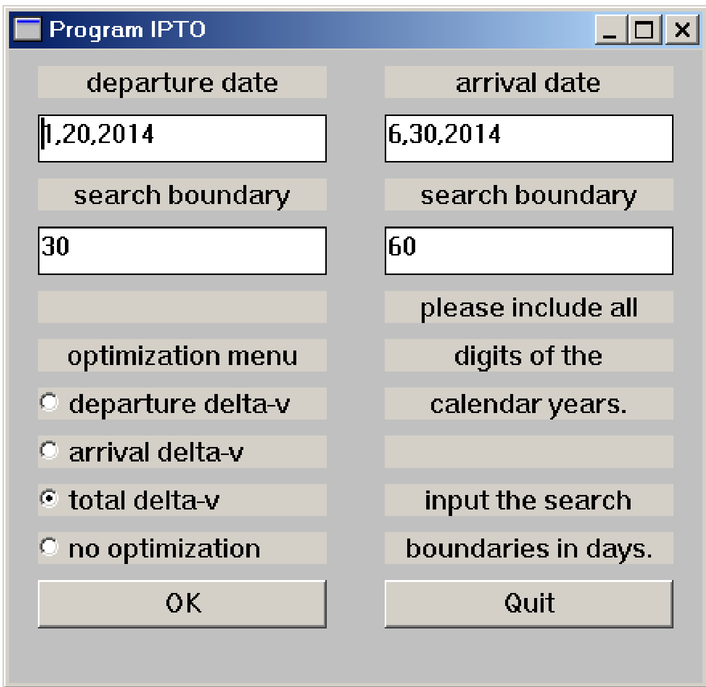
A Computer Program for Patched-Conic Trajectory Design and Optimization

This interactive Windows XP/Vista computer program (`ipto.exe`) can be used to determine the characteristics of *patched-conic* transfer trajectories between any two planets of our solar system. The software allows the user to minimize the launch, arrival or total delta-V required for the interplanetary transfer. The user can also create a graphics display of the interplanetary transfer trajectory.

The program begins by displaying the following *main menu* screen:



This screen allows the user to input an initial guess for the departure and arrival calendar dates. The user must also input values for the search bounds, in days, for the launch and arrival dates. The software will restrict its search for the optimum launch date D_L and arrival date D_A as follows:

$$D_{L_g} - \Delta D_L \le D_L \le D_{L_g} + \Delta D_L$$

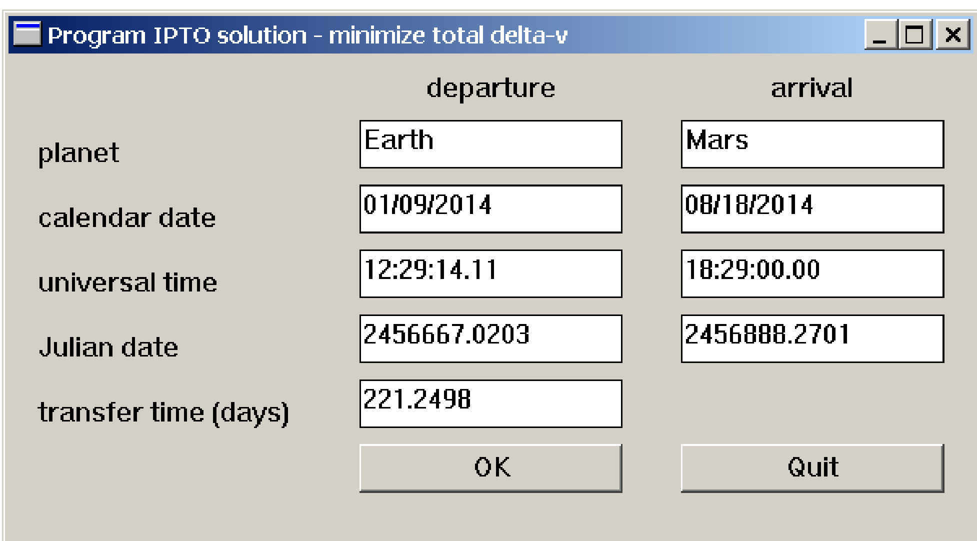
$$D_{A_g} - \Delta D_A \le D_A \le D_{A_g} + \Delta D_A$$

where D_{L_g} and D_{A_g} are the user's initial guess for launch and arrival dates, and ΔD_L and ΔD_A are the user-specified bounds on the launch and arrival dates, respectively. This screen also allows the user select the type of trajectory optimization to perform.

If the solution found by the software "bumps up" against one or both of the bounds, you might want to consider increasing one or both bounds.

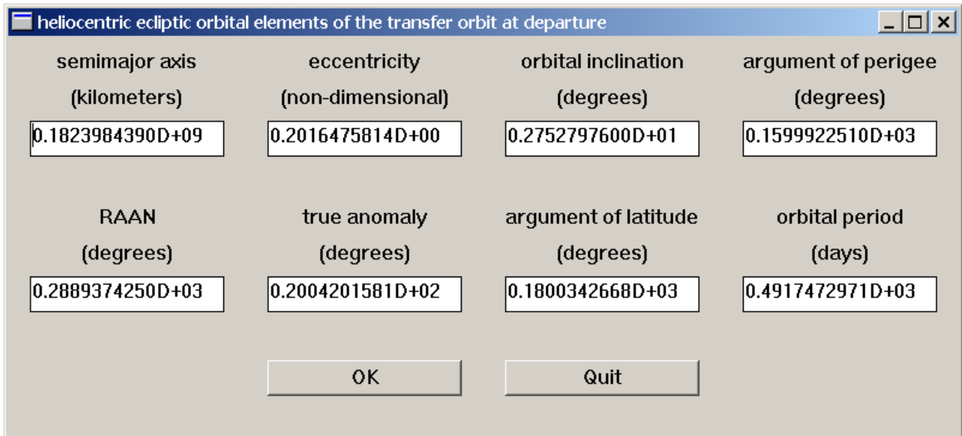
The next screen displayed by the software allows the user to select the departure and arrival planets. This screen appears as follows:

The first output screen created by the software is similar to the following:

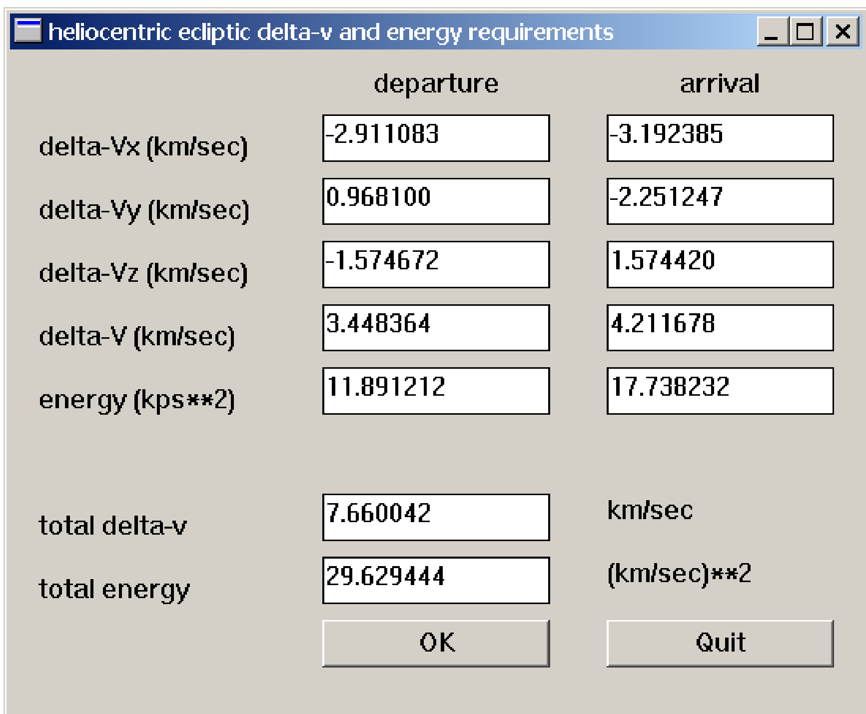


This screen displays the calendar date, universal time and Julian date of the departure and arrival. It also shows the transfer time in days.

The second screen displays the *heliocentric ecliptic* orbital elements of the transfer orbit at departure. This screen has the following appearance.

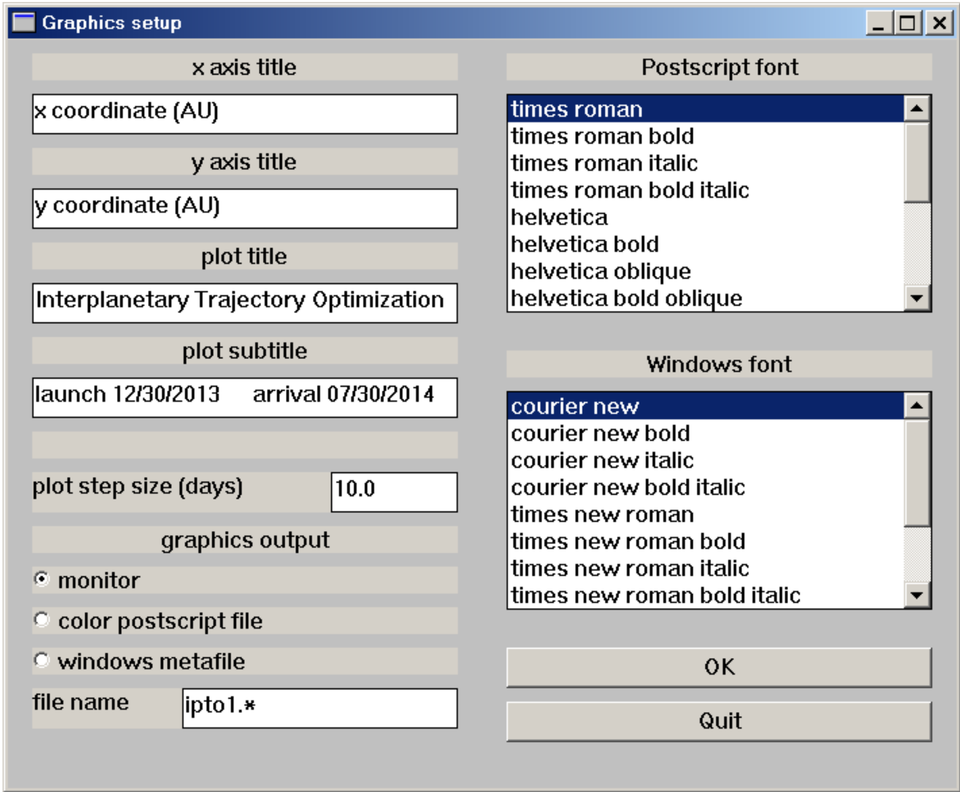


The third data screen summarizes the departure and arrival delta-v vectors and magnitude, and the specific orbital energy requirements for the mission. It is similar to the following:



After all trajectory data is computed and displayed, the software will ask the user if he or she would like to create a graphics display of the trajectory. This prompt is as follows:

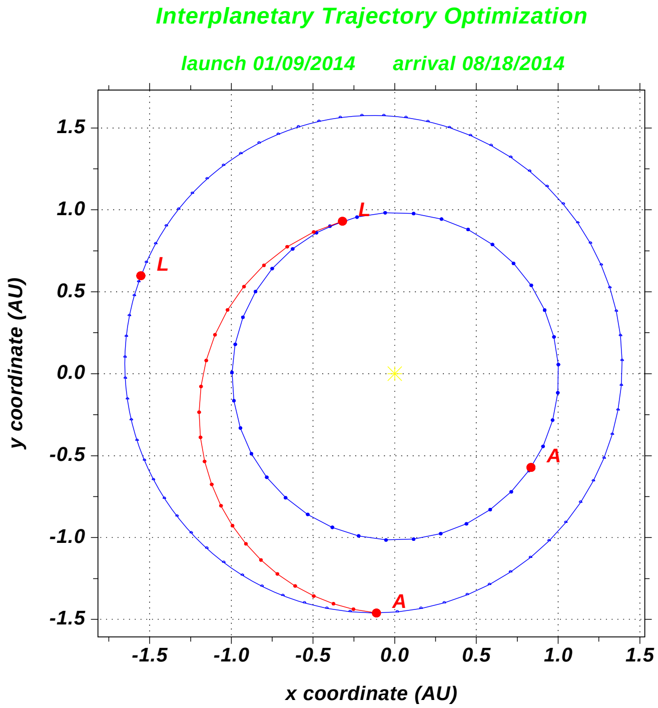
If the user clicks the Yes button, the program will display the following *graphics setup* screen.



This screen allows the user to create custom axes and plot labels and specify the destination of the graphics output. The software will automatically create a plot subtitle consisting of the calendar dates at launch and arrival. You can also specify the font to use and the step size for the trajectory plot. Please note that the Windows font is valid for both monitor and Windows metafile graphics. A plot step size between 5 and 10 days is recommended.

The following is a typical heliocentric graphics display created with this computer program. The locations of the departure and arrival planets at launch are labeled with the letter L. The locations of the departure and arrival planets at arrival are marked with the letter A. This particular trajectory is one of the Manned Mission-to-Mars Design Reference Missions (DRM). In this view, we are looking from the north ecliptic pole at the transfer trajectory and planetary orbits projected onto the ecliptic plane.

After viewing the graphics display, the user can press the right mouse button to return to the main program menu.



Technical Discussion

A patched-conic trajectory ignores the gravitational effect of both the launch and arrival planets on the heliocentric trajectory. This numerical technique involves the solution of Lambert's two-body problem relative to the Sun. Patched-conic trajectories are suitable for preliminary mission design of interplanetary trajectories.

The $\Delta V's$ or velocity impulses required at launch and arrival are simply the differences between the velocity on the transfer trajectory determined by the solution of Lambert’s problem and the heliocentric velocities of the two planets. If we treat each planet as a point mass and assume *impulsive* maneuvers, the magnitude and direction of the required maneuvers are given by the following two vector equations:

$$\Delta \mathbf{V}_L = \mathbf{V}_{T_L} - \mathbf{V}_{P_L}$$

$$\Delta \mathbf{V}_A = \mathbf{V}_{T_A} - \mathbf{V}_{P_A}$$

where

$\mathbf{V}_{T_L}$ = heliocentric velocity vector of the transfer trajectory at launch

$\mathbf{V}_{T_A}$ = heliocentric velocity vector of the transfer trajectory at arrival

$\mathbf{V}_{P_L}$ = heliocentric velocity vector of the launch planet at launch

$\mathbf{V}_{P_A}$ = heliocentric velocity vector of the arrival planet at arrival

The scalar magnitude of each maneuver is also called the "hyperbolic excess velocity" or V_∞ at launch and arrival. The hyperbolic excess velocity is the speed of a spacecraft relative to each planet at an *infinite* distance from the planet. Furthermore, the *specific orbital energy* at launch or arrival is equal to V_∞^2 for the respective maneuver.

In the terminology of numerical optimization, this computer program treats the launch and arrival dates as *control or optimization variables* and attempts to minimize the launch, arrival or sum of launch and arrival scalar $\Delta V's$. The scalar magnitude of the selected ΔV is called *the objective function*.

The software can solve for the following types of interplanetary space missions:

- minimum launch ΔV
- minimum arrival ΔV
- minimum total ΔV

An initial guess for the launch and arrival impulsive delta-v vectors can be determined from the solution of the Lambert two-point boundary-value problem (TPBVP). The Lambert solution that initializes the software uses the user's initial guess for the launch and arrival dates.

Lambert's Problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamic problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis *a* of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length *c* of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof\left(r_i + r_f, c, a\right)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}}\left(E - e\sin E\right)$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}}\left[E - E_0 - e\left(\sin E - \sin E_0\right)\right]$$

where E is the eccentric anomaly associated with radius r, E_0 is the eccentric anomaly at r_0, and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sun and difference identities:

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha-\beta}{2}\sin\frac{a+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t = \sqrt{\frac{a^3}{\mu}}\left\{E - E_0 - 2\sin\frac{E-E_0}{2}\left(e\cos\frac{E+E_0}{2}\right)\right\}$$

With the two substitutions given by

$$e\cos\frac{E+E_0}{2} = \cos\frac{\alpha+\beta}{2}$$

$$\sin\frac{E-E_0}{2} = \sin\frac{\alpha-\beta}{2}$$

the time equation becomes

$$t = \sqrt{\frac{a^3}{\mu}\left\{(\alpha-\beta) - 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}\right\}}$$

From the elliptic relationships given by

$$r = a(1 - e\cos E)$$

$$x = a(\cos E - e)$$

$$y = a\sin E\sqrt{1-e^2}$$

and some more manipulation, we have the following equations:

$$\cos\alpha = \left(1 - \frac{r + r_0}{2a}\right) - \frac{c}{2a} = 1 - \frac{r + r_0 + c}{2a} = 1 - \frac{s}{a}$$

$$\sin\beta = \left(1 - \frac{r + r_0}{2a}\right) + \frac{c}{2a} = 1 - \frac{r + r_0 - c}{2a} = 1 - \frac{s - c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos\frac{\alpha - \beta}{2}\cos\frac{\alpha + \beta}{2} = 1 - \frac{r + r_0}{2}$$

$$\sin\frac{\alpha - \beta}{2}\sin\frac{\alpha + \beta}{2} = \sin\frac{E - E_0}{2}\sqrt{1 - \left(e\cos\frac{E + E_0}{2}\right)^2}$$

$$\left(\sin\frac{\alpha - \beta}{2}\sin\frac{\alpha + \beta}{2}\right)^2 = \left(\frac{x - x_0}{2a}\right)^2 + \left(\frac{y - y_0}{2a}\right)^2 = \left(\frac{c}{2a}\right)^2$$

With the use of the half angle formulas given by

$$\sin\frac{\alpha}{2} = \sqrt{\frac{s}{2a}} \qquad \sin\frac{\beta}{2} = \sqrt{\frac{s - c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert's theorem

$$t = \sqrt{\frac{a^3}{\mu}}\left[(\alpha - \beta) - (\sin\alpha - \sin\beta)\right]$$

A discussion about the angles α and β can be found in "Geometrical Interpretation of the Angles α and β in Lambert's Problem" by J. E. Prussing, AIAA *Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this computer program is based on the method described in "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem" by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

Planetary Ephemeris

The planetary ephemeris used in this computer program is based on the JPL DE421 binary ephemeris file (`de421.bin`). This ephemeris is valid between January 1, 1900 and December 31, 2049. It must be in the same directory as the `ipto.exe` executable.