

Impulsive Hyperbolic Injection from a Circular Park Orbit

This document is the user's guide for a Matlab script named `hyper1.m` which can be used to determine the characteristics of the single impulsive maneuver required to transfer a spacecraft from an initial circular park orbit to a departure hyperbola. The algorithm implemented in this script is based on the equations derived in Chapter 4 of Richard Battin's classic text, *Astronautical Guidance*, and Chapter 11 of *An Introduction to the Mathematics and Methods of Astrodynamics*, also written by Professor Battin and published by the American Institute of Aeronautics and Astronautics (AIAA).

The Earth departure trajectory for interplanetary missions is usually defined by a "targeting specification" which consists of twice the specific (per unit mass) orbital energy C_3 , and the right ascension α_∞ (RLA) and declination δ_∞ (DLA) of the outgoing asymptote. These targets may be supplied by a spacecraft customer or determined with a patched-conic or more sophisticated trajectory analysis computer program that solves Lambert's problem for an interplanetary mission.

The `hyper1.m` Matlab script determines the orbital elements and state vectors of the park orbit and departure hyperbola at injection, and the injection delta-v vector and magnitude for one or two possible interplanetary injection opportunities. This information can be used as initial guesses for other trajectory simulations.

This computer program assumes that the hyperbolic targets and orbital characteristics are in the same Earth-centered-inertial (ECI) coordinate system. For example, targeting specs are often provided or computed in an Earth mean equator and equinox of J2000 coordinate system (EME2000). For this situation, the state vectors and orbital elements computed by this Matlab script will also be with respect to the EME2000 coordinate system.

Running the Script

The script will interactively prompt the user for the park orbit altitude and orbital inclination, and the departure hyperbola characteristics. These prompts appear as follows;

```
please input the altitude of the circular park orbit (kilometers)
? 185.2

please input the orbital inclination of the park orbit (degrees)
(0 <= inclination <= 180)
? 28.5

please input the C3 of the departure hyperbola (km^2/sec^2)
(C3 > 0)
? 9.28

please input the right ascension of the outgoing asymptote (degrees)
(0 <= right ascension <= 360)
? 352.59

please input the declination of the outgoing asymptote (degrees)
? 2.27
```

Please note the proper units and valid data range for each input.

Script Output

The following is the script output for this example. For a simulation where the absolute value of the declination is equal to the park orbit inclination, the script will display a single opportunity.

```
-----  
Interplanetary Injection from a Circular Park Orbit  
-----  
  
departure hyperbola characteristics  
-----  
  
c3                      9.280000  km^2/sec^2  
  
asymptote right ascension    352.590000  degrees  
  
asymptote declination        2.270000  degrees  
  
orbital elements and state vector of park orbit at injection - opportunity #1  
-----  
  
      sma (km)      eccentricity      inclination (deg)      argper (deg)  
+6.5633400000e+003 +0.0000000000e+000 +2.8500000000e+001 +0.0000000000e+000  
  
      raan (deg)      true anomaly (deg)      arglat (deg)      period (min)  
+1.7677673367e+002 +2.5074779915e+001 +2.5074779915e+001 +8.8195627034e+001  
  
      rx (km)      ry (km)      rz (km)      rmag (km)  
-6.0728215133e+003 -2.1063485210e+003 +1.3272402690e+003 +6.5633400000e+003  
  
      vx (kps)      vy (kps)      vz (kps)      vmag (kps)  
+2.9486811758e+000 -6.3790889123e+000 +3.3680638606e+000 +7.7930321568e+000  
  
orbital elements and state vector of hyperbola at injection - opportunity #1  
-----  
  
      sma (km)      eccentricity      inclination (deg)      argper (deg)  
-4.2952640086e+004 +1.1528041114e+000 +2.8500000000e+001 +2.5074779915e+001  
  
      raan (deg)      true anomaly (deg)      arglat (deg)  
+1.7677673367e+002 +3.6000000000e+002 +2.5074779915e+001  
  
      rx (km)      ry (km)      rz (km)      rmag (km)  
-6.0728215133e+003 -2.1063485210e+003 +1.3272402690e+003 +6.5633400000e+003  
  
      vx (kps)      vy (kps)      vz (kps)      vmag (kps)  
+4.3264339149e+000 -9.3596780970e+000 +4.9417705223e+000 +1.1434277432e+001  
  
injection delta-v vector and magnitude - opportunity #1  
-----  
  
x-component of delta-v      1377.752739  meters/second  
y-component of delta-v      -2980.589185  meters/second  
z-component of delta-v      1573.706662  meters/second  
  
delta-v magnitude      3641.245275  meters/second  
  
orbital elements and state vector of park orbit at injection - opportunity #2  
-----
```

sma (km)	eccentricity	inclination (deg)	argper (deg)
+6.5633400000e+003	+0.0000000000e+000	+2.8500000000e+001	+0.0000000000e+000
raan (deg)	true anomaly (deg)	arglat (deg)	period (min)
+3.4840326633e+002	+2.1459790192e+002	+2.1459790192e+002	+8.8195627034e+001
rx (km)	ry (km)	rz (km)	rmag (km)
-5.9507486892e+003	-2.1222246783e+003	-1.7782531903e+003	+6.5633400000e+003
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+3.2013960364e+000	-6.4119556941e+000	-3.0609210692e+000	+7.7930321568e+000

orbital elements and state vector of hyperbola at injection - opportunity #2

sma (km)	eccentricity	inclination (deg)	argper (deg)
-4.2952640086e+004	+1.1528041114e+000	+2.8500000000e+001	+2.1459790192e+002
raan (deg)	true anomaly (deg)	arglat (deg)	
+3.4840326633e+002	+0.0000000000e+000	+2.1459790192e+002	
rx (km)	ry (km)	rz (km)	rmag (km)
-5.9507486892e+003	-2.1222246783e+003	-1.7782531903e+003	+6.5633400000e+003
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
+4.6972282050e+000	-9.4079016759e+000	-4.4911171927e+000	+1.1434277432e+001

injection delta-v vector and magnitude - opportunity #2

x-component of delta-v	1495.832169	meters/second
y-component of delta-v	-2995.945982	meters/second
z-component of delta-v	-1430.196124	meters/second
delta-v magnitude	3641.245275	meters/second

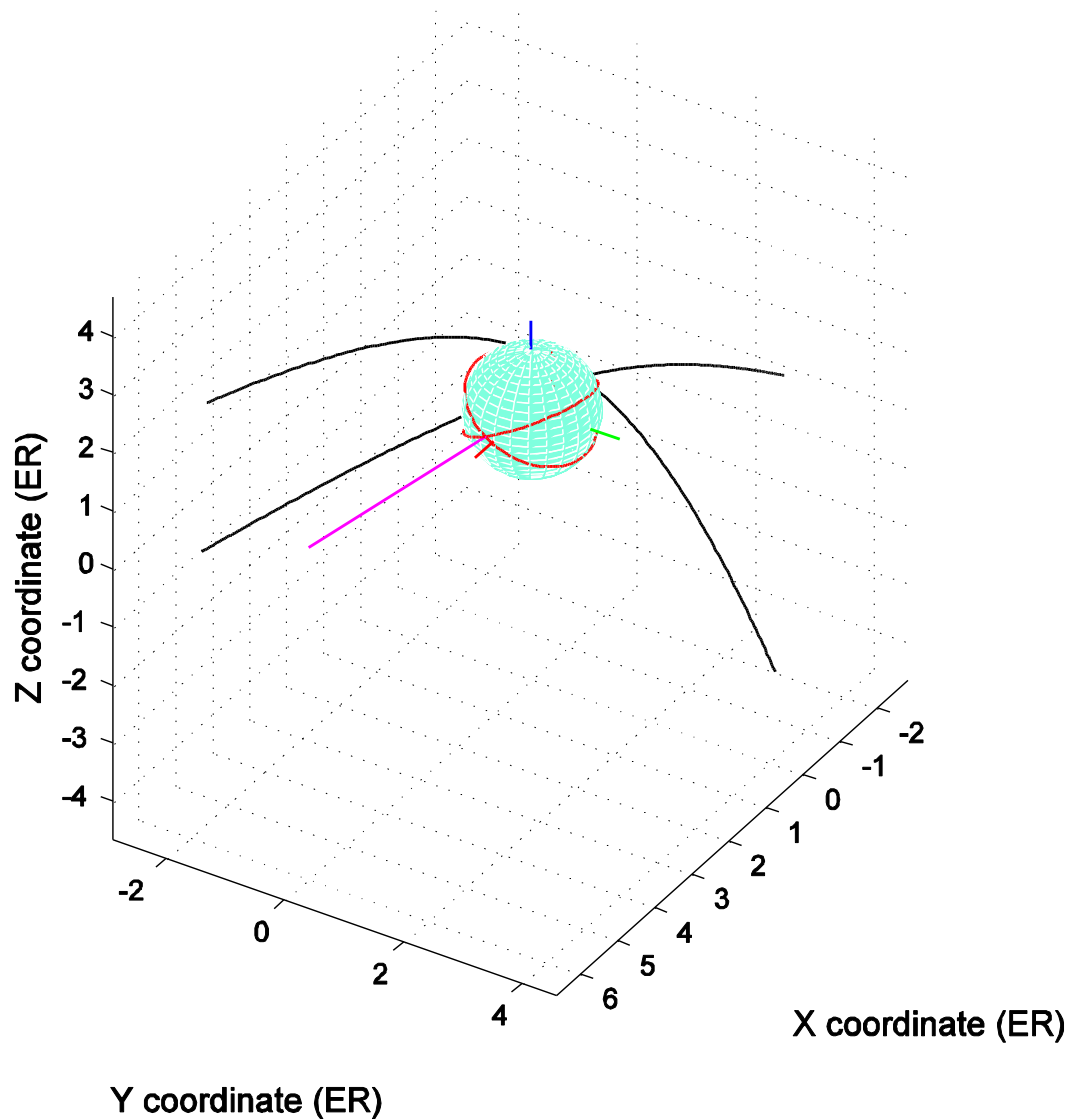
Trajectory Graphics

The `hyper1.m` Matlab script will also create a graphics display of the park orbit and departure hyperbola for the possible opportunities. The interactive graphic features of Matlab permit the user to rotate and zoom this display. These capabilities allow the user to interactively find the best viewpoint as well as verify basic orbital geometry of the park orbit and departure trajectory.

This script will also create a disk file of this graphics display called `hyper1.eps`. This file is a color Postscript image with a TIFF preview. The name and other characteristics of this file can be edited in the following line of Matlab source code;

```
print -depsc -tiff -r300 hyper1.eps
```

The following is the display for this example. This display is labeled with an Earth centered, inertial coordinate system. The x-axis of this system is red, the y-axis is green and the z-axis is blue. The outgoing asymptote is colored magenta, the park orbit traces are red, and the hyperbolic trajectories are black. Please note the units for each coordinate axis are Earth radii (ER).



Technical Discussion

This section describes the numerical algorithms implemented in this Matlab script. The script assumes that injection occurs impulsively at perigee of the departure hyperbola.

This MATLAB script is valid for geocentric orbit inclinations that satisfy the following geometric constraint

$$i \geq |\delta_\infty|$$

where i is the orbital inclination of the park orbit.

Whenever $i > |\delta_\infty|$, there will be two opportunities to establish a departure hyperbola that will satisfy the energy and orientation of the outgoing asymptote. One injection opportunity will occur while the spacecraft is ascending and the other while the spacecraft is descending along the park orbit. For the case where $i = |\delta_\infty|$, there will be a single injection opportunity.

A geocentric unit vector in the direction of the departure asymptote is given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos \delta_{\infty} \cos \alpha_{\infty} \\ \cos \delta_{\infty} \sin \alpha_{\infty} \\ \sin \delta_{\infty} \end{Bmatrix}$$

where

α_{∞} = right ascension of departure asymptote

δ_{∞} = declination of departure asymptote

The angle between the outgoing asymptote and the spin axis of the Earth is given by

$$\beta = \cos^{-1}(\hat{\mathbf{s}} \cdot \hat{\mathbf{z}})$$

where $\hat{\mathbf{z}} = [0 \ 0 \ 1]^T$. Note that $\beta = 90^\circ - \delta_{\infty}$.

Departure delta-V

The velocity vector at any geocentric position vector \mathbf{r} required to achieve a launch hyperbola defined by V_{∞} , α_{∞} and δ_{∞} is given by

$$\mathbf{v}_h = \left(d + \frac{1}{2}V_{\infty}\right)\hat{\mathbf{s}} + \left(d - \frac{1}{2}V_{\infty}\right)\hat{\mathbf{r}}$$

where

$$d = \sqrt{\frac{\mu}{(1 + \cos \psi)r_p} + \frac{V_{\infty}^2}{4}}$$

and ψ is the angle between the spacecraft's position vector and the departure asymptote unit vector which can be computed using

$$\cos \psi = \hat{\mathbf{s}} \cdot \hat{\mathbf{r}}$$

The position vector of the spacecraft prior to and immediately after the injection maneuver consists of the following three Cartesian components;

$$r_x = r[\cos \Omega \cos \theta - \sin \Omega \cos i \sin \theta]$$

$$r_y = r[\sin \Omega \cos \theta + \cos \Omega \cos i \sin \theta]$$

$$r_z = r \sin i \sin \theta$$

where r is the geocentric radius at injection, Ω is the right ascension of the ascending node, i is the orbital inclination and θ is the true anomaly.

The injection $\Delta \mathbf{v}$ vector can be determined from the following expression

$$\Delta \mathbf{v} = \mathbf{v}_h - \mathbf{v}_p$$

where \mathbf{v}_p is the inertial velocity vector in the park orbit prior to injection and $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$.

Finally, the scalar injection delta-v is $\Delta v = |\Delta \mathbf{v}|$.

Orientation of the park orbit and departure hyperbola

This section summarizes the equations used to determine the right ascension of the ascending node (RAAN) of the park orbit and the injection true anomaly on the circular park orbit.

The park orbit right ascension of the ascending node for each opportunity can be determined from

$$\Omega_1 = 180^\circ + \alpha_\infty + \sin^{-1} \left(\frac{\cot \beta}{\tan i} \right)$$

$$\Omega_2 = 360^\circ + \alpha_\infty - \sin^{-1} \left(\frac{\cot \beta}{\tan i} \right)$$

The true anomaly on the park orbit for each injection opportunity can be determined from

$$\theta_1 = \cos^{-1} \left(\frac{\cos \beta}{\sin i} \right) - \eta$$

$$\theta_2 = -\cos^{-1} \left(\frac{\cos \beta}{\sin i} \right) - \eta$$

where

$$\eta = \sin^{-1} \left(\frac{1}{1 + r_p V_\infty^2 / \mu} \right)$$

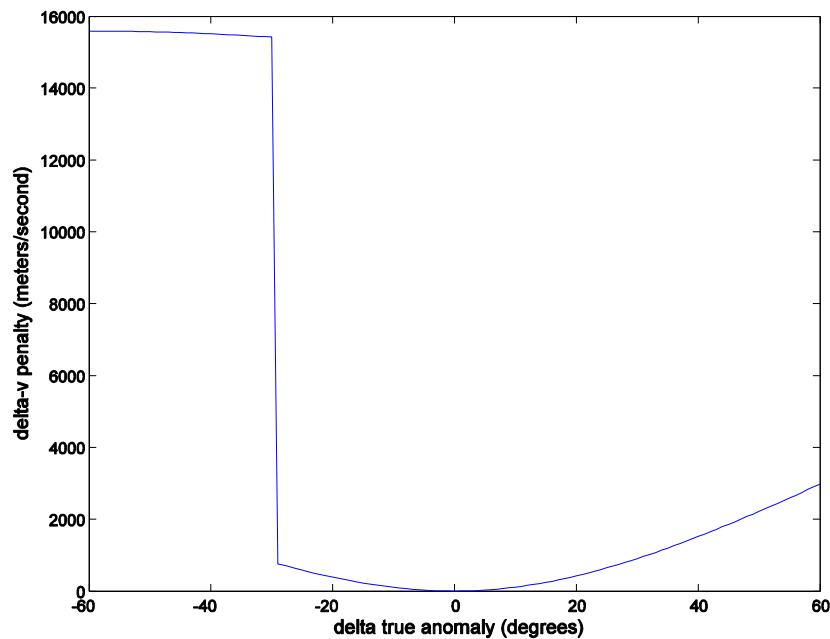
In the last equation, r_p is the geocentric radius of the circular park orbit and μ is the gravitational constant of the Earth. The speed at infinity V_∞ is determined from $V_\infty = \sqrt{C_3}$.

For an impulsive injection maneuver that occurs at perigee of the hyperbola, the true anomaly on the hyperbola is zero by definition. Furthermore, since the orbit transfer is coplanar, the right ascension of the ascending node computed above should be the same for both the Earth park orbit and the launch hyperbola. This can be verified by examining the hyperbola's RAAN which is computed using the state vector at injection.

Delta-v penalty for off-nominal injection

The velocity-required equation given above can also be used to access the delta-v penalty for off-nominal injection. Such things as ignition timing errors and other spacecraft contingencies may result in an injection maneuver that does not occur at the optimal true anomaly on the park orbit.

The following plot illustrates how the injection delta-v penalty changes as the true anomaly at injection is displaced from the optimal.



Algorithm Resources

“Design of Lunar and Interplanetary Ascent Trajectories”, Victor C. Clarke, Jr., JPL Technical Report No. 32-30, March 15, 1962.

An Introduction to the Mathematics and Methods of Astrodynamics, Richard H. Battin, AIAA Education Series, 1987.

Analytical Mechanics of Space Systems, Hanspeter Schaub and John L. Junkins, AIAA Education Series, 2003.

Spacecraft Mission Design, Charles D. Brown, AIAA Education Series, 1992.

Orbital Mechanics, Vladimir A. Chobotov, AIAA Education Series, 2002.

“A Computer Simulation of the Orbital Launch Window Problem”, Archie C. Young and Pat R. Odom, AIAA 67-615, 1967.

“Launch Parameters for Interplanetary Flights”, W. C. Riddell, *American Rocket Society Journal*, December 1960.