

A MATLAB Script for Designing Interplanetary Launch Trajectories

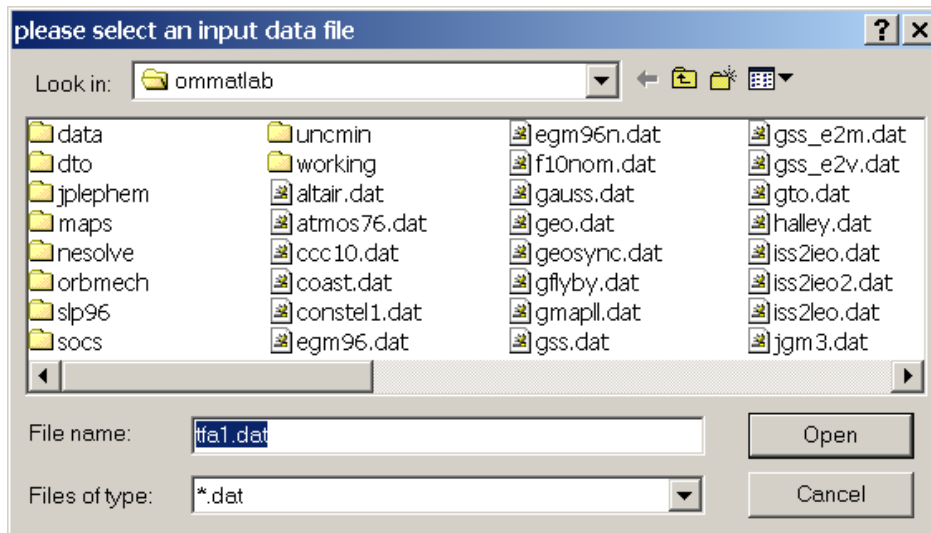
This document describes a MATLAB script named `launch1.m` that can be used for the preliminary design and analysis of launch trajectories for interplanetary spacecraft. Many of the algorithms implemented in this script are based on the classic reports, “Design of Lunar and Interplanetary Ascent Trajectories”, by Victor C. Clarke, Jr., JPL Technical Report No. 32-30, March 15, 1962, and “Interplanetary Mission Design Handbook, Volume 1, Part 2”, JPL Publication 82-43, September 15, 1983.

The Earth departure trajectory for interplanetary missions is usually defined by a “targeting specification” which consists of twice the specific (per unit mass) orbital energy C_3 , and the right ascension α_∞ and declination δ_∞ of the outgoing asymptote. The launch azimuth, park orbit altitude, and launch site coordinates are also specified. In order to estimate the park orbit coast duration, the user must also provide estimates of typical Earth central angles for launch-to-park-orbit-inject, and one or more propulsive and intermediate coasting maneuvers. The current version of this MATLAB script provides at most two injection propulsive maneuvers separated by a short coast. However, the software can be easily extended to include additional events.

This MATLAB script determines the launch universal time, orbital elements of the departure hyperbola, injection delta-v, and estimates park orbit coast times for both short and long coast injection opportunities. This information can be used to create initial guesses for other trajectory simulations.

Running the Script

The `launch1` script is “data-driven” by a single ASCII input file created by the user. The software will prompt you for the name of this file with a screen display similar to the following:



The default filename extension is `.dat`. However, the script will read a compatible data file with any filename extension the user prefers.

Input Data Files

This section describes a typical input data file for the software. In the following discussion the actual input file contents are in *courier* font and all explanations are in *times italic* font. The following is an input data file for the Mars Exploration Rover mission. The information for this file was extracted from “Mission Design Overview for the Mars Exploration Rover Mission”, by Ralph B. Roncoli and Jan M. Ludwinski, AIAA 2002-4823.

Each data item within a `launch1` input file is preceded by one or more lines of *annotation text*. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input. ASCII text input is not case sensitive but must be spelled correctly.

The first four lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with four and only four initial text lines.

```
*****
interplanetary injection data file
MER-A mission - mer-a.dat
*****
```

The first input item is the name of the simulation. This helps with traceability and will be displayed with the screen display of the solution.

```
mission name
-----
MER-A Mission
```

The next input is the launch calendar date of the simulation. Please include all 4 digits of the calendar year.

```
launch calendar date (month, day, year)
5,30,2003
```

The next data item is the launch azimuth in degrees.

```
launch azimuth (degrees)
93
```

The next data item is the geodetic latitude of the launch site in degrees.

```
launch site geodetic latitude (degrees)
28.285579
```

The next data item is the east longitude of the launch site in degrees.

```
launch site east longitude (degrees)
279.434701
```

The next data item is the altitude of the circular park orbit.

circular park orbit altitude (kilometers)
185.2

The next data item is the launch energy C_3 in the unit of km^2/sec^2 .

C_3 (twice specific orbital energy; $(\text{km}/\text{sec})^2$)
9.28

The next data item is the declination of the launch hyperbola, DLA, in degrees.

DLA (declination of launch hyperbola; degrees)
2.27

The next data item is the right ascension of the launch hyperbola, RLA, in degrees.

RLA (right ascension of launch hyperbola; degrees)
352.59

The next data item is an estimate of the Earth central angle from liftoff to park orbit injection, in degrees.

central angle from launch to park orbit inject (degrees)
24.0

The next data item is an estimate of the Earth central angle of the first and perhaps only interplanetary injection propulsive maneuver, in degrees.

central angle for first injection maneuver (degrees)
9.0

The next data item is an estimate of the Earth central angle of a short coast between a first and second propulsive maneuver. If there is no second maneuver, set this value to 0.

central angle between first and second injection maneuvers (degrees)
7.0

The next data item is an estimate of the Earth central angle for a second propulsive maneuver, in degrees. If there is no second maneuver, set this value to 0

central angle for second injection maneuver (degrees)
8.0

The final data item is an estimate of the injection true anomaly, in degrees. For an impulsive maneuver, this value is usually 0. For a finite-burn maneuver, this value is non-zero and usually positive.

injection true anomaly (degrees)
8.0

The injection true anomaly could also include an additional central angle increment that represents a short coast to a target interface point (TIP) which is often part of a target specification. Some spacecraft missions often require that C_3 , DLA and RLA be enforced at a TIP and not necessarily at final stage burnout.

Typical Script Simulation Example

This section is a typical output provided by the `launch1` MATLAB script. It illustrates the characteristics for both a short and long park orbit coast mission.

```
-----
launch1.m - interplanetary launch characteristics
-----

MER-A Mission

ascending injection
-----

launch calendar date           30-May-2003
launch universal time          18:30:17.499

launch azimuth                 93.000000  degrees
launch site geodetic latitude  28.446462  degrees
launch site geocentric declination 28.285533  degrees
launch site east longitude     279.434701  degrees

launch hyperbola characteristics
-----

c3                             9.280000  (km/sec)^2
asymptote declination          2.270000  degrees
asymptote right ascension      352.590000  degrees
semimajor axis                 -42952.633782 kilometers
eccentricity                   1.15280406
inclination                    28.431108  degrees
argument of perigee            214.608494  degrees
raan                           348.391213  degrees
orbital period                 87.912441  minutes
first park orbit coast time    19.332650  minutes
first park orbit coast angle   78.912735  degrees
range angle                    269.076398  degrees
inject argument of latitude    95.554959  degrees
```

rasc of launch site at 0 hr	166.529456	degrees
rasc of launch site at launch	84.862306	degrees
arglat of launch site at launch	95.554959	degrees
greenwich sidereal time at 0 hr	247.094755	degrees
asymptote true anomaly	150.163663	degrees
asymptote argument of latitude	4.772157	degrees
ascending node to launch site	96.311049	degrees
ascending node to asymptote	4.198787	degrees
local circular velocity	7793.033366	meters/second
injection velocity	11434.279080	meters/second
injection delta-v	3641.245714	meters/second
descending injection		

launch calendar date	30-May-2003	
launch universal time	07:06:40.208	
launch azimuth	93.000000	degrees
launch site geodetic latitude	28.446462	degrees
launch site geocentric declination	28.285533	degrees
launch site east longitude	279.434701	degrees
launch hyperbola characteristics		

c3	9.280000	(km/sec)^2
asymptote declination	2.270000	degrees
asymptote right ascension	352.590000	degrees
semimajor axis	-42952.633782	kilometers
eccentricity	1.15280406	
inclination	28.431108	degrees
argument of perigee	25.064180	degrees
raan	176.788787	degrees

orbital period	87.912441	minutes
first park orbit coast time	61.042400	minutes
first park orbit coast angle	249.165157	degrees
range angle	79.328821	degrees
inject argument of latitude	95.554959	degrees
rasc of launch site at 0 hr	166.529456	degrees
rasc of launch site at launch	273.489022	degrees
arglat of launch site at launch	95.554959	degrees
greenwich sidereal time at 0 hr	247.094755	degrees
asymptote true anomaly	150.163663	degrees
asymptote argument of latitude	175.227843	degrees
ascending node to launch site	96.311049	degrees
ascending node to asymptote	175.801213	degrees
local circular velocity	7793.033366	meters/second
injection velocity	11434.279080	meters/second
injection delta-v	3641.245714	meters/second

In the launch1 solution display screens,

rasc = right ascension

arglat = argument of latitude

raan = right ascension of the ascending node

The output shown for ascending node to launch site and ascending node to asymptote are the angles measured from the ascending node *along the equator* to the launch site and asymptote, respectively. The argument of latitude is equal to the sum of the argument of perigee and the true anomaly.

TECHNICAL DISCUSSION

This section is a summary of the geometry and orbital mechanics involved in the design and analysis of interplanetary injection trajectories.

Departure Trajectory Orientation

The transfer trajectory must contain the launch site, the outgoing asymptote of the departure hyperbola and the center of the Earth at the moment of launch. The constraining equation for this orientation is

$$\hat{\mathbf{h}} \cdot \hat{\mathbf{s}} = \hat{h}_x \hat{s}_x + \hat{h}_y \hat{s}_y + \hat{h}_z \hat{s}_z = 0$$

where $\hat{\mathbf{h}}$ is the unit angular momentum of the launch hyperbola and $\hat{\mathbf{s}}$ is a unit vector in the direction of the outgoing asymptote. This constraint forces the angular momentum vector of the departure trajectory to be perpendicular to the outgoing asymptote.

The departure asymptote unit vector given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos \delta_\infty \cos \alpha_\infty \\ \cos \delta_\infty \sin \alpha_\infty \\ \sin \delta_\infty \end{Bmatrix}$$

where

α_∞ = right ascension of departure asymptote

δ_∞ = declination of departure asymptote

Furthermore, the angular momentum unit vector must also satisfy the following dot product constraint

$$\hat{\mathbf{h}} \cdot \hat{\mathbf{h}} = \hat{h}_x^2 + \hat{h}_y^2 + \hat{h}_z^2 = 1$$

These two equations can be solved simultaneously to yield the following two equations for the x and y components of the unit angular momentum vector.

$$\hat{h}_x = -\frac{(\hat{h}_y \hat{s}_y + \hat{h}_z \hat{s}_z)}{\hat{s}_x}$$
$$\hat{h}_y = -\frac{\hat{h}_z \hat{s}_y \hat{s}_z \pm \hat{s}_x \sqrt{1 - \hat{s}_z^2 - \hat{h}_z^2}}{\hat{s}_x^2 + \hat{s}_y^2}$$

The positive and negative sign in front of the square root in the second equation indicates that there are two possible launch time solutions. One solution corresponds to a short transfer and the other represents a long transfer mission.

The z-component of the unit angular momentum vector can be determined from the orbital inclination i or the flight azimuth Σ_L and launch site geocentric declination δ_L according to

$$\hat{h}_z = \cos i = \cos \delta_L \sin \Sigma_L$$

Notice that it is physically impossible to have an orbit in which

$$h_z^2 > 1 - s^2$$

If the declination of the launch hyperbola is greater than the launch site latitude, there is a range of azimuths (symmetric about east) at which launching is not possible.

Therefore, launch azimuth is restricted such that the following constraint must be satisfied

$$\sin^2 \Sigma_L \leq \frac{\cos^2 \delta_\infty}{\cos^2 \delta_L}$$

Furthermore, the launch azimuth *sector* is symmetric about east with the following lower and upper limits

$$\sin^2 \Sigma_{L\text{LMIT}} = \pm \frac{\cos^2 \delta_\infty}{\cos^2 \delta_L}$$

Launch Time

When the launch site passes through the departure trajectory plane, the following two geometric conditions must be satisfied

$$\hat{\mathbf{r}}_L \cdot \hat{\mathbf{h}} = \hat{r}_{L_x} \hat{h}_x + \hat{r}_{L_y} \hat{h}_y + \hat{r}_{L_z} \hat{h}_z = 0$$

$$(\hat{\mathbf{h}} \times \hat{\mathbf{r}}_L) \cdot \hat{\mathbf{i}}_z = \cos \sigma = \sqrt{\cos^2 \delta_L - \hat{h}_z^2}$$

where $\hat{\mathbf{r}}_L$ is the inertial unit position vector of the launch site and $\hat{\mathbf{i}}_z$ is a unit vector aligned with the Earth's spin axis and is given by $\hat{\mathbf{i}}_z = [0, 0, 1]^T$.

The inertial unit position vector of the launch site can be determined from

$$\hat{\mathbf{r}}_L = \begin{Bmatrix} \cos \delta_L \cos \theta_L \\ \cos \delta_L \sin \theta_L \\ \sin \delta_L \end{Bmatrix}$$

where θ_L is the *local sidereal time* of the launch site at the time of launch t_L . The local sidereal time at the launch time can be determined from the following expression

$$\theta_L = \theta_{g_0} + \lambda_L^E + \omega_e t_L$$

where

$$\theta_{g_0} = \text{Greenwich sidereal time at 0 hours UT}$$

$$\lambda_L^E = \text{east longitude of launch site}$$

$$\omega_e = \text{inertial rotation rate of the Earth}$$

The local sidereal time is identical to the right ascension of a ground site.

The previous two constraint equations can be solved simultaneously to yield the following equations for the right ascension of the launch site at the launch time

$$\sin \alpha_L = \frac{\hat{h}_y \hat{h}_z \sin \delta_L - \hat{h}_x \sqrt{\cos^2 \delta_L - \hat{h}_z^2}}{(\hat{h}_z^2 - 1) \cos \delta_L}$$

$$\cos \alpha_L = \frac{\hat{h}_z \hat{h}_x \sin \delta_L + \hat{h}_y \sqrt{\cos^2 \delta_L - \hat{h}_z^2}}{(\hat{h}_z^2 - 1) \cos \delta_L}$$

These two equations should be evaluated with a four quadrant inverse tangent function to determine the right ascension of the launch site. The right ascension of the launch site as a function of launch azimuth is given by these next two equations

$$\sin \alpha_L = \frac{\hat{h}_y \sin \delta_L \sin \Sigma_L - \hat{h}_x \cos \Sigma_L}{\hat{h}_z^2 - 1}$$

$$\cos \alpha_L = \frac{\hat{h}_x \sin \delta_L \sin \Sigma_L + \hat{h}_y \cos \Sigma_L}{\hat{h}_z^2 - 1}$$

The three components of the unit angular momentum vector are functions of the launch azimuth and the components of the outgoing asymptote unit vector. Therefore, for a given geographic launch site, the launch time is determined by the launch azimuth and the declination and right ascension of the outgoing asymptote.

The right ascension of the launch site at the launch time is also given by

$$\alpha_L = \theta_{g_0} + \lambda_L^E + \omega_e t_L$$

Finally, this equation can be solved for the launch time to yield

$$t_L = \frac{\alpha_L - \theta_{g_0} - \lambda_L^E}{\omega_e}$$

Launch Azimuth

For a given launch site, launch time and outgoing asymptote, the corresponding flight azimuth can be computed from the following series of equations:

$$\sin \Sigma_L = \frac{\cos i}{\cos \delta_L}$$

$$\cos \Sigma_L = \sin i \cos(\alpha_{\mathcal{O}} - \alpha_L)$$

$$\Sigma_L = \tan^{-1}(\sin \Sigma_L, \cos \Sigma_L)$$

where $\alpha_{\mathcal{O}}$ is the right ascension of the *descending* node which can be determined from the following expression

$$\alpha_{\mathcal{O}} = \tan^{-1}(\hat{n}_y, \hat{n}_x)$$

In this equation, $\hat{\mathbf{n}}$ is a unit vector pointing in the direction of the descending node. This vector can be computed from the following cross product

$$\hat{\mathbf{n}} = \hat{\mathbf{h}} \times \hat{\mathbf{i}}_z$$

where $\hat{\mathbf{i}}_z$ is a unit vector aligned with the Earth's spin axis and is given by $\hat{\mathbf{i}}_z = [0, 0, 1]^T$.

Range Angle and Park Orbit Coast Time Estimate

The range angle measured in the orbit plane from the launch site to the outgoing asymptote can be determined from

$$\sin \eta = \frac{\sin(\alpha_{\infty} - \alpha_L) \cos \delta_{\infty}}{\sin \Sigma_L}$$

$$\cos \eta = \sin \delta_{\infty} \sin \phi_L + \cos \delta_{\infty} \cos \phi_L \cos(\alpha_{\infty} - \alpha_L)$$

and the four quadrant inverse tangent

$$\eta = \tan^{-1}(\sin \eta, \cos \eta)$$

The range angle is also equal to the following combination of Earth central angles and true anomalies

$$\eta = \phi_1 + \sum_{i=2}^N \phi_i + \phi_c + \theta_\infty - \theta_I$$

where

ϕ_1 = launch-to-park orbit inject Earth central angle

ϕ_i = Earth central angle for i^{th} propulsive or coast maneuver

ϕ_c = park orbit coast Earth central angle

θ_∞ = true anomaly of outgoing asymptote

θ_I = true anomaly at interplanetary injection

and N is the total number of additional propulsive or coasting maneuvers. The Earth central angle is the geocentric angle measured along the orbit plane from the beginning to the end of an orbital event such as a coast or maneuver.

The park orbit coast Earth central angle is computed from

$$\phi_c = \eta - \phi_1 - \sum_{i=2}^n \phi_i - \theta_\infty + \theta_I$$

The true anomaly of the outgoing asymptote is given by

$$\theta_\infty = \cos^{-1}\left(-\frac{1}{e}\right)$$

and determines where perigee of the departure trajectory is located relative to the outgoing asymptote.

Finally, the coast time can be estimated from the following expression

$$t_c = \frac{\theta_c}{n}$$

where n is the mean motion of the park orbit and is equal to $\sqrt{\mu/r_{po}^3}$.

Departure Hyperbola Orbital Characteristics

This section describes the algorithm used to determine the Earth-centered-inertial (ECI) state vector of a departure hyperbola for interplanetary missions. In the discussion that follows, interplanetary injection is assumed to occur *impulsively* at perigee of the geocentric departure hyperbola. The method described here is based on the fundamental characteristics of the B-plane coordinate system.

The departure trajectory for interplanetary missions is specified by the orbital energy C_3 , and the right ascension α_∞ and declination δ_∞ of the outgoing asymptote. The perigee radius of the departure hyperbola is specified and the orbital inclination is computed from the user-defined launch azimuth Σ_L and launch site geocentric latitude ϕ_L from the equation

$$i = \cos^{-1}(\cos \phi_L \sin \Sigma_L)$$

The algorithm used to design the departure hyperbola only works for geocentric orbit inclinations that satisfy the following constraint

$$|i| > |\delta_\infty|$$

A unit vector in the direction of the departure asymptote is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos \delta_\infty \cos \alpha_\infty \\ \cos \delta_\infty \sin \alpha_\infty \\ \sin \delta_\infty \end{Bmatrix}$$

where

α_∞ = right ascension of departure asymptote

δ_∞ = declination of departure asymptote

The T-axis direction of the B-plane coordinate system is determined from the following vector cross product:

$$\hat{\mathbf{T}} = \hat{\mathbf{S}} \times \hat{\mathbf{u}}_z$$

where $\hat{\mathbf{u}}_z = [0 \ 0 \ 1]^T$ is a unit vector perpendicular to the Earth's equator.

The following cross product operation completes the B-plane coordinate system.

$$\hat{\mathbf{R}} = \hat{\mathbf{S}} \times \hat{\mathbf{T}}$$

The B-plane angle is determined from the orbital inclination of the departure hyperbola i and the declination of the outgoing asymptote according to

$$\cos \theta = \frac{\cos i}{\cos \delta_\infty}$$

The unit angular momentum vector of the departure hyperbola is given by

$$\hat{\mathbf{h}} = \hat{\mathbf{T}} \sin \theta - \hat{\mathbf{R}} \cos \theta$$

The sine and cosine of the true anomaly at infinity are given by the next two equations

$$\cos \theta_\infty = -\frac{\mu}{r_p V_\infty^2 + \mu}$$

$$\sin \theta_\infty = \sqrt{1 - \cos^2 \theta_\infty}$$

where $V_\infty = \sqrt{C_3} = V_L - V_p$ is the spacecraft's velocity at infinity, V_L is the heliocentric departure velocity determined from the Lambert solution, V_p is the heliocentric velocity of the departure planet, and r_p is the user-specified perigee radius of the departure hyperbola.

A unit vector in the direction of perigee of the departure hyperbola is determined from

$$\hat{\mathbf{r}}_p = \hat{\mathbf{S}} \cos \theta_\infty - (\hat{\mathbf{h}} \times \hat{\mathbf{S}}) \sin \theta_\infty$$

The ECI position vector at perigee is

$$\mathbf{r}_p = r_p \hat{\mathbf{r}}_p$$

The scalar magnitude of the perigee velocity can be determined from

$$V_p = \sqrt{\frac{2\mu}{r_p} + V_\infty^2}$$

A unit vector aligned with the velocity vector at perigee is

$$\hat{\mathbf{v}}_p = \hat{\mathbf{h}} \times \hat{\mathbf{r}}_p$$

The ECI velocity vector at perigee of the departure hyperbola is given by

$$\mathbf{v}_p = V_p \hat{\mathbf{v}}_p$$

Finally, the classical orbital elements of the departure hyperbola can be determined from the position and velocity vectors at perigee. The injection delta-v vector and magnitude can be determined from the velocity difference between the park orbit and departure hyperbola at the orbital location of the impulsive maneuver.

An Alternative Algorithm for Computing Launch Time

This section describes a slightly different algorithm for computing launch time. It is based on informal notes provided by Johnny Kwok at JPL.

The equations for computing the classical orbital elements are as follows:

semimajor axis

$$a = -\frac{\mu}{C_3}$$

orbital eccentricity

$$e = 1 - \frac{r_{po}}{a}$$

orbital inclination

$$i = \cos^{-1}(\cos \phi_L \sin \Sigma_L)$$

argument of perigee

$$\omega = c - \theta_\infty$$

for injection during the *ascending* part of the orbit

$$c = \sin^{-1}\left(\frac{\sin \delta_L}{\sin i}\right)$$

for injection during the *descending* part of the orbit

$$c = 180^\circ - c$$

right ascension of the ascending node

$$\Omega = \alpha_\infty - h$$

where h is the angle from the ascending node to the launch asymptote measured along the equator and determined from

$$h = \tan^{-1}(\sin c \sin i, \cos c)$$

The launch time can be determined from this next set of calculations:

The launch site sidereal time at 0 hours UT is computed using

$$\alpha_0 = \theta_{g0} + \lambda_L^E$$

The launch site sidereal time at launch is given by

$$\alpha = \alpha_\infty - h + g$$

where g is the angle from the ascending node to the launch site, measured along the equator and determined from

$$g = \tan^{-1}(\sin d \cos i, \cos d)$$

For a launch azimuth $\Sigma_L < 90^\circ$,

$$d = \sin^{-1}\left(\frac{\sin \phi_0}{\sin i}\right)$$

otherwise $d = 180^\circ - d$.

Finally, the launch time can be determined from

$$t_L = \frac{\alpha - \alpha_0}{\omega_e}$$

Geocentric Declination of the Launch Site

This MATLAB script uses the following equation to calculate the geocentric declination of the launch site given the geodetic latitude ϕ_L and geodetic altitude h .

$$\delta_L = \phi_L + \left(\frac{-\sin 2\phi_L}{\hat{h} + 1}\right) f + \left\{ \frac{-\sin 2\phi_L}{2(\hat{h} + 1)^2} + \left[\frac{1}{4(\hat{h} + 1)^2} + \frac{1}{4(\hat{h} + 1)} \right] \sin 4\phi_L \right\} f^2$$

In this equation, the geocentric distance r and geodetic altitude h have been normalized by $\hat{\rho} = r / r_{eq}$ and $\hat{h} = h / r_{eq}$, respectively. r_{eq} is the equatorial radius of the Earth and f is the flattening factor.

Spherical Equations

This section provides additional equations that may be useful for interplanetary mission design and analysis. For example, they can be used to determine the characteristics of a launch hyperbola from the classical orbital elements at the target interface point (TIP).

The asymptote unit vector in terms of the classical orbital elements of a hyperbolic orbit is given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos \Omega \cos(\omega + \theta) - \sin \Omega \sin(\omega + \theta) \cos i \\ \sin \Omega \cos(\omega + \theta) + \cos \Omega \sin(\omega + \theta) \cos i \\ \sin(\omega + \theta) \sin i \end{Bmatrix}$$

In this expression, Ω is the right ascension of the ascending node (RAAN), ω is the argument of periapsis and θ is the true anomaly.

The declination of the asymptote (DLA) is given by

$$\delta_{\infty} = \sin^{-1} [\sin(\omega + \theta_{\infty}) \sin i] = \sin^{-1} [\sin(u_{\infty}) \sin i]$$

where $u_{\infty} = \omega + \theta_{\infty}$ is the argument of latitude of the launch asymptote. In this expression θ_{∞} is the true anomaly of the launch hyperbola “at infinity” and is a function of the orbital eccentricity e of the hyperbola according to $\theta_{\infty} = \cos^{-1}(-1/e)$.

From the following two expressions

$$\sin(\alpha_{\infty} - \Omega) = \frac{\tan \delta_{\infty}}{\tan i}$$

$$\cos(\alpha_{\infty} - \Omega) = \frac{\cos u_{\infty}}{\cos \delta_{\infty}}$$

the right ascension of the asymptote (RLA) can be determined from

$$\alpha_{\infty} = \Omega + \tan^{-1} \left(\frac{\tan \delta_{\infty}}{\tan i}, \frac{\cos u_{\infty}}{\cos \delta_{\infty}} \right)$$

where the inverse tangent in this equation is a four quadrant operation.

References and Bibliography

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- (6) “A Computer Simulation of the Orbital Launch Window Problem”, Archie C. Young and Pat R. Odom, AIAA 67-615, 1967.
- (7) “Launch Parameters for Interplanetary Flights”, W. C. Riddell, *American Rocket Society Journal*, December 1960.