

## Impulsive Hyperbolic Injection from a Circular Park Orbit

This document is the user's guide for a Matlab script named `hyper1.m` which can be used to determine the characteristics of a single impulsive maneuver from a circular park orbit to a departure hyperbola. The algorithm implemented in this script is based on the equations derived in Chapter 4 of Richard Battin's classic text, *Astronautical Guidance*, and Chapter 11 of *An Introduction to the Mathematics and Methods of Astrodynamics*, also written by Professor Battin and published by the American Institute of Aeronautics and Astronautics (AIAA). This script is valid for both tangential and non-tangential interplanetary injection.

The Earth departure trajectory for interplanetary missions is usually defined by a "targeting specification" which consists of twice the specific (per unit mass) orbital energy  $C_3$ , and the right ascension  $\alpha_\infty$  (RLA) and declination  $\delta_\infty$  (DLA) of the outgoing asymptote. These numbers may be supplied by a spacecraft customer or determined with a patched-conic or more sophisticated trajectory analysis computer program that solves Lambert's problem for an interplanetary mission.

The `hyper1.m` Matlab script determines the orbital elements and state vectors of the park orbit and departure hyperbola at injection, and the injection delta-v vector and magnitude. This information can be used as initial guesses for other trajectory simulations.

This computer program assumes that the hyperbolic targets and orbital characteristics are in the same Earth-centered-inertial (ECI) coordinate system. For example, targeting specs are often provided or computed in an Earth mean equator and equinox of J2000 coordinate system (EME2000). For this situation, the state vectors and orbital elements computed by this Matlab script will also be with respect to the EME2000 coordinate system.

### Running the Script

The script will interactively prompt the user for the park orbit altitude and orbital inclination, and the departure hyperbola characteristics. These prompts appear as follows;

```
please input the altitude of the circular park orbit (kilometers)
? 185.2

please input the orbital inclination of the park orbit (degrees)
(0 <= inclination <= 180)
? 28.5

please input the C3 of the departure hyperbola (km^2/sec^2)
(C3 > 0)
? 9.28

please input the right ascension of the outgoing asymptote (degrees)
(0 <= right ascension <= 360)
? 352.59

please input the declination of the outgoing asymptote (degrees)
? 2.27
```

Please note the proper units and valid data range for each input.

## Script Output

The following is the script output for this example. For a non-tangential injection case or a simulation where the declination is equal to the park orbit inclination, the script will display a single opportunity.

```
-----  
Interplanetary Injection from a Circular Park Orbit  
-----
```

```
departure hyperbola characteristics  
-----
```

```
c3                9.280000  km^2/sec^2  
asymptote right ascension    352.590000  degrees  
asymptote declination        2.270000  degrees
```

```
orbital elements and state vector of park orbit at injection - opportunity #1  
-----
```

```
      sma (km)      eccentricity      inclination (deg)      argper (deg)  
+6.5633400000e+003 +0.0000000000e+000 +2.8500000000e+001 +0.0000000000e+000  
      raan (deg)      true anomaly (deg)      arglat (deg)      period (min)  
+1.7677673367e+002 +2.5074779915e+001 +2.5074779915e+001 +8.8195627034e+001  
      rx (km)      ry (km)      rz (km)      rmag (km)  
-6.0728215133e+003 -2.1063485210e+003 +1.3272402690e+003 +6.5633400000e+003  
      vx (kps)      vy (kps)      vz (kps)      vmag (kps)  
+2.9486811758e+000 -6.3790889123e+000 +3.3680638606e+000 +7.7930321568e+000
```

```
orbital elements and state vector of hyperbola at injection - opportunity #1  
-----
```

```
      sma (km)      eccentricity      inclination (deg)      argper (deg)  
-4.2952640086e+004 +1.1528041114e+000 +2.8500000000e+001 +2.5074779915e+001  
      raan (deg)      true anomaly (deg)      arglat (deg)      period (min)  
+1.7677673367e+002 +3.6000000000e+002 +2.5074779915e+001 +0.0000000000e+000  
      rx (km)      ry (km)      rz (km)      rmag (km)  
-6.0728215133e+003 -2.1063485210e+003 +1.3272402690e+003 +6.5633400000e+003  
      vx (kps)      vy (kps)      vz (kps)      vmag (kps)  
+4.3264339149e+000 -9.3596780970e+000 +4.9417705223e+000 +1.1434277432e+001
```

```
injection delta-v vector and magnitude - opportunity #1  
-----
```

```
x-component of delta-v    1377.752739  meters/second  
y-component of delta-v    -2980.589185 meters/second  
z-component of delta-v    1573.706662  meters/second  
  
delta-v magnitude        3641.245275  meters/second
```

orbital elements and state vector of park orbit at injection - opportunity #2

```
-----  
      sma (km)      eccentricity      inclination (deg)      argper (deg)  
+6.5633400000e+003 +0.0000000000e+000 +2.8500000000e+001 +0.0000000000e+000  
  
      raan (deg)      true anomaly (deg)      arglat (deg)      period (min)  
+3.4840326633e+002 +2.1459790192e+002 +2.1459790192e+002 +8.8195627034e+001  
  
      rx (km)      ry (km)      rz (km)      rmag (km)  
-5.9507486892e+003 -2.1222246783e+003 -1.7782531903e+003 +6.5633400000e+003  
  
      vx (kps)      vy (kps)      vz (kps)      vmag (kps)  
+3.2013960364e+000 -6.4119556941e+000 -3.0609210692e+000 +7.7930321568e+000
```

orbital elements and state vector of hyperbola at injection - opportunity #2

```
-----  
      sma (km)      eccentricity      inclination (deg)      argper (deg)  
-4.2952640086e+004 +1.1528041114e+000 +2.8500000000e+001 +2.1459790192e+002  
  
      raan (deg)      true anomaly (deg)      arglat (deg)      period (min)  
+3.4840326633e+002 +0.0000000000e+000 +2.1459790192e+002 +0.0000000000e+000  
  
      rx (km)      ry (km)      rz (km)      rmag (km)  
-5.9507486892e+003 -2.1222246783e+003 -1.7782531903e+003 +6.5633400000e+003  
  
      vx (kps)      vy (kps)      vz (kps)      vmag (kps)  
+4.6972282050e+000 -9.4079016759e+000 -4.4911171927e+000 +1.1434277432e+001
```

injection delta-v vector and magnitude - opportunity #2

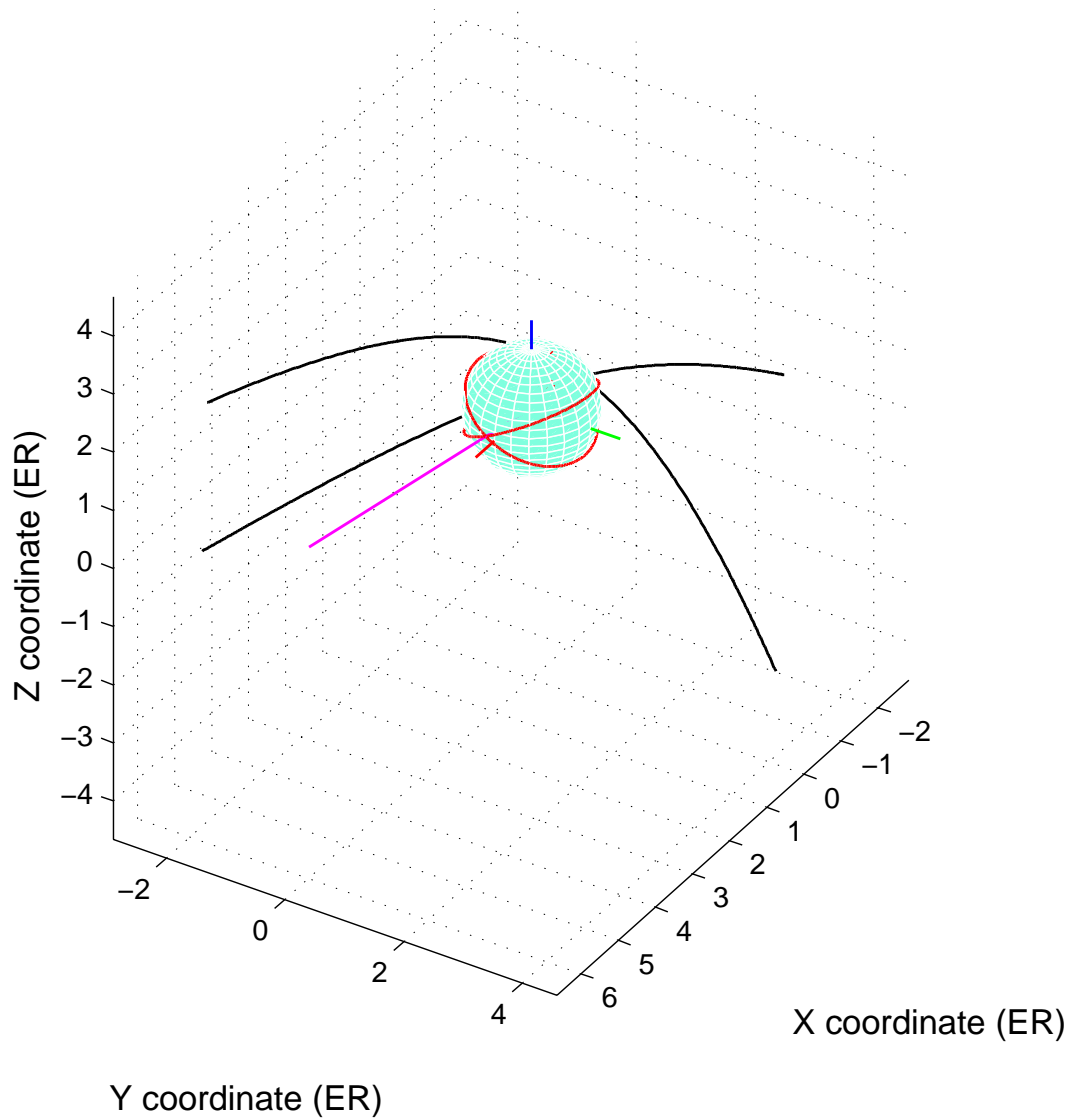
```
-----  
x-component of delta-v      1495.832169      meters/second  
y-component of delta-v      -2995.945982      meters/second  
z-component of delta-v      -1430.196124      meters/second  
  
delta-v magnitude      3641.245275      meters/second
```

## Trajectory Graphics

The `hyper1.m` Matlab script will also create a graphics display of the park orbit and departure hyperbola for the possible opportunities. The interactive graphic features of Matlab permit the user to rotate and zoom this display. These capabilities allow the user to interactively find the best viewpoint as well as verify basic orbital geometry of the park orbit and departure trajectory.

This script will also create a disk file of this display called `hyper1.eps`. This file is a color Postscript image with a TIFF preview.

The following is the display for this example. This display is labeled with an Earth centered, inertial coordinate system. The x-axis of this system is red, the y-axis is green and the z-axis is blue. The outgoing asymptote is colored magenta, the park orbit traces are red, and the hyperbolic trajectories are black. Please note the units for each coordinate axis are Earth radii (ER).



### Technical Discussion

This section describes the numerical algorithms implemented in this Matlab script. The discussion describes computations for both tangential and non-tangential injection. The first part explains calculations that are common to both types of interplanetary injection. The script assumes that injection occurs impulsively at perigee of the departure hyperbola.

A geocentric unit vector in the direction of the departure asymptote is given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos \delta_{\infty} \cos \alpha_{\infty} \\ \cos \delta_{\infty} \sin \alpha_{\infty} \\ \sin \delta_{\infty} \end{Bmatrix}$$

where

$\alpha_\infty$  = right ascension of departure asymptote

$\delta_\infty$  = declination of departure asymptote

The angle between the outgoing asymptote and the spin axis of the Earth is given by

$$\beta = \cos^{-1}(\hat{\mathbf{s}} \cdot \hat{\mathbf{z}})$$

where  $\hat{\mathbf{z}} = [0 \ 0 \ 1]^T$ . Note that  $\beta = 90^\circ - \delta_\infty$ .

### *Departure delta-V*

The velocity vector at any geocentric position vector  $\mathbf{r}$  required to achieve a launch hyperbola defined by  $V_\infty$ ,  $\alpha_\infty$  and  $\delta_\infty$  is given by

$$\mathbf{v}_h = \left( d + \frac{1}{2} V_\infty \right) \hat{\mathbf{s}} + \left( d - \frac{1}{2} V_\infty \right) \hat{\mathbf{r}}$$

where

$$d = \sqrt{\frac{\mu}{(1 + \cos \psi) r_p} + \frac{V_\infty^2}{4}}$$

and  $\psi$  is the angle between the spacecraft's position vector and the departure asymptote unit vector which can be computed using

$$\cos \psi = \hat{\mathbf{s}} \cdot \hat{\mathbf{r}}$$

The injection  $\Delta \mathbf{v}$  vector can be determined from the following expression

$$\Delta \mathbf{v} = \mathbf{v}_h - \mathbf{v}_p$$

where  $\mathbf{v}_p$  is the inertial velocity vector in the park orbit prior to injection and  $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$ .

Finally, the scalar injection delta-v is  $\Delta v = |\Delta \mathbf{v}|$ .

### ***Tangential injection*** ( $i \geq |\delta_\infty|$ )

This part of the script is valid for geocentric orbit inclinations that satisfy the following geometric constraint

$$i \geq |\delta_\infty|$$

where  $i$  is the orbital inclination of the park orbit.

Whenever  $i > |\delta_\infty|$ , there will be two opportunities to establish a departure hyperbola that will satisfy the energy and orientation of the outgoing asymptote. One injection opportunity will occur while the spacecraft is ascending and the other while the spacecraft is descending along the park orbit. For the case where  $i = |\delta_\infty|$ , there will be a single injection opportunity.

### *Orientation of the park orbit and departure hyperbola*

This section summarizes the equations used to determine the right ascension of the ascending node (RAAN) of the park orbit and the injection true anomaly on the park orbit.

The park orbit right ascension of the ascending node for each opportunity can be determined from

$$\Omega_1 = 180^\circ + \alpha_\infty + \sin^{-1}\left(\frac{\cot \beta}{\tan i}\right)$$

$$\Omega_2 = 360^\circ + \alpha_\infty - \sin^{-1}\left(\frac{\cot \beta}{\tan i}\right)$$

The true anomaly on the park orbit for each injection opportunity can be determined from

$$\theta_1 = \cos^{-1}\left(\frac{\cos \beta}{\sin i}\right) - \eta$$

$$\theta_2 = -\cos^{-1}\left(\frac{\cos \beta}{\sin i}\right) - \eta$$

where

$$\eta = \sin^{-1}\left(\frac{1}{1 + r_p V_\infty^2 / \mu}\right)$$

In the last equation,  $r_p$  is the geocentric radius of the park orbit and  $\mu$  is the gravitational constant of the Earth. The speed at infinity  $V_\infty$  is determined from  $V_\infty = \sqrt{C_3}$ .

For a tangential impulsive injection maneuver that occurs at perigee of the hyperbola, the true anomaly on the hyperbola is zero. Furthermore, since the orbit transfer is coplanar, the right ascension of the ascending node computed above should be the same for both the park orbit and the launch hyperbola. This can be verified by examining the hyperbola's RAAN which is computed using the state vector at injection.

### ***Non-tangential injection*** ( $\delta_\infty \geq i$ )

This case involves high declination trajectories that require a non-tangential impulsive because the angle between the park orbit and departure hyperbola orbit planes is nonzero.

A unit vector normal to the park orbit plane can be computed from

$$\hat{\mathbf{h}} = \frac{\sin(i + \beta)}{\sin \beta} \hat{\mathbf{z}} - \frac{\sin i}{\sin \beta} \hat{\mathbf{s}}$$

A unit vector in the direction of the ascending node of the park orbit is given by

$$\hat{\mathbf{n}} = \frac{\hat{\mathbf{z}} \times \hat{\mathbf{h}}}{\sin i}$$

The right ascension of the ascending node of the park orbit can be computed from the x and y components of the node vector using a four quadrant inverse tangent according to

$$\Omega = \tan^{-1}(n_y, n_x)$$

Finally, the true anomaly of the injection impulse on the park orbit is given by

$$\theta = 360^\circ - \sin^{-1} \left\{ \frac{\sin \eta}{\sin(i + \beta)} \right\}$$

The following is the script output and the graphics display for a typical non-tangential injection using the same park orbit, C3 and RLA as the previous example..

```
-----
Interplanetary Injection from a Circular Park Orbit
-----
```

```
departure hyperbola characteristics
-----
```

```
c3                9.280000  km^2/sec^2
asymptote right ascension    352.590000  degrees
asymptote declination        35.000000  degrees
```

```
orbital elements and state vector of park orbit at injection - opportunity #1
-----
```

```

      sma (km)      eccentricity      inclination (deg)      argper (deg)
+6.5633400000e+003  +0.0000000000e+000  +2.8500000000e+001  +0.0000000000e+000

      raan (deg)    true anomaly (deg)    arglat (deg)      period (min)
+2.6259000000e+002  +2.9918353533e+002  +2.9918353533e+002  +8.8195627034e+001

      rx (km)      ry (km)      rz (km)      rmag (km)
-5.4064898347e+003  -2.5241545283e+003  -2.7342171713e+003  +6.5633400000e+003

      vx (kps)      vy (kps)      vz (kps)      vmag (kps)
+2.4340956904e+000  -7.1776672373e+000  +1.8131799130e+000  +7.7930321568e+000
```

orbital elements and state vector of hyperbola at injection - opportunity #1

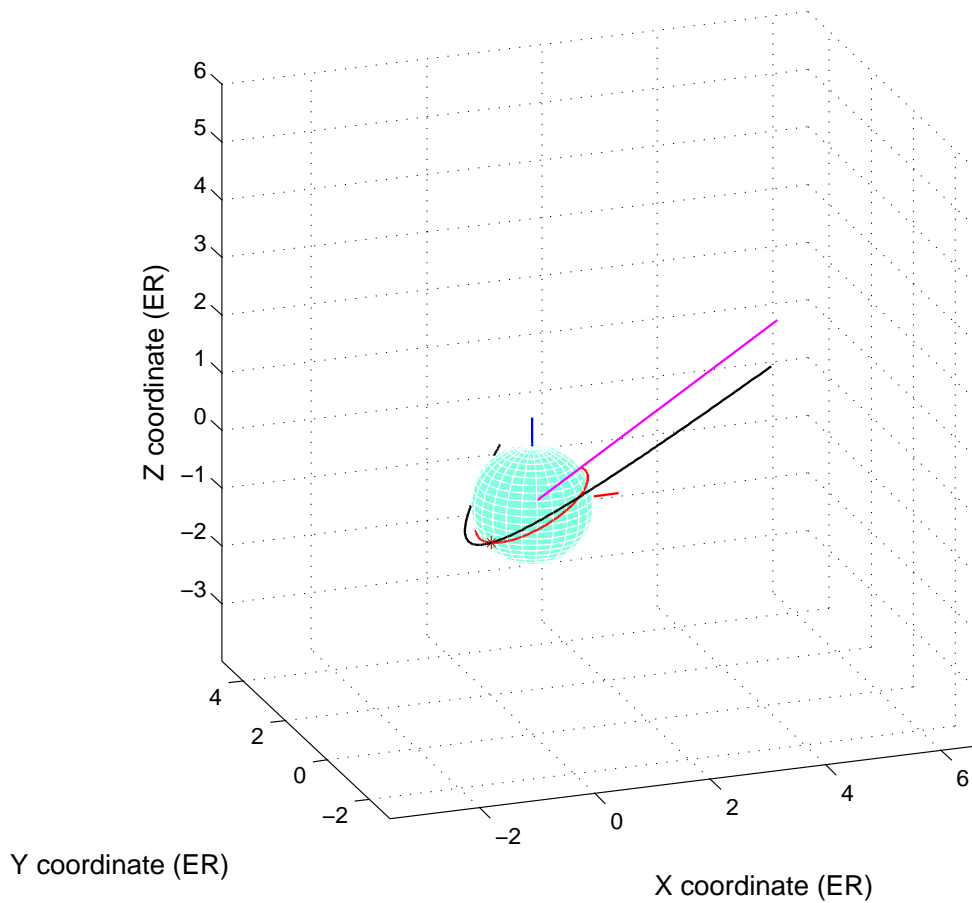
```
-----
```

sma (km)	eccentricity	inclination (deg)	argper (deg)
-4.2952640086e+004	+1.1528041114e+000	+3.6599116896e+001	+3.1567510430e+002
raan (deg)	true anomaly (deg)	arglat (deg)	period (min)
+2.4312764530e+002	+3.6000000000e+002	+3.1567510430e+002	+0.0000000000e+000
rx (km)	ry (km)	rz (km)	rmag (km)
-5.4064898347e+003	-2.5241545283e+003	-2.7342171713e+003	+6.5633400000e+003
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+2.2466896700e+000	-1.0095049278e+001	+4.8769935414e+000	+1.1434277432e+001

injection delta-v vector and magnitude - opportunity #1

```
-----
```

x-component of delta-v	-187.406020	meters/second
y-component of delta-v	-2917.382040	meters/second
z-component of delta-v	3063.813628	meters/second
delta-v magnitude	4234.760080	meters/second



## Delta-v penalty for off-nominal injection

The velocity-required equation given above can also be used to access the delta-v penalty for off-nominal injection. Such things as ignition timing errors and other spacecraft contingencies may result in an injection maneuver that does not occur at the optimal true anomaly on the park orbit.

The following plot illustrates how the injection delta-v penalty changes as the true anomaly at injection is displaced from the optimal.

