593

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.863336645053D+00    0.158375605679D+00    0.847000468985D+01    0.336511764234D+03

    raan (deg)         true anomaly (deg)       arglat (deg)         period (days)
 0.306994503352D+03    0.203493455512D+03    0.180005219745D+03    0.293000721522D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.886527921296D+08    0.117647654908D+09    -.197668503679D+04    0.147310177047D+09

     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 -.206867751604D+02    -.181549902253D+02    -.408728428686D+01    0.278253882176D+02


spacecraft heliocentric coordinates prior to the flyby
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

calendar date          June  2, 2009

TDB time               08:16:39.207

TDB Julian date        2454984.84489823

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.863336645053D+00    0.158375605679D+00    0.847000468985D+01    0.336511764234D+03

    raan (deg)         true anomaly (deg)       arglat (deg)         period (days)
 0.306994503352D+03    0.960346937280D+01    0.346115233606D+03    0.293000721522D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 0.429730470229D+08    -.999964511310D+08    -.384938831214D+07    0.108907257791D+09

     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 0.345439803029D+02    0.137054680170D+02    0.533671905703D+01    0.375447333061D+02


spacecraft heliocentric coordinates after the flyby
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

calendar date          June  2, 2009

TDB time               08:16:39.207

TDB Julian date        2454984.84489823

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.118238783618D+01    0.390747964979D+00    0.246861766312D+01    0.320550544364D+03

    raan (deg)         true anomaly (deg)       arglat (deg)         period (days)
 0.348376600956D+03    0.344303310897D+03    0.304853855261D+03    0.469611103430D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 0.429730470229D+08    -.999964511310D+08    -.384938831214D+07    0.108907257791D+09

     vx (kps)              vy (kps)              vz (kps)              vmag (kps)
 0.363852756910D+02    0.190194212704D+02    0.111920193150D+01    0.410716360221D+02


heliocentric coordinates of planet at flyby
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------
```

calendar date	June 2, 2009
TDB time	08:16:39.207
TDB Julian date	2454984.84489823

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.723333437840D+00	0.679370756530D-02	0.339452767942D+01	0.547914013544D+02
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.766523712877D+02	0.161859809222D+03	0.216651210577D+03	0.224701638719D+03
rx (km)	ry (km)	rz (km)	rmag (km)
0.429730470229D+08	-.999964511310D+08	-.384938831214D+07	0.108907257791D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.319383575150D+02	0.137084253921D+02	-.165553830354D+01	0.347954079522D+02

spacecraft heliocentric coordinates prior to the second impulse
(Earth mean ecliptic and equinox of J2000)
----

calendar date	January 14, 2010
TDB time	03:31:03.776
TDB Julian date	2455210.64657148

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.118238783618D+01	0.390747964979D+00	0.246861766312D+01	0.320550544364D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.348376600956D+03	0.173664986659D+03	0.134215531023D+03	0.469611103430D+03
rx (km)	ry (km)	rz (km)	rmag (km)
-.132024670603D+09	0.206292687992D+09	0.756455886054D+07	0.245039607650D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.160233513521D+02	-.871216493709D+01	-.507079377338D+00	0.182457319924D+02

spacecraft heliocentric coordinates after the second impulse
(Earth mean ecliptic and equinox of J2000)
----

calendar date	January 14, 2010
TDB time	03:31:03.776
TDB Julian date	2455210.64657148

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.152370504377D+01	0.933397005685D-01	0.184890378064D+01	0.286556582002D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.495254216471D+02	0.146545017461D+03	0.731015994633D+02	0.686989002161D+03
rx (km)	ry (km)	rz (km)	rmag (km)
-.132024670603D+09	0.206292687992D+09	0.756455886054D+07	0.245039607650D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.194905032425D+02	-.110014953848D+02	0.248080858077D+00	0.223824520877D+02

heliocentric coordinates of celestial body at arrival
(Earth mean ecliptic and equinox of J2000)
----

calendar date	January 14, 2010
TDB time	03:31:03.776
TDB Julian date	2455210.64657148

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.152370504377D+01	0.933397005685D-01	0.184890378064D+01	0.286556582002D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.495254216471D+02	0.146545017461D+03	0.731015994633D+02	0.686989002161D+03
rx (km)	ry (km)	rz (km)	rmag (km)
-.132024670603D+09	0.206292687992D+09	0.756455886054D+07	0.245039607650D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.194905032425D+02	-.110014953848D+02	0.248080858077D+00	0.223824520877D+02

FLYBY CONDITIONS
================

planet-centered conditions at periapsis
(Earth mean ecliptic and equinox J2000)
----

calendar date	June 2, 2009
TDB time	08:16:39.207
TDB Julian date	2454984.84489823

sma (km)	eccentricity	inclination (deg)	argper (deg)
-.583428694155D+04	0.212299927406D+01	0.725810929720D+02	0.389614366799D+02
raan (deg)	true anomaly (deg)	arglat (deg)	
0.237280331704D+03	0.000000000000D+00	0.389614366799D+02	
rx (km)	ry (km)	rz (km)	rmag (km)
-.171615807934D+04	-.495280869537D+04	0.393088807827D+04	0.655190000000D+04
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.666631746444D+01	0.501735047803D+01	0.923212030684D+01	0.124436987956D+02

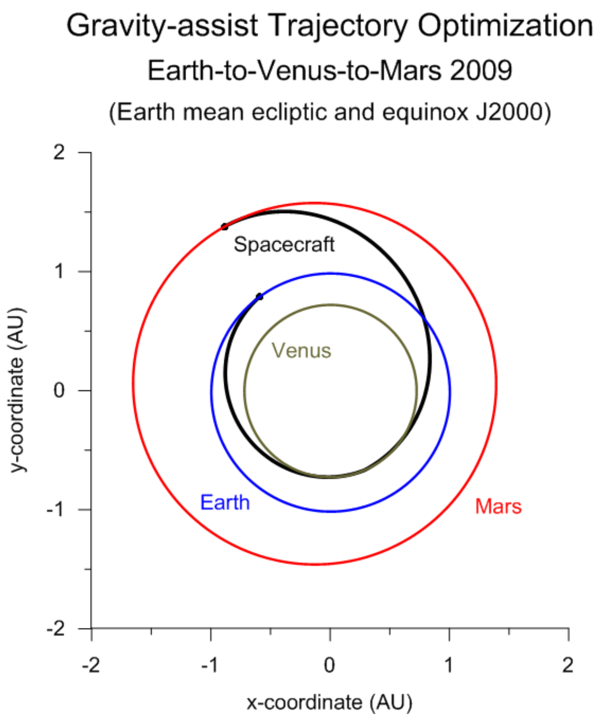
flyby altitude	500.000000003548	kilometers
incoming v-infinity	7461.96635311907	meters/second
outgoing v-infinity	7461.96635313622	meters/second
maximum turn angle	58.7923996173451	degrees
actual turn angle	56.2025771701917	degrees
heliocentric delta-v	7029.64569447678	meters/second
max heliocentric delta-v	7326.58018700546	meters/second

```
planet-centered b-plane coordinates at flyby
(Earth mean ecliptic and equinox J2000)
--------------------------------------

b-magnitude                 10926.0570580046      kilometers
b dot r                    -5625.06099043639
b dot t                     3274.30321169388
theta                       300.203344102918      degrees
v-infinity                  7461.96635311907      meters/second
r-periapsis                 6551.90000000355      kilometers
decl-asymptote              69.5623804114816      degrees
rasc-asymptote              359.934969467458      degrees


departure-to-flyby time     126.397422302049      days

flyby-to-arrival time       225.801673250739      days

total mission duration      352.199095552787      days


HYPERBOLIC TRAJECTORY CHARACTERISTICS
(Earth mean equator and equinox of J2000)
-----------------------------------------

departure hyperbola
-------------------

c3                    29.6591013674340      km**2/sec**2

v-infinity            5446.01701865079      meters/second

decl-asymptote       -44.1318625409487      degrees

rasc-asymptote        22.9942696246057      degrees


arrival hyperbola
-----------------

c3                    17.8324431109914      km**2/sec**2

v-infinity            4222.84774897123      meters/second

decl-asymptote        2.95638980672858      degrees

rasc-asymptote        34.7003674941741      degrees


orbital elements and state vector of park orbit at injection
(Earth mean equator and equinox of J2000)
-----------------------------------------

     sma (km)         eccentricity       inclination (deg)     argper (deg)
 0.6563460000D+04    0.0000000000D+00    0.2850000000D+02    0.0000000000D+00

    raan (deg)      true anomaly (deg)     arglat (deg)        period (min)
 0.1129942696D+03    0.1469097953D+03    0.1469097953D+03    0.8819805229D+02

     rx (km)             ry (km)             rz (km)            rmag (km)
 -.7508132559D+03    -.6292196164D+04    0.1709840360D+04    0.6563460000D+04

     vx (kps)            vy (kps)            vz (kps)           vmag (kps)
 0.6943949956D+01    -.1675158168D+01    -.3115386823D+01    0.7792960344D+01
```

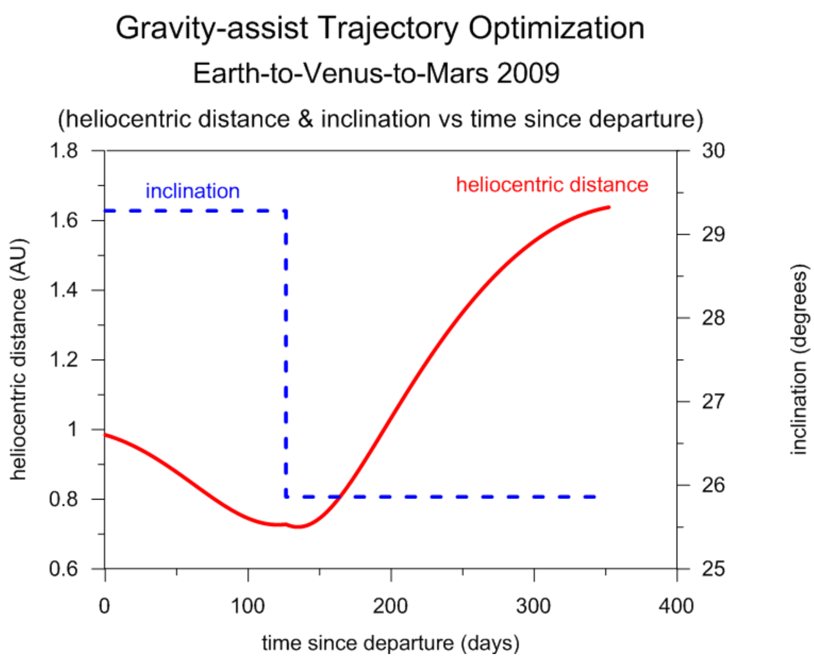
```
orbital elements and state vector of hyperbola at injection
(Earth mean equator and equinox of J2000)
-----------------------------------------

     sma (km)          eccentricity        inclination (deg)      argper (deg)
 -.1343939712D+05    0.1483331743D+01    0.4501480756D+02    0.1477189209D+03

    raan (deg)       true anomaly (deg)      arglat (deg)         period (min)
 0.9884098100D+02    0.1066902371D+02    0.1583879446D+03    0.1666666667D+98

     rx (km)             ry (km)             rz (km)             rmag (km)
 -.7508132559D+03    -.6292196164D+04    0.1709840360D+04    0.6563460000D+04

     vx (kps)            vy (kps)            vz (kps)            vmag (kps)
 0.8469388164D+01    -.4520952621D+01    -.7677891296D+01    0.1229306972D+02


hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
-----------------------------------------

delta-vx                   1525.43820830718      meters/seconds
delta-vx                  -2845.79445217781      meters/seconds
delta-vx                  -4562.50447358561      meters/seconds

delta-v magnitude          5589.45031849269      meters/seconds
```

The algorithm used to determine the trajectory characteristics of the park orbit at injection and the departure hyperbola is summarized in Appendix C.

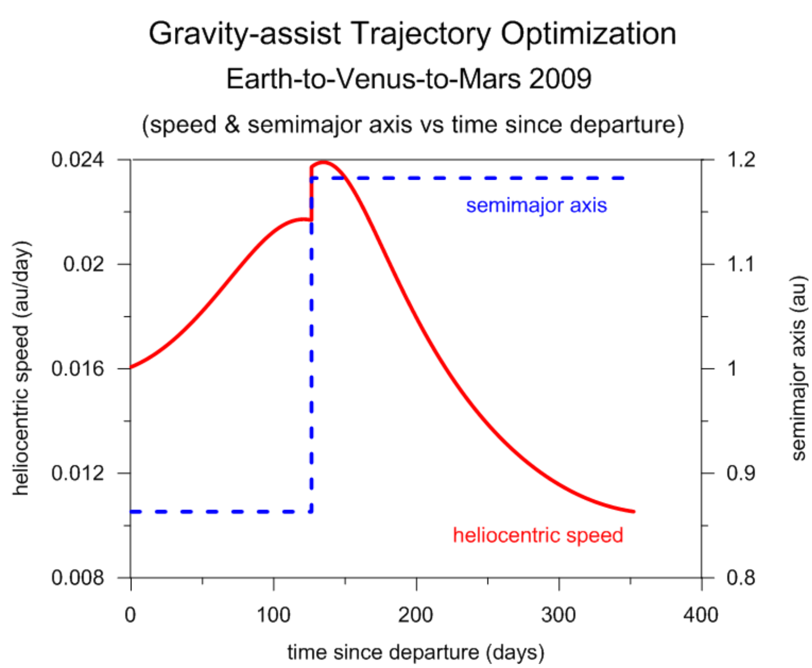
The `flyby_ftn` computer program will also produce a comma-separated-variable (CSV) data file named `flyby.csv` that contains important trajectory characteristics. An explanation of the screen display and data file contents can be found in Appendix A. The following graphs were produced using the contents of the `flyby.csv` data file for this gravity assist example. The first plot is a north ecliptic view of the planetary orbits and the heliocentric transfer trajectory in the Earth mean ecliptic and equator of J2000 system.



This next plot illustrates the effect of the gravity assist on the heliocentric distance and orbital inclination of the spacecraft's trajectory. In this plot, the orbital inclination is measured with respect to the mean ecliptic of J2000.



This next plot illustrates the effect of the gravity assist on the heliocentric speed and semimajor axis of the spacecraft's trajectory.



Technical discussion

The computational steps used to create an initial guess for the spacecraft state, and the departure and arrival delta-v characteristics are as follows:

(1) compute the state vector of the departure planet at the departure date initial guess

(2) compute the state vector of the flyby planet at the flyby date initial guess

(3) compute the state vector of the arrival planet at the arrival date initial guess

(4) solve Lambert's problem for the departure-to-flyby leg and determine the initial velocity vector of this leg

(5) compute the departure delta-v vector from the departure planet’s velocity vector and the initial velocity vector of the first leg of the transfer trajectory

(6) solve Lambert's problem for the second heliocentric leg and determine the initial and final velocity vectors of the second leg

(7) compute the arrival delta-v vector from the arrival planet’s velocity vector and the final velocity vector of the second leg of the transfer trajectory

(8) estimate the flyby incoming v-infinity vector from the flyby planet’s velocity vector and the final velocity vector of the first leg of the transfer trajectory

Gravity-assist flight mechanics

The vector relationships between the incoming v-infinity vector $\mathbf{v}_\infty^-$, the outgoing v-infinity vector $\mathbf{v}_\infty^+$ and the two legs of the heliocentric transfer orbit are as follows:

$$\mathbf{v}_\infty^- = \mathbf{v}_{fb} - \mathbf{v}_{to_1}$$

$$\mathbf{v}_\infty^+ = \mathbf{v}_{to_2} - \mathbf{v}_{fb}$$

where

$\mathbf{v}_{fb} =$ heliocentric velocity vector of the flyby planet at the flyby date

$\mathbf{v}_{to_1} =$ heliocentric velocity vector of the first transfer orbit at the flyby date

$\mathbf{v}_{to_2} =$ heliocentric velocity vector of the second transfer orbit at the flyby date

The turn angle of the planet-centered trajectory during the flyby is determined from

$$\delta = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\left|\mathbf{v}_\infty^- \times \mathbf{v}_\infty^+\right|}{\left|\mathbf{v}_\infty^-\right|\left|\mathbf{v}_\infty^+\right|}\right) = 2\sin^{-1}\left(\frac{1}{1 + r_p v_\infty^2 / \mu}\right)$$

where r_p is the periapsis radius of the flyby hyperbola, v_∞ is the magnitude of the incoming (or outgoing) v-infinity vector and μ is the gravitational constant of the flyby planet.

The maximum turn angle possible during a gravity assist flyby occurs when the spacecraft just gazes the planet's surface. It is given by

$$\delta_{\max} = 2\sin^{-1}\left(\frac{1}{1 + r_s v_\infty^2 / \mu}\right)$$

where r_s is the radius of the flyby planet. The semimajor axis and orbital eccentricity of the flyby hyperbola are given by

$$a = -\mu / \left|\mathbf{v}_\infty^-\right|^2 = -\mu / \left|\mathbf{v}_\infty^+\right|^2$$

$$e = -1/\cos\theta_\infty = 1 - \frac{r_p}{a} = 1 + \frac{r_p v_\infty^2}{\mu}$$

where θ_∞ is the true anomaly at infinity which is determined from the following expression:

$$\theta_\infty = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\left|\mathbf{v}_\infty^- \times \mathbf{v}_\infty^+\right|}{\left|\mathbf{v}_\infty^-\right|\left|\mathbf{v}_\infty^+\right|}\right)$$

The periapsis radius of the flyby hyperbola is determined from the expression $r = a(1-e)$ and the flyby altitude is $h = r - r_p$.

The heliocentric speed gained during the flyby and the heliocentric delta-v vector caused by the close encounter can be determined from the following two equations:

$$\Delta v_{fb} = 2 v_\infty / e$$

$$\Delta \mathbf{v}_{fb} = \mathbf{v}_h^- - \mathbf{v}_h^+$$

where e is the orbital eccentricity of the hyperbolic flyby trajectory.

In the second equation $\mathbf{v}_h^-$ is the heliocentric velocity vector of the spacecraft prior to the flyby and $\mathbf{v}_h^+$ is the heliocentric velocity vector after the flyby. For any planet it can be shown that the *maximum* heliocentric delta-v possible is given by the expression

$$\Delta v_{\max} = \sqrt{\frac{\mu}{r_s}}$$

This corresponds to a "grazing" flyby at the planet's surface and is equal to the "local circular velocity" for the flyby planet at the surface.

During the optimization analysis, the software enforces the following two nonlinear equality *point constraints*:

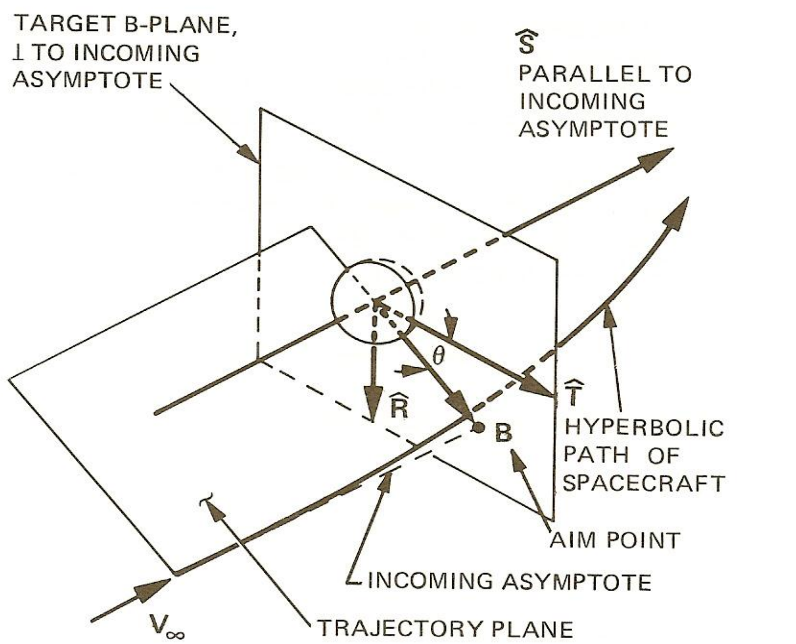
$$\left|\mathbf{v}_{\infty}^{-}\right| - \left|\mathbf{v}_{\infty}^{+}\right| = 0$$

$$h_{fb} - h_t = 0$$

The first equation is the v-infinity matching constraint and the second equation is the (positive) flyby altitude constraint. In the second expression h_{fb} is the actual flyby altitude and h_t is the user-defined or "targeted" flyby altitude.

The B-plane

The derivation of B-plane coordinates is described in the classic JPL reports, "A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories" and "Some Orbital Elements Useful in Space Trajectory Calculations", both by William Kizner. The following diagram illustrates the fundamental geometry of the B-plane coordinate system.



The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_{\infty}\cos\alpha_{\infty} \\ \cos\delta_{\infty}\sin\alpha_{\infty} \\ \sin\delta_{\infty} \end{Bmatrix}$$

where δ_{∞} and α_{∞} are the declination and right ascension of the asymptote of the incoming or outgoing hyperbola.

The following computational steps summarize the calculation of the B-plane vector from a planet-centered position vector **r** and velocity vector **v** at closest approach to the flyby planet.

angular momentum vector

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

radius rate

$$\dot{r} = \frac{\mathbf{r} \bullet \mathbf{v}}{|\mathbf{r}|}$$

semiparameter

$$p = \frac{h^2}{\mu}$$

semimajor axis

$$a = \frac{r}{\left(2 - \frac{r v^2}{\mu}\right)}$$

orbital eccentricity

$$e = \sqrt{1 - p/a}$$

true anomaly

$$\cos\theta = \frac{p - r}{e r} \qquad \sin\theta = \frac{\dot{r} h}{e\mu}$$

B-plane magnitude

$$B = \sqrt{p|a|}$$

fundamental vectors

$$\hat{\mathbf{z}} = \frac{r\,\mathbf{v} - \dot{r}\,\mathbf{r}}{h}$$

$$\hat{\mathbf{p}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\mathbf{z}} \qquad \hat{\mathbf{q}} = \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\mathbf{z}}$$

S vector

$$\mathbf{S} = -\frac{a}{\sqrt{a^2 + b^2}}\hat{\mathbf{p}} + \frac{b}{\sqrt{a^2 + b^2}}\hat{\mathbf{q}}$$

B vector

$$\mathbf{B} = \frac{b^2}{\sqrt{a^2 + b^2}}\hat{\mathbf{p}} + \frac{ab}{\sqrt{a^2 + b^2}}\hat{\mathbf{q}}$$

T vector

$$\mathbf{T} = \frac{\left(S_y^2, -S_x^2, 0\right)^T}{\sqrt{S_x^2 + S_y^2}}$$

R vector

$$\mathbf{R} = \mathbf{S} \times \mathbf{T} = \left(-S_z T_y,\ S_z T_x,\ S_x T_y - S_y T_x\right)^T$$

The orientation of the departure hyperbola is specified in terms of the right ascension (RLA) and declination (DLA) of the outgoing asymptote. These coordinates can be calculated using the Cartesian components of the corresponding V_∞ velocity vector.

The right ascension of the asymptote is determined from

$$\alpha = \tan^{-1}\left(\Delta V_y, \Delta V_z\right)$$

and the geocentric declination of the asymptote is given by

$$\delta = 90^0 - \cos^{-1}\left(\Delta \hat{V}_z\right)$$

where $\Delta \hat{V}_x$, $\Delta \hat{V}_y$ and $\Delta \hat{V}_z$ are the x, y and z components of the unit ΔV vector. The right ascension is computed using a four quadrant inverse tangent function.

In this computer program the heliocentric planetary coordinates and therefore the ΔV vectors are computed in the Earth mean ecliptic and equinox of J2000 coordinate system. In order to determine the orientation of the departure hyperbola, the ΔV vector must be transformed to the Earth mean equator and equinox of J2000 (EME2000) coordinate frame.

The required transformation is given by

$$\Delta \mathbf{V}_{eq} = \begin{bmatrix} 1 & -0.000000479966 & 0 \\ 0.000000440360 & 0.917482137087 & 0.397776982902 \\ -0.000000190919 & -0.397776982902 & 0.917482137087 \end{bmatrix} \Delta \mathbf{V}_{ec}$$

where $\Delta \mathbf{V}_{ec}$ is the delta-velocity vector in the ecliptic frame, and $\Delta \mathbf{V}_{eq}$ is the delta-velocity vector in the equatorial frame.

The transformation of vectors from the equatorial to the ecliptic system involves the transpose of this matrix according to

$$\Delta \mathbf{V}_{ec} = \begin{bmatrix} 1 & -0.000000479966 & 0 \\ 0.000000440360 & 0.917482137087 & 0.397776982902 \\ -0.000000190919 & -0.397776982902 & 0.917482137087 \end{bmatrix}^T \Delta \mathbf{V}_{eq}$$

The `flyby_ftn` software models the planetary coordinates using the DE421 model from JPL. These coordinates are first computed in the EME2000 system and then converted to the ecliptic system.

Important Note

The binary ephemeris file provided with this computer program was created for use on Windows compatible computers. For other platforms, you will need to create or obtain binary files specific to that system. Information and computer programs for creating these files can be found at the JPL solar system FTP site located at ftp://ssd.jpl.nasa.gov/pub/eph/export/. This site provides ASCII data files and Fortran computer programs for creating a binary file. A program for testing the user's ephemeris is also provided along with documentation.

The DE421 ephemeris and the declination and right ascension of the departure v-infinity vector are modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time (TT).

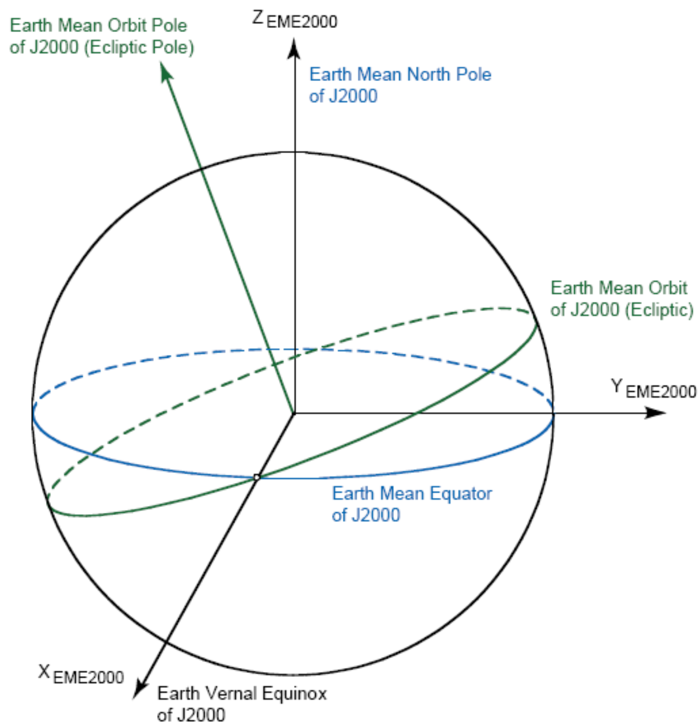


Figure 1. Earth mean equator and equinox of J2000 coordinate system

In order to model ballistic interplanetary missions involving asteroids and comets, the classical orbital elements of an asteroid or comet relative to the mean ecliptic and equinox of J2000 coordinate system must be provided by the user. These elements can be obtained from the JPL Near Earth Object (NEO) website (http://neo.jpl.nasa.gov).

These orbital elements consist of the following items:

- TDB calendar date of perihelion passage
- perihelion distance (AU)
- orbital eccentricity (non-dimensional)
- orbital inclination (degrees)
- argument of perihelion (degrees)
- longitude of ascending node (degrees)

The software determines the mean anomaly of the asteroid or comet at any simulation time using the following equation:

$$M = \sqrt{\frac{\mu_s}{a^3}}\, t_{pp} = \sqrt{\frac{\mu_s}{a^3}}\left(JD - JD_{pp}\right)$$

where μ_s is the gravitational constant of the sun, a is the semimajor axis of the celestial body's heliocentric orbit, and t_{pp} is the time since perihelion passage.

The semimajor axis is determined from the perihelion distance r_p and orbital eccentricity e according to

$$a = \frac{r_p}{\left(1-e\right)}$$

This solution of Kepler's equation in this computer program is based on a numerical solution devised by Professor J.M.A. Danby at North Carolina State University. Additional information about this algorithm can be found in "The Solution of Kepler's Equation", *Celestial Mechanics*, **31** (1983) 95-107, 317-328 and **40** (1987) 303-312.

The initial guess for Danby's method for elliptic orbits is

$$E_0 = M + 0.85\,\text{sign}\left(\sin M\right) e$$

The fundamental equation we want to solve is

$$f\left(E\right) = E - e\sin E - M = 0$$

which has the first three derivatives given by

$$f'\left(E\right) = 1 - e\cos E \qquad f''\left(E\right) = e\sin E \qquad f'''\left(E\right) = e\cos E$$

The iteration for an updated eccentric anomaly based on a current value E_n is given by the next four equations:

$$\Delta(E_n) = -\frac{f}{f'}$$

$$\Delta^*(E_n) = -\frac{f}{f' + \frac{1}{2}\Delta f''}$$

$$\Delta_n(E_n) = -\frac{f}{f' + \frac{1}{2}\Delta f'' + \frac{1}{6}\Delta^{*2} f'''}$$

$$E_{n+1} = E_n + \Delta_n$$

This algorithm provides quartic convergence of Kepler's equation. This process is repeated until the following convergence test involving the fundamental equation is satisfied:

$$\left| f(E) \right| \le \varepsilon$$

where ε is the convergence tolerance. This tolerance is hardwired in the software to ε `= 1.0e-10`.

Finally, the true anomaly of the celestial body can be calculated with the following two equations

$$\sin\theta = \sqrt{1-e^2}\sin E \qquad \cos\theta = \cos E - e$$

and the four quadrant inverse tangent given by

$$\theta = \tan^{-1}(\sin\theta, \cos\theta)$$

If the orbit is hyperbolic, the initial guess is

$$H_0 = \log\left(\frac{2M}{e} + 1.8\right)$$

where H_0 is the hyperbolic anomaly. The fundamental equation and first three derivatives for this case are as follows:

$$f(H) = e\sinh H - H - M$$

$$f'(H) = e\cosh H - 1$$

$$f''(H) = e\sinh H$$

$$f'''(H) = e\cosh H$$

Otherwise, the iteration loop which calculates Δ, Δ^*, and so forth is the same. The true anomaly for hyperbolic orbits is determined with this next set of equations

$$\sin\theta = \sqrt{e^2 - 1}\sinh H \qquad \cos\theta = e - \cosh H$$

The true anomaly is then determined from a four quadrant inverse tangent evaluation of these last two equations.

Lambert's Problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamic problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis *a* of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length *c* of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof\left(r_i + r_f, c, a\right)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}\left(E - e\sin E\right)}$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}}\left[E - E_0 - e\left(\sin E - \sin E_0\right)\right]$$

where *E* is the eccentric anomaly associated with radius *r*, E_0 is the eccentric anomaly at r_0, and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sun and difference identities:

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha-\beta}{2}\sin\frac{a+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t = \sqrt{\frac{a^3}{\mu}}\left\{E - E_0 - 2\sin\frac{E-E_0}{2}\left(e\cos\frac{E+E_0}{2}\right)\right\}$$

With the two substitutions given by

$$e\cos\frac{E+E_0}{2}=\cos\frac{\alpha+\beta}{2}$$

$$\sin\frac{E-E_0}{2}=\sin\frac{\alpha-\beta}{2}$$

the time equation becomes

$$t=\sqrt{\frac{a^3}{\mu}}\left\{(\alpha-\beta)-2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}\right\}$$

From the elliptic relationships given by

$$r=a(1-e\cos E)$$

$$x=a(\cos E-e)$$

$$y=a\sin E\sqrt{1-e^2}$$

and some more manipulation, we have the following equations:

$$\cos\alpha=\left(1-\frac{r+r_0}{2a}\right)-\frac{c}{2a}=1-\frac{r+r_0+c}{2a}=1-\frac{s}{a}$$

$$\sin\beta=\left(1-\frac{r+r_0}{2a}\right)+\frac{c}{2a}=1-\frac{r+r_0-c}{2a}=1-\frac{s-c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}=1-\frac{r+r_0}{2}$$

$$\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}=\sin\frac{E-E_0}{2}\sqrt{1-\left(e\cos\frac{E+E_0}{2}\right)^2}$$

$$\left(\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\right)^2=\left(\frac{x-x_0}{2a}\right)^2+\left(\frac{y-y_0}{2a}\right)^2=\left(\frac{c}{2a}\right)^2$$

With the use of the half angle formulas given by

$$\sin\frac{\alpha}{2}=\sqrt{\frac{s}{2a}}\qquad\qquad \sin\frac{\beta}{2}=\sqrt{\frac{s-c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert’s theorem

$$t = \sqrt{\frac{a^3}{\mu}}\left[(\alpha - \beta) - (\sin\alpha - \sin\beta)\right]$$

A discussion about the angles α and β can be found in "Geometrical Interpretation of the Angles α and β in Lambert's Problem" by J. E. Prussing, AIAA *Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this Fortran computer program is based on the method described in "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem" by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

In this computer program, the spacecraft's heliocentric trajectory is modeled as two-body, Keplerian motion. The equations of motion include only the point-mass gravity of the sun.

Algorithm Resources

"Gravity-Assist Trajectory Design with MATLAB", C. David Eagle, AAS 99-138, AAS/AIAA Space Flight Mechanics Meeting, Breckenridge, CO, February 7-10, 1999.

"Optimal Interplanetary Orbit Transfers by Direct Transcription", John T. Betts, *The Journal of the Astronautical Sciences*, Vol. 42, No. 3, July-September 1994, pp. 247-268.

"Multiple Gravity Assist Interplanetary Trajectories", A. V. Labunsky, O. V. Papkov, and K. G. Sukhanov, Gordon and Breach Science Publishers, 1998.

"Modern Astrodynamics", Victor R. Bond and Mark C. Allman, Princeton University Press, 1996.

"Automated Design of Gravity-Assist Trajectories to Mars and the Outer Planets", J.M. Longuski and S.N. Williams, *Celestial Mechanics and Dynamical Astronomy*, **52**: 207-220, 1991.

"Gravitational Assist in Celestial Mechanics – A Tutorial", J. A. Van Allen, *American Journal of Physics*, **71** (5), May 2003.

"Notes for the Gravitational Assisted Trajectories Lectures", E. Barrabes, G. Gomez, and J. Rodriquez-Canabal, Advanced Topics in Astrodynamics Summer Course, Barcelona, July 2004.

"Graphical Method for Gravity-Assist Trajectory Design", Nathan J. Strange and James M. Longuski, *AIAA Journal of Spacecraft and Rockets*, Vol. 39, No. 1, January-February 2002.

"Optimization of Interplanetary Trajectories with Unpowered Planetary Swingbys", Carl G. Sauer, Jr., AAS 87-424.

APPENDIX A

Contents of the Simulation Summary and CSV Data File

This appendix is a brief summary of the information contained in the simulation summary screen displays and the comma-separated-variable (csv) data file produced by the `flyby_ftn` software.

The simulation summary screen display contains the following information:

calendar date = calendar date of trajectory event

TDB time = TDB time of trajectory event

TDB julian date = julian date of trajectory event on TDB time scale

sma (au) = semimajor axis in astronomical unit

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of periapsis in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (days) = orbital period in days

rx (km) = x-component of the spacecraft's position vector in kilometers

ry (km) = y-component of the spacecraft's position vector in kilometers

rz (km) = z-component of the spacecraft's position vector in kilometers

rmag (km) = scalar magnitude of the spacecraft's position vector in kilometers

vx (kps) = x-component of the spacecraft's velocity vector in kilometers per second

vy (kps) = y-component of the spacecraft's velocity vector in kilometers per second

vz (ksp) = z-component of the spacecraft's velocity vector in kilometers per second

vmag (kps) = scalar magnitude of the spacecraft's velocity vector in kilometers per second

b-magnitude = magnitude of the b-plane vector

b dot r = dot product of the b-vector and r-vector

b dot t = dot product of the b-vector and t-vector

theta = orientation of the b-plane vector in degrees

v-infinity = magnitude of incoming v-infinity vector in kilometers/second

r-periapsis = periapsis radius of incoming hyperbola in kilometers

decl-asy = declination of incoming v-infinity vector in degrees

rasc-asy = right ascension of incoming v-infinity vector in degrees

The `flyby.csv` disk file contains the following information:

time (days) = simulation time since departure in days

rs2sc-x (au) = x-component of the spacecraft's heliocentric position vector in astronomical units

rs2sc-y (au) = y-component of the spacecraft's heliocentric position vector in astronomical units

rs2sc-z (au) = z-component of the spacecraft's heliocentric position vector in astronomical units

rs2sc-mag (au) = the spacecraft's heliocentric radius in astronomical units

vs2sc-y (km/sec) = y-component of the spacecraft's heliocentric velocity vector in astronomical units per day

vs2sc-z (km/sec) = z-component of the spacecraft's heliocentric position vector in astronomical units per day

vs2sc-mag (km/sec) = the spacecraft's heliocentric speed in astronomical units per day

rs2p1-x (au) = x-component of the departure planet's heliocentric position vector in astronomical units

rs2p1-y (au) = y-component of the departure planet's heliocentric position vector in astronomical units

rs2p1-z (au) = z-component of the departure planet's heliocentric position vector in astronomical units

rs2p1-mag (au) = departure planet heliocentric radius in astronomical units

vs2p1-x (au) = x-component of the departure planet's heliocentric velocity vector in astronomical units per day

vs2p1-y (au) = y-component of the departure planet's heliocentric velocity vector in astronomical units per day

vs2p1-z (au) = z-component of the departure palnet's heliocentric velocity vector in astronomical units per day

vs2p1-mag (au) = departure planet's heliocentric speed in astronomical units per day
rs2p2-x (au) = x-component of the flyby planet's heliocentric position vector in astronomical units

rs2p2-y (au) = y-component of the flyby planet's heliocentric position vector in astronomical units

rs2p2-z (au) = z-component of the flyby planet's heliocentric position vector in astronomical units

rs2p2-mag (au) = flyby planet heliocentric radius in astronomical units

vs2p2-x (au) = x-component of the flyby planet's heliocentric velocity vector in astronomical units per day

vs2p2-y (au) = y-component of the flyby planet's heliocentric velocity vector in astronomical units per day

vs2p2-z (au) = z-component of the flyby planet's heliocentric velocity vector in astronomical units per day

vs2p2-mag (au) = flyby planet heliocentric speed in astronomical units per day

rs2p3-x (au) = x-component of the arrival body's heliocentric position vector in astronomical units

rs2p3-y (au) = y-component of the arrival body's heliocentric position vector in astronomical units

rs2p3-z (au) = z-component of the arrival body's heliocentric position vector in astronomical units

rs2p3-mag (au) = arrival body heliocentric radius in astronomical units

vs2p3-x (au) = x-component of the arrival body's heliocentric velocity vector in astronomical units per day

vs2p3-y (au) = y-component of the arrival body's heliocentric velocity vector in astronomical units per day

vs2p3-z (au) = z-component of the arrival body's heliocentric velocity vector in astronomical units per day

vs2p3-mag (au) = arrival body heliocentric speed in astronomical units per day

sma-heo (au) = heliocentric semimajor axis of the spacecraft in astronomical units

ecc-heo = heliocentric orbital eccentricity of the spacecraft(non-dimensional)

inc-heo (deg) = heliocentric orbital inclination of the spacecraft in degrees

argper-heo (deg) = heliocentric argument of perigee of the spacecraft in degrees

raan-heo (deg) = heliocentric right ascension of the ascending node of the spacecraft in degrees

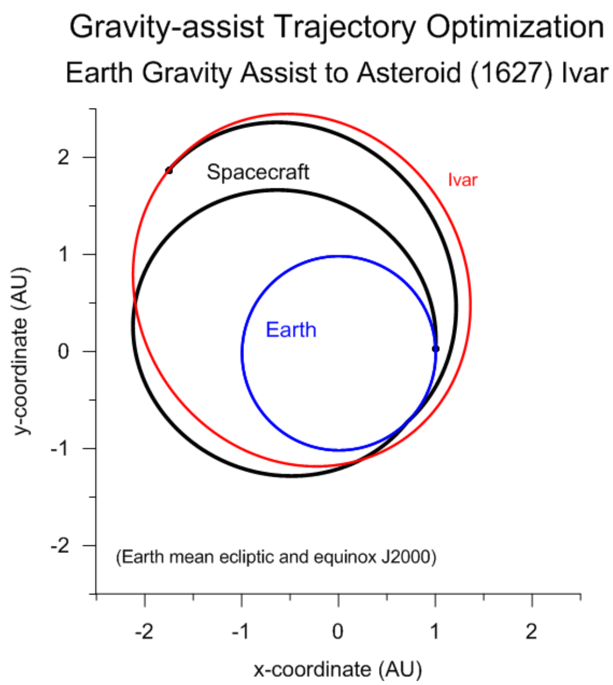
tanom-heo (deg) = heliocentric true anomaly of the spacecraft in degrees

APPENDIX B

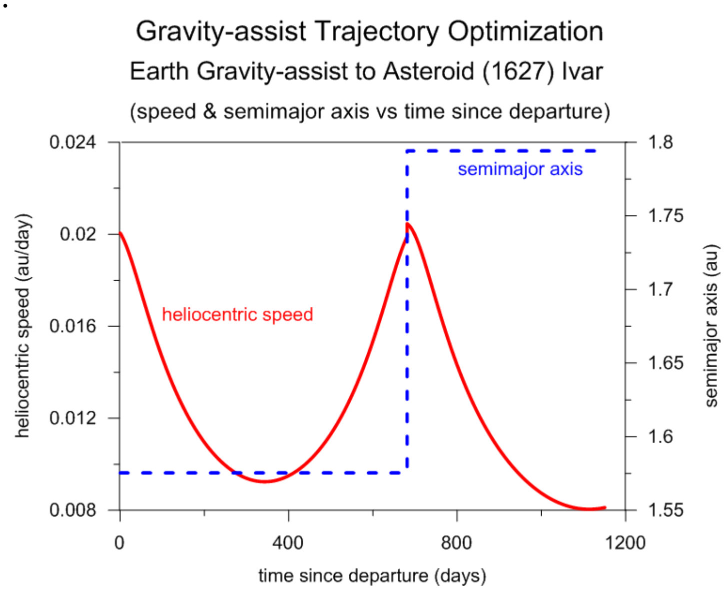
An Earth Gravity-assist Trajectory to the Asteroid (1627) Ivar

This appendix summarizes the trajectory characteristics for a patched-conic, ballistic interplanetary trajectory from the Earth to the asteroid (1627) Ivar. This example uses a single Earth gravity-assist during the trajectory optimization. The initial guess for this trajectory was extracted from the technical paper "The design of transfer trajectory for Ivar asteroid exploration mission", by D. Qiao, H. Cui, and P. Cui, *Acta Astronautica*, 65 (2009), 1553-1560.

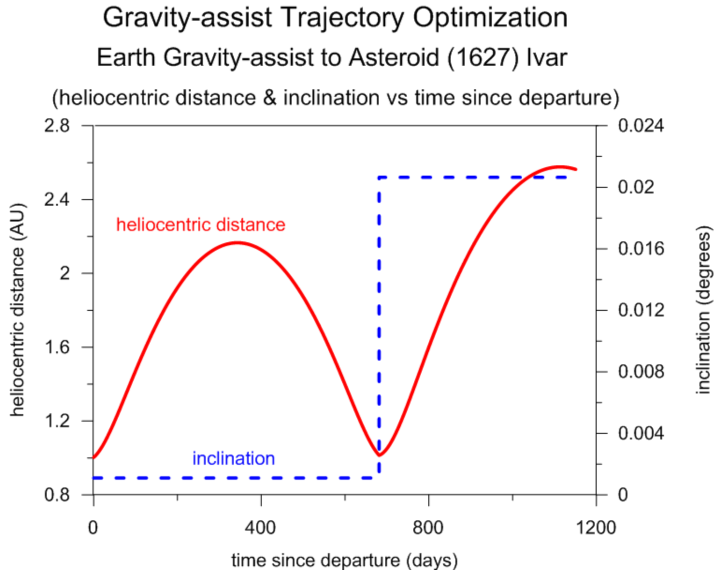
The following is a graphics display of the heliocentric transfer trajectory and the orbits of Earth and Ivar.



This next plot illustrates the behavior of the heliocentric speed and semimajor axis of the spacecraft's trajectory.



This final plot depicts the evolution of the spacecraft's heliocentric distance and ecliptic orbital inclination as a function of time since Earth departure.



Here's the `flyby_ftn` input data file for this example.

```
********************************************
patched-conic, gravity-assist interplanetary
trajectory optimization
Earth-to-Earth-to-Ivar 2006 - ega_ivar.in
December 1, 2011
********************************************

*******************
* simulation type *
*******************
 1 = minimize departure delta-v
 2 = minimize arrival delta-v
 3 = minimize total delta-v
---------------------------
1

departure calendar date initial guess (month, day, year)
9, 12, 2006

departure calendar date search boundary (days)
-60, +60

flyby calendar date initial guess (month, day, year)
8, 4, 2008

flyby calendar date search boundary (days)
-90, +90

arrival calendar date initial guess (month, day, year)
3, 19, 2010

arrival calendar date search boundary (days)
-180, +180

flyby altitude (kilometers)
5000.0
```

```
*************************
park orbit characteristics
*************************

altitude of circular park orbit (kilometers)
200.0d0

inclination of circular park orbit (degrees)
28.5d0

********************
* departure planet *
********************
 1 = Mercury
 2 = Venus
 3 = Earth
 4 = Mars
 5 = Jupiter
 6 = Saturn
 7 = Uranus
 8 = Neptune
 9 = Pluto
----------
3

****************
* flyby planet *
****************
 1 = Mercury
 2 = Venus
 3 = Earth
 4 = Mars
 5 = Jupiter
 6 = Saturn
 7 = Uranus
 8 = Neptune
 9 = Pluto
----------
3

*********************************
* arrival planet/asteroid/comet *
*********************************
 1 = Mercury
 2 = Venus
 3 = Earth
 4 = Mars
 5 = Jupiter
 6 = Saturn
 7 = Uranus
 8 = Neptune
 9 = Pluto
10 = asteroid/comet
-------------------
10

***********************************
* asteroid/comet orbital elements *
* (heliocentric, ecliptic J2000)  *
***********************************

asteroid/comet name
Ivar

calendar date of perihelion passage (month, day, year)
12, 21.22534918, 2010
```

```
perihelion distance (au)
1.123573843203009

orbital eccentricity (nd)
0.3969340266260379

orbital inclination (degrees)
8.447932354337741

argument of perihelion (degrees)
167.654127085269

longitude of the ascending node (degrees)
133.1744377376822
```

Here's the `flyby_ftn` solution for this asteroid rendezvous example.

```
program flyby_ftn - gravity assist mission design
-------------------------------------------------

minimize departure delta-v


departure heliocentric delta-v vector and magnitude
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

x-component of delta-v     3756.40867846233       meters/second
y-component of delta-v     4987.11443536249       meters/second
z-component of delta-v    -1.06470644910030       meters/second

delta-v magnitude          6243.55008665965       meters/second


arrival heliocentric delta-v vector and magnitude
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

x-component of delta-v    -236.504802232583       meters/second
y-component of delta-v     829.437112353355       meters/second
z-component of delta-v    -2143.40711307338       meters/second

delta-v magnitude          2310.43166901809       meters/second


heliocentric coordinates of the planet at departure
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

calendar date           September 25, 2006

TDB time                07:24:21.011

TDB Julian date         2454003.80857652

     sma (au)           eccentricity      inclination (deg)      argper (deg)
 0.100072516884D+01    0.167959030465D-01    0.773909527366D-03    0.979997937626D+02

    raan (deg)        true anomaly (deg)       arglat (deg)         period (days)
 0.253357022029D+01    0.261472992238D+03    0.359472786001D+03    0.365654279731D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 0.149945798569D+09    0.525287755257D+07    -.186477466905D+02    0.150037779346D+09
```

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.153426397601D+01	0.296684118206D+02	0.401263591097D-03	0.297080565852D+02

spacecraft heliocentric coordinates after the first impulse
(Earth mean ecliptic and equinox of J2000)
----

calendar date September 25, 2006

TDB time 07:24:21.011

TDB Julian date 2454003.80857652

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.157540716609D+01	0.374874022668D+00	0.109932537653D-02	0.159400748316D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.181635207659D+03	0.209704002466D+02	0.180371148562D+03	0.722250168446D+03
rx (km)	ry (km)	rz (km)	rmag (km)
0.149945798569D+09	0.525287755257D+07	-.186477466905D+02	0.150037779346D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.222214470245D+01	0.346555262560D+02	-.663442858003D-03	0.347266961803D+02

spacecraft heliocentric coordinates prior to the flyby
(Earth mean ecliptic and equinox of J2000)
----

calendar date August 6, 2008

TDB time 14:49:31.908

TDB Julian date 2454685.11773041

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.157540716609D+01	0.374874022668D+00	0.109932537653D-02	0.159400748316D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.181635207659D+03	0.333348559979D+03	0.132749308295D+03	0.722250168446D+03
rx (km)	ry (km)	rz (km)	rmag (km)
0.106126103714D+09	-.108431012767D+09	0.213770755787D+04	0.151723546043D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.214111660334D+02	0.269786922115D+02	-.505702346788D-03	0.344425298811D+02

spacecraft heliocentric coordinates after the flyby
(Earth mean ecliptic and equinox of J2000)
----

calendar date August 6, 2008

TDB time 14:49:31.908

TDB Julian date 2454685.11773041

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.179416156158D+01	0.436335242381D+00	0.206481968577D-01	0.354368296722D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.312143891242D+03	0.787232814020D+01	0.224062486216D+01	0.877790242511D+03

```
     rx (km)                ry (km)               rz (km)              rmag (km)
 0.106126103714D+09    -.108431012767D+09    0.213770755787D+04    0.151723546043D+09

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 0.263280907143D+02    0.237018342182D+02    0.127664589489D-01    0.354252095117D+02


heliocentric coordinates of planet at flyby
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

calendar date           August  6, 2008

TDB time                14:49:31.908

TDB Julian date         2454685.11773041

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.100041683289D+01    0.165572932949D-01    0.868320498586D-03    0.215281738304D+03

    raan (deg)         true anomaly (deg)       arglat (deg)         period (days)
 0.245998063559D+03    0.213104714097D+03    0.683864524019D+02    0.365485298754D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 0.106126103714D+09    -.108431012767D+09    0.213770755787D+04    0.151723546043D+09

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 0.208008688212D+02    0.207355660163D+02    0.160154017469D-03    0.293707310389D+02


spacecraft heliocentric coordinates prior to the second impulse
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

calendar date           November 18, 2009

TDB time                19:48:14.710

TDB Julian date         2455154.32517025

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.179416156158D+01    0.436335242381D+00    0.206481968577D-01    0.354368296722D+03

    raan (deg)         true anomaly (deg)       arglat (deg)         period (days)
 0.312143891242D+03    0.186659755473D+03    0.181028052195D+03    0.877790242511D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.262395846895D+09    0.279697536651D+09    -.247975579743D+04    0.383513092974D+09

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 -.935650415305D+01    -.104924438635D+02    -.503733061717D-02    0.140582919862D+02


spacecraft heliocentric coordinates after the second impulse
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

calendar date           November 18, 2009

TDB time                19:48:14.710

TDB Julian date         2455154.32517025

     sma (au)             eccentricity       inclination (deg)       argper (deg)
 0.186310269989D+01    0.396934026626D+00    0.844793235434D+01    0.167654127085D+03
```

```
    raan (deg)         true anomaly (deg)      arglat (deg)         period (days)
 0.133174437738D+03    0.192343351187D+03    0.359997478272D+03    0.928867183395D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.262395846895D+09    0.279697536651D+09    -.247975579743D+04    0.383513092974D+09

     vx (kps)               vy (kps)              vz (kps)             vmag (kps)
 -.911999935081D+01    -.113218809759D+02    0.213836978246D+01    0.146946249464D+02


heliocentric coordinates of celestial body at arrival
(Earth mean ecliptic and equinox of J2000)
-----------------------------------------

calendar date           November 18, 2009

TDB time                19:48:14.710

TDB Julian date         2455154.32517025

     sma (au)             eccentricity        inclination (deg)       argper (deg)
 0.186310269989D+01    0.396934026626D+00    0.844793235434D+01    0.167654127085D+03

    raan (deg)         true anomaly (deg)      arglat (deg)         period (days)
 0.133174437738D+03    0.192343351187D+03    0.359997478272D+03    0.928867183395D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.262395846895D+09    0.279697536651D+09    -.247975579743D+04    0.383513092974D+09

     vx (kps)               vy (kps)              vz (kps)             vmag (kps)
 -.911999935081D+01    -.113218809759D+02    0.213836978246D+01    0.146946249464D+02


FLYBY CONDITIONS
================

planet-centered conditions at periapsis
(Earth mean ecliptic and equinox J2000)
--------------------------------------

calendar date           August  6, 2008

TDB time                14:49:31.908

TDB Julian date         2454685.11773041

     sma (km)             eccentricity        inclination (deg)       argper (deg)
 -.101298521410D+05    0.212322863568D+01    0.179857227729D+03    0.295656541145D+03

    raan (deg)         true anomaly (deg)      arglat (deg)
 0.819753517765D+02    0.000000000000D+00    0.295656541145D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.946813107158D+04    0.630998500032D+04    -.255571461654D+02    0.113781400000D+05

     vx (kps)               vy (kps)              vz (kps)             vmag (kps)
 0.580084762021D+01    0.870421625658D+01    0.112854593962D-01    0.104600832270D+02


flyby altitude             5000.00000003827      kilometers

incoming v-infinity        6272.88512726392      meters/second

outgoing v-infinity        6272.88512727867      meters/second

maximum turn angle         75.7053471939008      degrees
```

```
actual turn angle          56.1959672798800      degrees

heliocentric delta-v       5908.81737543844      meters/second

max heliocentric delta-v   7905.36385686889      meters/second


planet-centered b-plane coordinates at flyby
(Earth mean ecliptic and equinox J2000)
--------------------------------------

b-magnitude                18973.1342044858      kilometers
b dot r                    47.2351687686644
b dot t                   -1843.91547582883
theta                      178.532587804828      degrees
v-infinity                 6272.88512726392      meters/second
r-periapsis                11378.1400000383      kilometers
decl-asymptote           -6.081852087604049E-003 degrees
rasc-asymptote             84.4167867010344      degrees


departure-to-flyby time    681.309153897688      days

flyby-to-arrival time      469.207439836580      days

total mission duration     1150.51659373427      days


--------------------------------------------------
park orbit and departure hyperbola characteristics
(Earth mean equator and equinox of J2000)
--------------------------------------------------

park orbit
----------

calendar date           September 25, 2006

TDB time                07:24:21.011

TDB Julian date         2454003.80857652

     sma (km)            eccentricity       inclination (deg)       argper (deg)
 0.657814000000D+04    0.982431794869D-16    0.285000000000D+02    0.000000000000D+00

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.125328762262D+02    0.274241709035D+03    0.274241709035D+03    0.884941166590D+02

     rx (km)               ry (km)              rz (km)              rmag (km)
 0.172598881444D+04    -.552219284340D+04    -.313021958851D+04    0.657814000000D+04

     vx (kps)             vy (kps)              vz (kps)              vmag (kps)
 0.746816157068D+01    0.217848364634D+01    0.274727222744D+00    0.778425997061D+01

departure hyperbola
-------------------

c3                         38.9819176846278      km**2/sec**2

v-infinity                 6243.55008665965      meters/second

decl-asymptote             18.5162092108578      degrees

rasc-asymptote             50.6176953774857      degrees
```

calendar date September 25, 2006

TDB time 07:24:21.011

TDB Julian date 2454003.80857652

sma (km)	eccentricity	inclination (deg)	argper (deg)
-.102252650761D+05	0.164332219762D+01	0.285000000000D+02	0.274241709035D+03

raan (deg)	true anomaly (deg)	arglat (deg)
0.125328762262D+02	0.000000000000D+00	0.274241709035D+03

rx (km)	ry (km)	rz (km)	rmag (km)
0.172598881444D+04	-.552219284340D+04	-.313021958851D+04	0.657814000000D+04

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.121419588498D+02	0.354184340260D+01	0.446659676803D+00	0.126558810150D+02

hyperbolic injection delta-v vector and magnitude
(Earth mean equator and equinox of J2000)
----

delta-vx	4673.79727907987	meters/seconds
delta-vx	1363.35975626720	meters/seconds
delta-vx	171.932454058516	meters/seconds
delta-v magnitude	4871.62104434467	meters/seconds

APPENDIX C

Interplanetary Injection from a Circular Park Orbit

The algorithm implemented in this scientific simulation assumes that the spacecraft is initially in a circular Earth park orbit. Furthermore, the orbital transfer maneuver is assumed to be impulsive which implies an instantaneous change in velocity but not change in position. In the following discussion, i is the orbital inclination of the initial circular Earth park orbit and δ_∞ is the declination of the outgoing or departure hyperbola.

Whenever $i > |\delta_\infty|$, there will be two coplanar opportunities to establish a departure hyperbola that will satisfy the energy and orientation of the outgoing asymptote. One injection opportunity will occur while the spacecraft is ascending and the other while the spacecraft is descending along the park orbit. For coplanar orbital transfer, the impulse is applied at the perigee of the departure hyperbola.

For the case where $|\delta_\infty| > i$, there will be a single *non-coplanar* injection opportunity.

Coplanar Transfer - Orientation of the park orbit and departure hyperbola

This section summarizes the equations used to determine the right ascension of the ascending node (RAAN) of the park orbit and the injection true anomaly on the park orbit.
A unit vector is the direction of the departure asymptote is given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where

$\alpha_\infty =$ right ascension of departure asymptote

$\delta_\infty =$ declination of departure asymptote

The angle between the outgoing asymptote and the spin axis of the Earth is given by

$$\beta = \cos^{-1}\left(\hat{\mathbf{s}} \bullet \hat{\mathbf{z}}\right)$$

where $\hat{\mathbf{z}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. Note that $\beta = 90^\circ - \delta_\infty$.

The park orbit right ascension of the ascending node for each opportunity can be determined from

$$\Omega_1 = 180^\circ + \alpha_\infty + \sin^{-1}\left(\frac{\cot\beta}{\tan i}\right)$$

$$\Omega_2 = 360^\circ + \alpha_\infty - \sin^{-1}\left(\frac{\cot\beta}{\tan i}\right)$$

The true anomaly on the park orbit for each injection opportunity can be determined from

$$\theta_1 = \cos^{-1}\left(\frac{\cos\beta}{\sin i}\right) - \eta$$

$$\theta_2 = -\cos^{-1}\left(\frac{\cos\beta}{\sin i}\right) - \eta$$

where

$$\eta = \sin^{-1}\left(\frac{1}{1 + r_p V_\infty^2 / \mu}\right)$$

In the last equation, r_p is the geocentric radius of the park orbit and μ is the gravitational constant of the Earth. The velocity vector at infinity V_∞ is determined from $V_\infty = \sqrt{C_3}$.

For a tangential impulsive injection maneuver that occurs at perigee of the hyperbola, the true anomaly on the hyperbola is zero. Furthermore, since the orbit transfer modeled by this software is coplanar, the right ascension of the ascending node computed above should be the same for both the park orbit and the departure hyperbola. This can be verified by examining the hyperbola's right ascension of the ascending node (RAAN) which is computed using the state vector at injection.

Coplanar Transfer - Departure delta-V

The velocity vector at any geocentric position vector $\mathbf{r}$ required to achieve a departure hyperbola defined by V_∞, α_∞ and δ_∞ is given by

$$\mathbf{v}_h = \left(d + \frac{1}{2}V_\infty\right)\hat{\mathbf{s}} + \left(d - \frac{1}{2}V_\infty\right)\hat{\mathbf{r}}$$

where

$$d = \sqrt{\frac{\mu}{(1 + \cos\psi) r_p} + \frac{V_\infty^2}{4}}$$

and ψ is the angle between the spacecraft's position vector and the departure asymptote unit vector which can be computed using

$$\cos\psi = \hat{\mathbf{s}} \bullet \hat{\mathbf{r}}$$

The injection $\Delta\mathbf{v}$ vector can be determined from the following expression

$$\Delta\mathbf{v} = \mathbf{v}_h - \mathbf{v}_p$$

where $\mathbf{v}_p$ is the inertial velocity vector in the park orbit prior to injection and $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$.

Finally, the scalar injection delta-v is $\Delta v = |\Delta \mathbf{v}|$. The injection delta-v is also given by

$$\Delta v = \sqrt{2\frac{\mu}{r_p} + V_\infty^2} - \sqrt{\frac{\mu}{r_p}}$$

Non-coplanar Transfer – Park Orbit Orientation and Departure delta-V

A geocentric unit vector is the direction of the departure asymptote is given by

$$\hat{\mathbf{i}}_\infty = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where

$\alpha_\infty =$ right ascension of departure asymptote

$\delta_\infty =$ declination of departure asymptote

The velocity vector of the spacecraft on the initial circular orbit is given by

$$\mathbf{v}_0 = \sqrt{\frac{\mu}{r}}\,\hat{\mathbf{i}}_\theta$$

The velocity vector at any geocentric position vector $\mathbf{r}$ required to achieve a departure hyperbola defined by v_∞, α_∞ and δ_∞ is given by

$$\mathbf{v}_1 = \frac{1}{2} v_\infty \left[(D+1)\hat{\mathbf{i}}_\infty + (D-1)\hat{\mathbf{i}}_r \right]$$

where

$$D = \sqrt{1 + \frac{4\mu}{r v_\infty^2 \left(1 + \hat{\mathbf{i}}_\infty \bullet \hat{\mathbf{i}}_r\right)}}$$

and

$$\hat{\mathbf{i}}_r = \begin{bmatrix} \cos\Omega\cos\theta - \sin\Omega\sin\theta\cos i \\ \sin\Omega\cos\theta + \cos\Omega\sin\theta\cos i \\ \sin\theta\sin i \end{bmatrix}$$

$$\hat{\mathbf{i}}_\theta = \begin{bmatrix} -\cos\Omega\sin\theta - \sin\Omega\cos\theta\cos i \\ -\sin\Omega\sin\theta + \cos\Omega\cos\theta\cos i \\ \cos\theta\sin i \end{bmatrix}$$

In these equations, Ω is the right ascension of the ascending node, *i* is the orbital inclination, θ is the true anomaly at injection, *r* is the geocentric radius of the park orbit and $v_\infty = \sqrt{C_3}$.

The injection $\Delta\mathbf{v}$ vector can be determined from the following expression

$$\Delta\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_0$$

Finally, the scalar injection delta-v is $\Delta v = \left|\Delta\mathbf{v}\right|$.

The orientation of the park orbit and departure hyperbola at injection is computed using a two-dimensional grid search involving the park orbit right ascension of the ascending node (RAAN) and the true anomaly of the impulsive maneuver on the park orbit. During the grid search, the `flyby_ftn` software uses a nonlinear programming (NLP) algorithm to find the current minimum delta-v and saves the RAAN and true anomaly values corresponding to the "best" delta-v.