Converting an ECI State Vector to Hyperbolic Coordinates

`hyperbola.exe` is a PC-compatible Fortran computer program that can be used to convert Earth-centered-inertial (ECI) position and velocity vectors to the corresponding hyperbolic coordinates RLA (right ascension of the asymptote), DLA (geocentric declination of the asymptote) and C3 (twice the specific orbital energy).

The software is invoked by typing the word hyperbola followed by the name of an input data file from a DOS command line. For example,

hyperbola hyper1.in

where `hyper1.in` is the name of an input data file.

Each data item within an input file is preceded by one or more lines of annotation text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input. ASCII text input is not case sensitive but must be spelled correctly.

The following are the contents of a typical data file (`hyper2.in`);

```
*****************************************************
input data file for program hyperbola.exe - hyper2.in
conversion of an ECI state vector to C3, RLA and DLA
*****************************************************

system of units (1 = english, 2 = metric)
-----------------------------------------
2

gravitational constant (ft**3/sec**2 or km**3/sec**2)
-----------------------------------------------------
398600.4415d0

x-component of ECI position vector (feet or kilometers)
-------------------------------------------------------
3138.76206D0

y-component of ECI position vector (feet or kilometers)
-------------------------------------------------------
-7739.06258D0

z-component of ECI position vector (feet or kilometers)
-------------------------------------------------------
1597.53617D0

x-component of ECI velocity vector (feet/second or kilometers/second)
---------------------------------------------------------------------
7.77691883D0

y-component of ECI velocity vector (feet/second or kilometers/second)
```

```
-------------------------------------------------------------------------
-1.20197070D0

z-component of ECI velocity vector (feet/second or kilometers/second)
-------------------------------------------------------------------------
6.95070671D0
```

The following is the program output for this example;

```
program hyperbola
=================

conversion of an ECI state vector to C3, RLA and DLA
----------------------------------------------------


gravitational constant    398600.441500000      km**3/sec**2


eci position vector and magnitude (kilometers)

reci-x                     3138.76206000000
reci-y                    -7739.06258000000
reci-z                     1597.53617000000

r-magnitude                8502.76653218880


eci velocity vector and magnitude (kilometers/second)

veci-x                     7.77691883000000
veci-y                    -1.20197070000000
veci-z                     6.95070671000000

v-magnitude                10.4994058794060


asymptote coordinates and energy
--------------------------------

right ascension (RLA)      29.6501671123395      degrees

declination (DLA)          41.3950361435085      degrees

orbital energy (C3)        16.4797004130248      (km/sec)**2


classical orbital elements
--------------------------

     sma (km)         eccentricity       inclination (deg)     argper (deg)
 -.2418735969D+05    0.1272135732D+01    0.4305914990D+02    0.3225983094D+03

    raan (deg)      true anomaly (deg)     arglat (deg)
 0.2802636503D+03    0.5337480766D+02    0.1597311705D+02
```

PROGRAM NOTES

(2) To invoke a DOS command window, click Start, then Programs, then Accessories and finally Command Prompt.

(2) If you do not provide the name of an input data file on the command line, the software will interactively prompt you for a name with the following

```
            ********************************
            *      program hyperbola       *
            *                              *
            *  conversion of an ECI state  *
            *  vector to C3, RLA and DLA   *
            *                              *
            *        March 1, 2007         *
            ********************************


    please input the name of a data file
```

(3) The input can be input either in the English or metric system. However, the units for the gravitational constant and ECI state vector must be consistent. C3 is always computed and displayed in the metric system.

(4) The angular elements RLA and DLA are displayed in the same coordinate system as the input ECI state vector. For example, if the ECI state vector is in the EME2000 system, RLA and DLA will also be in this system.

(5) If the orbit is not hyperbolic, the software will display the following error message

```
    subroutine asympt - orbit is not hyperbolic
```

(6) In the displayed output, sma = semimajor axis, raan = right ascension of the ascending node, argper = argument of perigee, and arglat = argument of latitude

Calculating C3, RLA and DLA

Using position and velocity vectors

The asymptote unit vector of a hyperbolic orbit is given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

In this expression, α_∞ is the right ascension of the asymptote (RLA), and δ_∞ is the declination of the asymptote (DLA).

The asymptote unit vector at any trajectory time can be computed from

$$\hat{\mathbf{s}} = \frac{1}{1 + C_3 \dfrac{h^2}{\mu^2}} \left\{ \left(\frac{\sqrt{C_3}}{\mu} \right) \mathbf{h} \times \mathbf{e} - \mathbf{e} \right\} = \frac{1}{1 + C_3 \dfrac{p}{\mu}} \left\{ \left(\frac{\sqrt{C_3}}{\mu} \right) \mathbf{h} \times \mathbf{e} - \mathbf{e} \right\}$$

where **h** and **e** are the angular momentum and orbital eccentricity vectors, respectively. In the second expression p is the semiparameter of the hyperbolic orbit which can be computed from

$$p = a\left(1 - e^2\right)$$

The angular momentum and eccentricity vectors are computed using the following equations;

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

$$\mathbf{e} = \frac{\mathbf{v} \times \mathbf{h}}{\mu} - \frac{\mathbf{r}}{r} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r} \right) \mathbf{r} - \left(\mathbf{r} \bullet \mathbf{v} \right) \mathbf{v} \right]$$

C3 is the "twice specific" orbital energy which is determined from the position **r** and velocity **v** vectors according to

$$C_3 = \left|\mathbf{v}\right|^2 - \frac{2\mu}{\left|\mathbf{r}\right|}$$

The right ascension and declination of the asymptote can be computed from components of the unit asymptote vector according to

$$\alpha_\infty = \tan^{-1}\left(s_x, s_y\right)$$

$$\delta_\infty = \sin^{-1}\left(s_z\right)$$

Using classical orbital elements

The asymptote unit vector in terms of the classical orbital elements of a hyperbolic orbit is given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos\Omega\cos\left(\omega+\theta\right) - \sin\Omega\sin\left(\omega+\theta\right)\cos i \\ \sin\Omega\cos\left(\omega+\theta\right) + \cos\Omega\sin\left(\omega+\theta\right)\cos i \\ \sin\left(\omega+\theta\right)\sin i \end{Bmatrix}$$

In this expression Ω is the right ascension of the ascending node (RAAN), ω is the argument of periapsis and θ is the true anomaly.

The declination of the asymptote (DLA) is given by

$$\delta_\infty = \sin^{-1}\left[\sin\left(\omega+\theta_\infty\right)\sin i\right] = \sin^{-1}\left[\sin\left(u_\infty\right)\sin i\right]$$

where $u_\infty = \omega + \theta_\infty$ is the argument of latitude of the launch asymptote. In this expression θ_∞ is the true anomaly of the launch hyperbola "at infinity" and is a function of the orbital eccentricity e of the hyperbola according to $\theta_\infty = \cos^{-1}\left(-1/e\right)$.

From the following two expressions

$$\sin\left(\alpha_\infty - \Omega\right) = \frac{\tan\delta_\infty}{\tan i}$$

$$\cos\left(\alpha_\infty - \Omega\right) = \frac{\cos u_\infty}{\cos\delta_\infty}$$

the right ascension of the asymptote (RLA) can be determined from

$$\alpha_\infty = \Omega + \tan^{-1}\left(\frac{\tan\delta_\infty}{\tan i}, \frac{\cos u_\infty}{\cos\delta_\infty}\right)$$

Please note that the inverse tangent in these expressions is a four quadrant calculation.