

A Computer Program for Creating Pork Chop Plots of Ballistic Earth-to-Mars Trajectories

This document describes a Windows compatible Fortran computer program called `porkchop_ftn` that can be used to create and display interplanetary “pork chop” plots for Type I and Type II Earth-to-Mars missions. These plots illustrate the behavior of departure energy (C3L), right ascension (RLA) and declination (DLA) of the departure v-infinity vector, time-of-flight, and arrival v-infinity for a range of departure and arrival calendar dates.

The data required for these contour plots is created by solving the heliocentric, two-body “patched-conic” Lambert problem. A patched-conic trajectory ignores the gravitational effect of both the departure and arrivals planets on the heliocentric trajectory. Type I trajectories are characterized by heliocentric Earth-to-Mars transfer angles which are less than 180 degrees while Type II trajectories have transfer angles greater than 180 degrees. Pork chop plots are typically used for preliminary interplanetary mission analysis.

The graphics displayed by this software were created using the DISLIN library (www.dislin.de) and version 11.1 of the Intel Visual Fortran compiler.

Running the software

An input file created by the user can be run from the command line or a simple batch file with a statement similar to the following:

```
porkchop_ftn mars05.in
```

If the software is executed without an input file on the command line, the computer program will display the following prompt:

```
*****
*      program porkchop_ftn      *
*                                *
* porkchop plots of Earth-to-Mars *
* interplanetary trajectories    *
*                                *
*      November 27, 2011        *
*****
please input the name of the simulation definition file
```

At this point the user should input the name of a valid input file, including the filename extension.

To create a DOS command window in Windows 7, select **start**, then **All Programs**, then **Accessories** and finally **Command Prompt**. The size, font and other characteristics of the screen can be controlled by the user with the `c:\` icon in the upper left corner of the window. To log into the subdirectory created during the installation of the Fortran executable and support files, type `root:\` and then `cd subdirectory` from the DOS command line where `root` is the name of the root directory, usually `c:`, and `subdirectory` is the name of the subdirectory created by the user. The DOS command line prompt looks similar to `C:\porkchop_ftn>_.`

Input data file

This section describes a typical input data file for the software. In the following discussion the actual input file contents are in *courier* font and all explanations are in *times italic* font. Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input.

The first six lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with six and only six initial text lines.

```
*****
** porkchop plot of Earth-to-Mars patched-conic
** ballistic interplanetary trajectories
** Mars 2028 example - mars28.in
** November 27, 2011
*****
```

The next three inputs specify the nominal departure calendar date. Be sure to include all four digits of the calendar year.

```
nominal departure calendar date (month, day, year)
11, 24, 2028
```

The next program input defines the time span about the nominal departure date.

```
departure date time span (days)
90
```

The next three inputs specify the nominal arrival calendar date. Be sure to include all four digits of the calendar year.

```
nominal arrival calendar date (month, day, year)
10, 30, 2029
```

The next program input defines the time span about the nominal arrival date.

```
arrival date time span (days)
180
```

This next item defines the step size to use during the creation of the graphics data file. A value between 2 and 5 days is recommended.

```
time span step size (days)
2
```

The last program option is an integer that specifies the type of graphics display to create.

```
graphics plot option
=====
1 = departure C3 and time-of-flight
2 = departure C3 and declination of departure v-infinity vector
3 = departure C3 and right ascension of departure v-infinity vector
4 = departure C3 and arrival v-infinity
-----
1
```

Pork chop graphics

The `porkchop_ftn` computer program creates a comma separated variable (csv) data file of the Earth-to-Mars trajectory characteristics. This data file is named `porkchp.csv`. The following are the first few lines of a typical data file. In the numerical data area of this file, column 1 is the departure scan data, column 2 is the arrival scan data, column 3 is the departure energy, column 4 is the time-of-flight data, column 5 is the declination of the departure v-infinity vector, column 6 is the right ascension of the departure v-infinity vector, and column 7 is the magnitude of the arrival v-infinity vector.

```
number of x-data values
  91

number of y-data values
 181

nominal departure date
 7,19,2005

nominal arrival date
 6, 4,2006

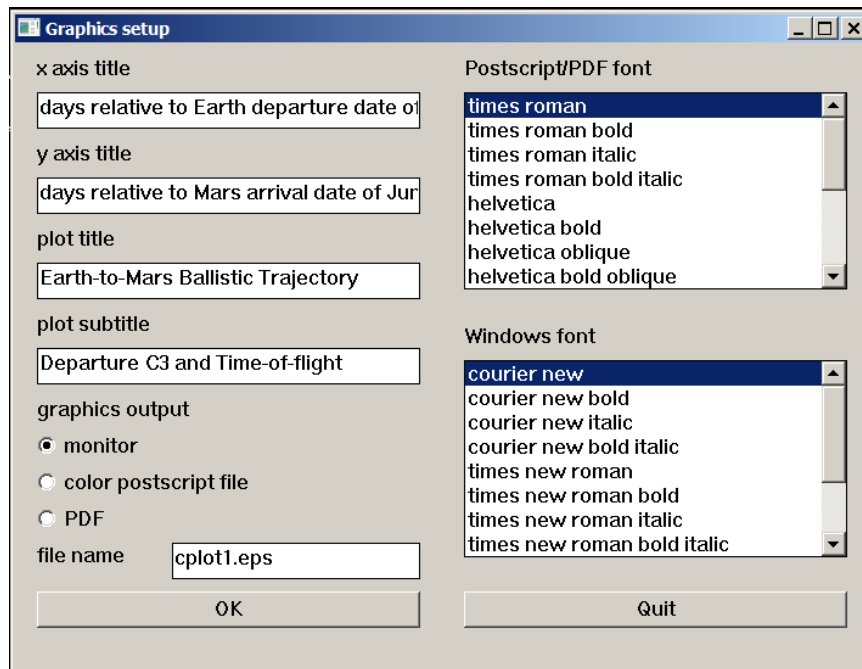
-.9000000000000000D+02, -.1800000000000000D+03, 0.5852003846861998D+02,
0.2300000000000000D+03, 0.3179805240340534D+01, 0.1486687765857488D+02,
0.3723252154551255D+01,
-.9000000000000000D+02, -.1780000000000000D+03, 0.5832877029721859D+02,
0.2320000000000000D+03, 0.3032262372461848D+01, 0.1498624040931991D+02,
0.3661620140332319D+01,
-.9000000000000000D+02, -.1760000000000000D+03, 0.5817121591595753D+02,
0.2340000000000000D+03, 0.2895684189711683D+01, 0.1510224239270248D+02,
0.3604452915329150D+01,
-.9000000000000000D+02, -.1740000000000000D+03, 0.5804649803210329D+02,
0.2360000000000000D+03, 0.2769240426592599D+01, 0.1521533255874905D+02,
0.3551711867432625D+01,
-.9000000000000000D+02, -.1720000000000000D+03, 0.5795381514713819D+02,
0.2380000000000000D+03, 0.2652207878935154D+01, 0.1532591117523477D+02,
0.3503355733768461D+01,
```

After the `porkchop.csv` data file is created, the `porkchop_ftn` software calls one of the following executable programs to actually create the graphics display.

```
cplot1.exe = departure C3 and time-of-flight
cplot2.exe = departure C3 and declination of departure v-infinity vector
cplot3.exe = departure C3 and right ascension of departure v-infinity vector
cplot4.exe = departure C3 and arrival v-infinity
```

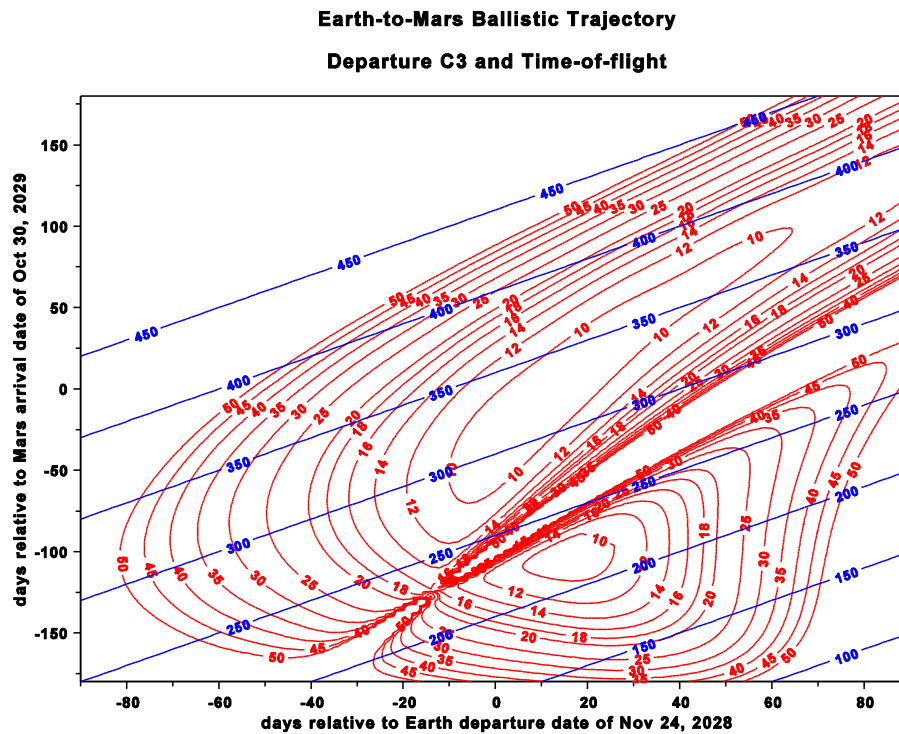
Please note that these programs can be run standalone from a DOS command line or by double clicking on the name after a data file is created.

Prior to creating a graphics display of the data computed by `porkchop_ftn`, the software will display the following interactive graphics setup screen.

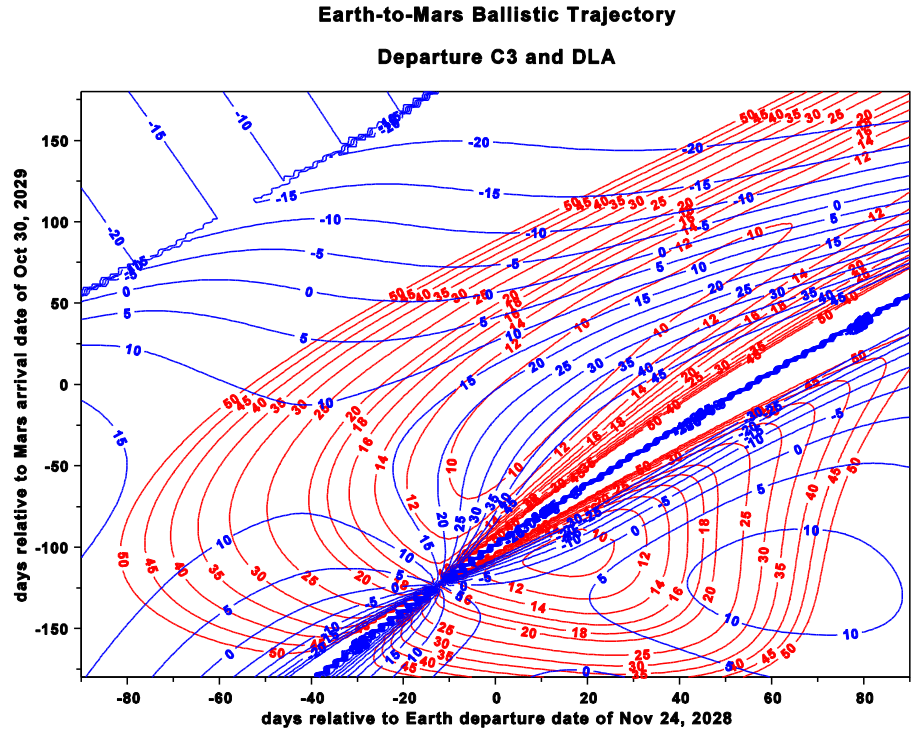


This screen allows the user to modify such items as the plot, subplot and axis titles. It also includes options for defining the type of graphics output, the font to use for text and numbers, and the name for graphic disk files.

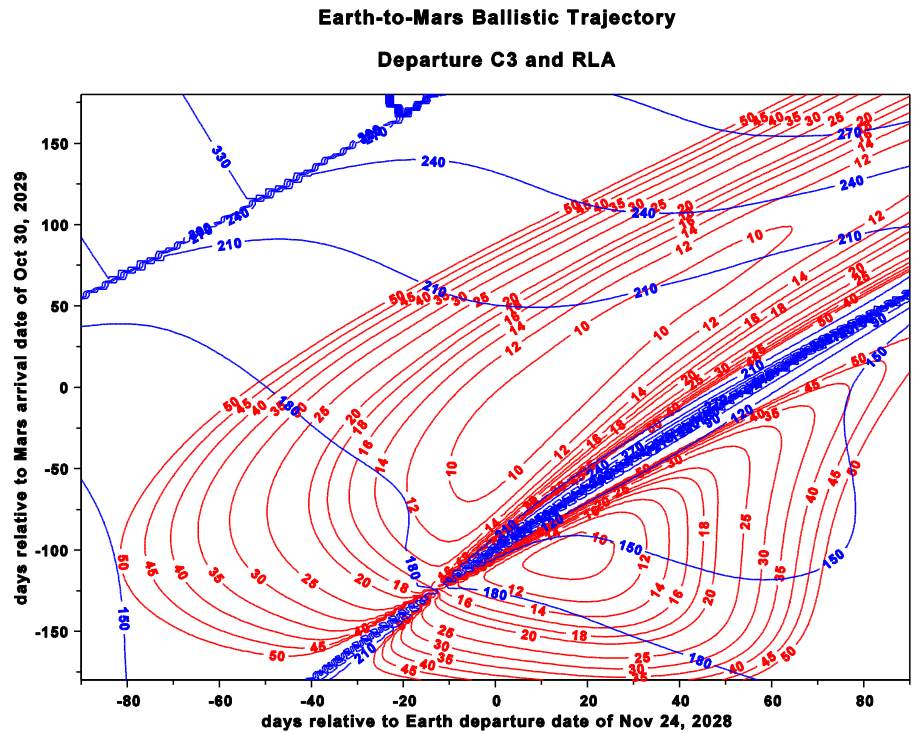
The following typical contour plot summarizes the behavior of the departure energy (C3L) in km^2/sec^2 and the time-of-flight in days. The C3L contours are red and the time-of-flight contours are blue.



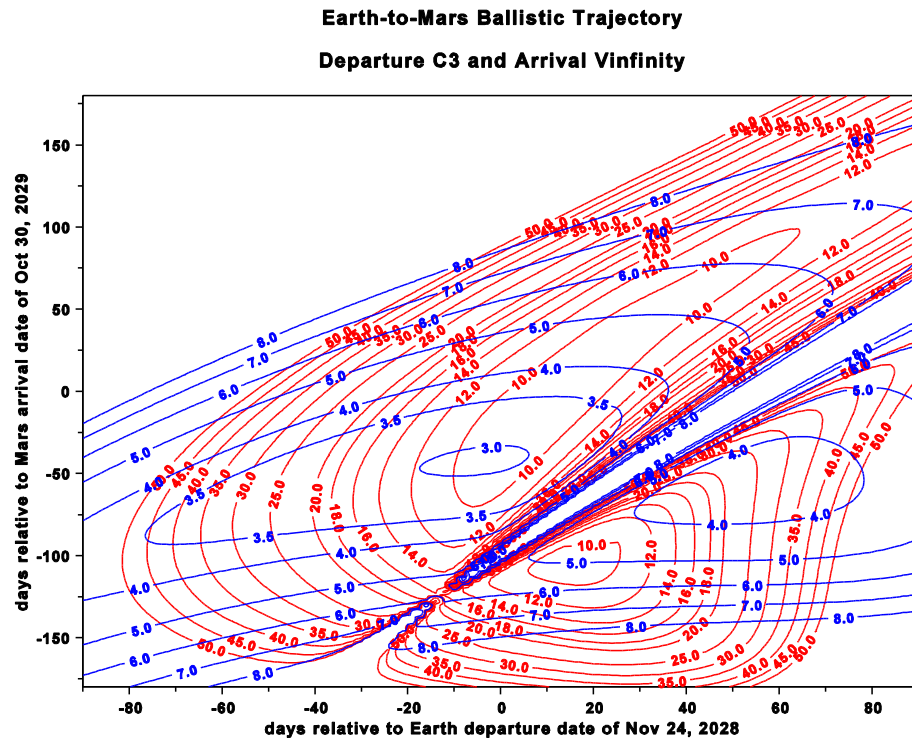
This next contour plot illustrates the behavior of the departure energy (C3L) and the geocentric declination of the departure v-infinity vector (DLA) in degrees relative to the Earth mean equator and equinox of J2000 (EME2000) coordinate system.



This next plot illustrates the behavior of the departure energy (C3L) and the right ascension of the departure v-infinity vector (RLA) in degrees relative to the Earth mean equator and equinox of J2000 (EME2000) coordinate system.



This final contour plot summarizes the behavior of the departure energy (C3L) and the magnitude of the arrival v-infinity vector in kilometers per second at Mars.



Technical Discussion

A solution for the departure and arrival impulsive delta-v vectors can be determined from the solution of the Lambert two-point boundary-value problem (TPBVP). Lambert’s Theorem states that the time to traverse a trajectory depends only upon the length of the semimajor axis a of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length c of the chord joining these two positions.

Lambert’s Problem

Lambert’s problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamic problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis a of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length c of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof(r_i + r_f, c, a)$$

From the following form of Kepler’s equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}} (E - e \sin E)$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}} [E - E_0 - e(\sin E - \sin E_0)]$$

where E is the eccentric anomaly associated with radius r , E_0 is the eccentric anomaly at r_0 , and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sum and difference identities:

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t = \sqrt{\frac{a^3}{\mu}} \left\{ E - E_0 - 2 \sin \frac{E - E_0}{2} \left(e \cos \frac{E + E_0}{2} \right) \right\}$$

With the two substitutions given by

$$e \cos \frac{E + E_0}{2} = \cos \frac{\alpha + \beta}{2}$$

$$\sin \frac{E - E_0}{2} = \sin \frac{\alpha - \beta}{2}$$

the time equation becomes

$$t = \sqrt{\frac{a^3}{\mu}} \left\{ (\alpha - \beta) - 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2} \right\}$$

From the elliptic relationships given by

$$r = a(1 - e \cos E)$$

$$x = a(\cos E - e)$$

$$y = a \sin E \sqrt{1 - e^2}$$

and some more manipulation, we have the following equations:

$$\cos \alpha = \left(1 - \frac{r + r_0}{2a}\right) - \frac{c}{2a} = 1 - \frac{r + r_0 + c}{2a} = 1 - \frac{s}{a}$$

$$\sin \beta = \left(1 - \frac{r + r_0}{2a}\right) + \frac{c}{2a} = 1 - \frac{r + r_0 - c}{2a} = 1 - \frac{s - c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2} = 1 - \frac{r + r_0}{2}$$

$$\sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2} = \sin \frac{E - E_0}{2} \sqrt{1 - \left(e \cos \frac{E + E_0}{2}\right)^2}$$

$$\left(\sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2}\right)^2 = \left(\frac{x - x_0}{2a}\right)^2 + \left(\frac{y - y_0}{2a}\right)^2 = \left(\frac{c}{2a}\right)^2$$

With the use of the half angle formulas given by

$$\sin \frac{\alpha}{2} = \sqrt{\frac{s}{2a}} \quad \sin \frac{\beta}{2} = \sqrt{\frac{s - c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert's theorem

$$t = \sqrt{\frac{a^3}{\mu}} [(\alpha - \beta) - (\sin \alpha - \sin \beta)]$$

A discussion about the angles α and β can be found in "Geometrical Interpretation of the Angles α and β in Lambert's Problem" by J. E. Prussing, *AIAA Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this computer program is based on the method described in "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem" by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48**: 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

The ΔV 's required at departure and arrival are simply the differences between the velocity on the transfer trajectory determined by the solution of Lambert's problem and the heliocentric velocities of the two planets. If we treat each planet as a point mass and assume *impulsive* maneuvers, the *planet-centered* magnitude and direction of the required maneuvers are given by the two vector equations:

$$\Delta \mathbf{V}_L = \mathbf{V}_{T_L} - \mathbf{V}_{P_L}$$

$$\Delta \mathbf{V}_A = \mathbf{V}_{T_A} - \mathbf{V}_{P_A}$$

where

\mathbf{V}_{T_L} = heliocentric velocity vector of the transfer trajectory at departure

\mathbf{V}_{T_A} = heliocentric velocity vector of the transfer trajectory at arrival

\mathbf{V}_{P_L} = heliocentric velocity vector of the departure planet

\mathbf{V}_{P_A} = heliocentric velocity vector of the arrival planet

The scalar magnitude of each maneuver is also called the “hyperbolic excess velocity” or V_∞ at departure and arrival. The hyperbolic excess velocity is the speed of the spacecraft relative to each planet at an *infinite* distance from the planet. Furthermore, the *energy* or C_3 at departure or arrival is equal to V_∞^2 for the respective maneuver. C_3 is also equal to twice the orbital energy per unit mass (the specific orbital energy).

The orientation of the departure and arrival hyperbolas is specified in terms of the right ascension and declination of the departure V_∞ velocity vector. These coordinates can be calculated using the x, y and z components of the departure V_∞ velocity vector.

The right ascension of the departure V_∞ velocity vector is determined from

$$\alpha = \tan^{-1}(\Delta V_y, \Delta V_z)$$

and the geocentric declination of the departure V_∞ velocity vector is given by

$$\delta = 90^\circ - \cos^{-1}(\Delta \hat{V}_z)$$

where $\Delta \hat{V}_z$ is the z-component of the unit ΔV vector.

In this computer program the heliocentric planetary coordinates and therefore the ΔV vectors are computed in the J2000 ecliptic and equinox coordinate system. In order to determine the orientation of the departure hyperbola, these ΔV vectors must be transformed to the equatorial frame.

The required transformation is given by

$$\Delta\mathbf{V}_{eq} = \begin{bmatrix} 1 & -0.000000479966 & 0 \\ 0.000000440360 & 0.917482137087 & 0.397776982902 \\ -0.000000190919 & -0.397776982902 & 0.917482137087 \end{bmatrix} \Delta\mathbf{V}_{ec}$$

where $\Delta\mathbf{V}_{ec}$ is the delta-velocity vector in the ecliptic frame, and $\Delta\mathbf{V}_{eq}$ is the delta-velocity vector in the equatorial frame.

This Fortran computer program models the planetary coordinates using the DE421 binary ephemeris from the Jet Propulsion Laboratory (JPL).

The DE421 ephemeris and the declination and right ascension of the departure v-infinity vector are modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time.

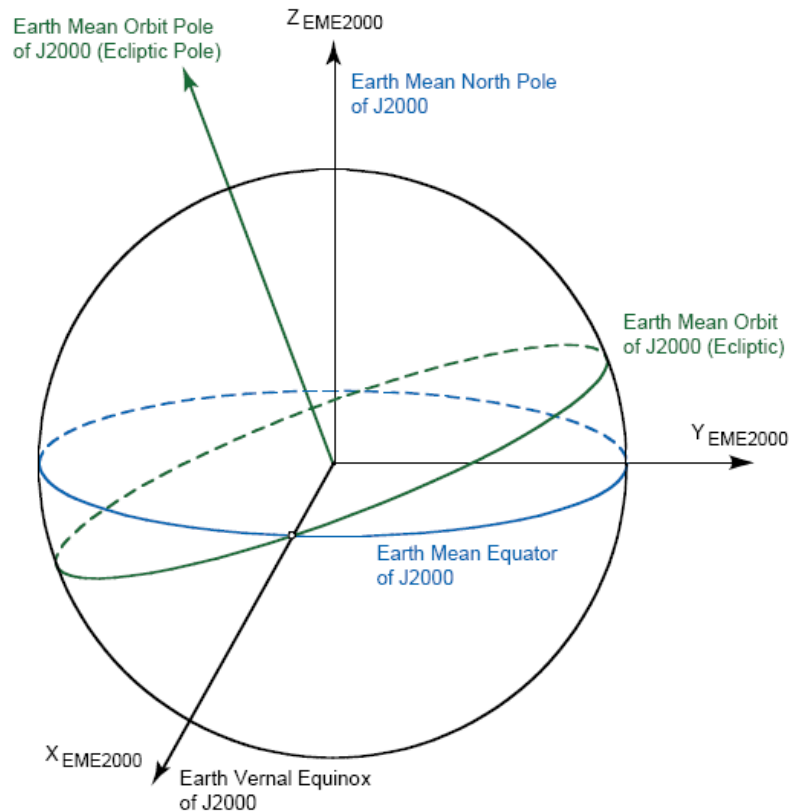


Figure 1. Earth mean equator and equinox of J2000 coordinate system

Algorithm Resources

“Modern Astrodynamics”, Victor R. Bond and Mark C. Allman, Princeton University Press, 1996.

“Interplanetary Mission Design Handbook, Volume 1, Part 2”, JPL Publication 82-43, September 15, 1983.

Explanatory Supplement to the Astronomical Almanac, Edited by P. K. Seidelmann, University Science Books, 1992.

“Update to Mars Coordinate Frame Definitions”, R. A. Mase, JPL IOM 312.B/015-99, 15 July 1999.

R. H. Battin, *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA, 1987.