A Computer Program for Propagating Spacecraft Trajectories from Earth to Mars

This document is the user's manual for a Windows compatible executable computer program called `pprop_e2m` that can be used to numerically integrate the orbital equations of motion of a spacecraft traveling from the Earth to Mars. This scientific simulation begins at a user-defined epoch and state vector somewhere within the Earth's sphere-of-influence (SOI) and ends at (1) closest approach to Mars, (2) a user-defined Mars-centered (areocentric) distance, or (3) at a user-defined final epoch.

This manual also includes a technical discussion that summarizes the numerical technique and methods implemented in this computer program. Barycentric Dynamical Time (TDB) is the fundamental time argument for this simulation and the fundamental solar, lunar and planetary ephemeris is DE421. This computer program also uses version 3.1 of the Naval Observatory Vector Astrometry Software (NOVAS) library for coordinate conversions. The software was created using version 11.1 of Intel Visual Fortran.

All internal calculations and the output provided by the `pprop_e2m` software are performed in the metric system. The geocentric equations of spacecraft motion include the non-spherical gravity effects of the Earth and the point-mass gravity of the sun, Moon and (optionally) the planets within the Earth's sphere-of-influence. During the interplanetary cruise the point-mass gravity of the sun, Moon and planets is included in the heliocentric equations of motion. The option to include the effect of solar radiation pressure in both the geocentric and heliocentric trajectory segments is also provided.

Input file format and contents

The `pprop_e2m` software is "data-driven" by a user-created text file. The following is a typical input file used by this computer program. Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or change the number of lines reserved for each comment and data item. However, you may change them to reflect your own explanation or information. The annotation line also includes the correct units and when appropriate, the valid range of the input data items. ASCII text input is <u>not</u> case sensitive but must be spelled correctly. In the following discussion, the actual input file contents are in **`bold courier`** font and all explanations are in *times italic* font.

The first four lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with four and only four initial text lines.

```
******************************************
* pprop_e2m input data file - pprop_e2m.in
* Earth-to-Mars trajectory example
******************************************
```

The first program input is the name of a "constants and models" data file. This ASCII data file contains user-defined astrodynamic constants and other information.

```
name of constants and models data file
--------------------------------------
pprop_e2m_cm.dat
```

The following is a typical constants and models data file. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment or data item. Also, please note the proper units for each data item.

```
****************************************
* pprop_e2m constants and models data file
****************************************

astronomical unit (kilometers)
------------------------------
149597870.691d0

speed of light (meters/second)
------------------------------
299792458.0d0

solar flux at 1 AU (watts/meters**2)
------------------------------------
1366.1d0

Earth gravitational constant (km**3/sec**2)
-------------------------------------------
398600.4415d0

Earth equatorial radius (kilometers)
------------------------------------
6378.14d0

Earth sphere-of-influence value (kilometers)
--------------------------------------------
925000.0d0

Moon gravitational constant (km**3/sec**2)
------------------------------------------
4902.800238d0

Mars gravitational constant (km**3/sec**2)
------------------------------------------
42828.376212d0

Mars equatorial radius (kilometers)
-----------------------------------
3396.2d0
```

The second program input is the difference between ephemeris time (Terrestrial Time) and Universal Coordinated Time (UTC) in seconds.

```
ET-UTC (seconds)
64.184d0
```

This next option specifies the type of final conditions of the propagated trajectory. Option 1 propagates to closest approach at Mars, option 2 propagates to a user-defined areocentric distance, and option 3 propagates to a user-defined final epoch.

```
type of propagation final condition
 1 = Mars closest approach
 2 = user-defined Mars-centered distance
 3 = user-defined final epoch
------------------------------
1
```

The next two sets of inputs define the calendar date and UTC time of the final epoch for option 3 described above. Be sure to include all four digits of the calendar year.

```
user-defined final calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
------------------------------------------------------
6, 20, 2003
```

```
user-defined final UTC
(0 <= hours <= 24, 0 <= minutes <= 60, 0 <= seconds <= 60)
-----------------------------------------------------------
14, 47, 23.918
```

The next input defines the final user-defined lunar distance for program option 2 described above.

```
user-defined lunar distance (kilometers)
10000.0d0
```

The next two inputs define the calendar date and UTC at the initial time.

```
initial calendar date
(1 <= month <= 12, 1 <= day <= 31, year = all digits!)
-------------------------------------------------------
6, 5, 2003

initial UTC
(0 <= hours <= 24, 0 <= minutes <= 60, 0 <= seconds <= 60)
-----------------------------------------------------------
14, 47, 23.918
```

The next three data items define the x, y, and z components of the geocentric position vector of the spacecraft at the initial time.

```
Earth departure geocentric position vector components
(mean equator and equinox j2000 - kilometers)
----------------------------------------------
-6272.07312607554d0
-1760.41957306828d0
-800.642741501510d0
```

The next three data items define the x, y, and z components of the geocentric velocity vector of the spacecraft at interplanetary injection. The position and velocity vectors must be specified relative to the Earth mean equator and equinox of J2000 (EME2000) coordinate system.

```
Earth departure geocentric velocity vector components
(mean equator and equinox j2000 - kilometers/second)
-----------------------------------------------------
3.35306759386591d0
-9.54013022081091d0
-5.29081568530960d0
```

The next three integers allow the user to specify what types of third body point-mass gravity perturbations are included during the geocentric trajectory propagation. To activate an option, the input should be set to `1`. Otherwise, the input should be `0`.

```
***************************
geocentric phase definition
***************************

include solar point-mass perturbation (1 = yes, 0 = no)
--------------------------------------------------------
1

include lunar point-mass perturbation (1 = yes, 0 = no)
--------------------------------------------------------
1

include planetary point-mass perturbations (1 = yes, 0 = no)
-------------------------------------------------------------
1
```

The name of the ASCII data file containing the Earth gravity model data is specified in the next line. Please see the Technical Discussion section later in this document for a description and format of the data in this file.

```
name of Earth gravity model data file
egm96.dat
```

The order (zonals) of the Earth gravity model is an integer defined in the next line.

```
order of the gravity model (zonals)
8
```

The degree (tesserals) of the Earth gravity model is an integer defined in this next line.

```
degree of the gravity model (tesserals)
8
```

The next series of inputs define the spacecraft characteristics used for solar radiation pressure perturbation calculations. These three items include the spacecraft's mass, reference cross-sectional area, and reflectivity coefficient. To exclude this perturbation, input a spacecraft mass of zero.

```
spacecraft mass (kilograms; input 0 to ignore SRP calculations)
---------------------------------------------------------------
0.0d0

SRP reference area (square meters)
----------------------------------
18.75

reflectivity coefficient (non-dimensional)
------------------------------------------
1.4d0
```

Program execution

The `pprop_e2m` computer program can be executed by typing the following from a DOS command line

```
pprop_e2m input_file
```

where `input_file` is the name of the input data file, including the file name extension.

If the user types `pprop_e2m` without a file name, the software will request a file name with the following interactive prompt,

```
please input the name of the simulation definition file
```

At this point the user should provide a compatible file name, complete with extension.

To create a DOS command window in Windows 7, select **start**, then **All Programs**, then **Accessories** and finally **Command Prompt**. The size, font and other characteristics of the screen can be controlled by the user with the **c:** icon in the upper left corner of the window. To log into the subdirectory created during the installation of the Fortran executable and support files, type **root:** and then **cd subdirectory** from the DOS command line where root is the name of the root directory, usually c:, and subdirectory is the name of the subdirectory created by the user.

The DOS command line prompt looks similar to **C:\pprop_e2m>_.**

Program example

The following is the program output for a typical closest approach simulation. Explanatory text is provided in italic times Roman font. Appendix A contains a brief explanation of these data items.

```
program pprop_e2m
-----------------

precision propagation from Earth to Mars

input file ==> pprop_e2m.in

models/constants file ==> pprop_e2m_cm.dat

DE421 ephemeris
```

The first part of the output summarizes the geocentric and heliocentric conditions at the initial time. It includes the orbital elements of the departure hyperbola along with the energy (C3) and right ascension (RLA) and declination (DLA) of the trajectory.

```
Initial time and conditions
(geocentric Earth mean equator and equinox of J2000)
----------------------------------------------------

calendar date           June  5, 2003

UTC time                14:47:23.918

UTC Julian date         2452796.11624905

TDB time                14:48:28.050

TDB Julian date         2452796.11699132


     sma (km)            eccentricity       inclination (deg)       argper (deg)
 -.453500501333D+05    0.114472883670D+01    0.286442848562D+02    0.194742119763D+03

    raan (deg)         true anomaly (deg)       arglat (deg)
 0.267494586008D+01    0.763333123551D-13    0.194742119763D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.627207312608D+04    -.176041957307D+04    -.800642741502D+03    0.656346000000D+04

     vx (kps)               vy (kps)              vz (kps)             vmag (kps)
 0.335306759387D+01    -.954013022081D+01    -.529081568531D+01    0.114127068452D+02


asymptote coordinates and specific orbital energy
(geocentric Earth mean equator and equinox of J2000)
----------------------------------------------------

right ascension      349.992647760703      degrees

declination         -6.83825103161977      degrees

orbital energy       8.78941567492588      (km/sec)**2


v-infinity           2.96469487045899      km/sec

ta-infinity          150.876105569553      degrees
```

```
time and conditions of the Earth
(heliocentric Earth mean equator and equinox of J2000)
------------------------------------------------------

calendar date          June  5, 2003

UTC time               14:47:23.918

UTC Julian date        2452796.11624905

TDB time               14:48:28.050

TDB Julian date        2452796.11699132

     sma (au)            eccentricity       inclination (deg)      argper (deg)
 0.100035811608D+01   0.162374958182D-01   0.234390545150D+02   0.102451915932D+03

    raan (deg)        true anomaly (deg)      arglat (deg)         period (days)
 0.722897068242D-03   0.152049044492D+03   0.254500960424D+03   0.365453122443D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.405597427281D+08   -.134200242066D+09   -.581820454662D+08   0.151789156778D+09

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 0.282280784458D+02   -.739734366114D+01   -.320725910415D+01   0.293569687969D+02

time and conditions of the Earth
(heliocentric Earth mean ecliptic and equinox of J2000)
-------------------------------------------------------

calendar date          June  5, 2003

UTC time               14:47:23.918

UTC Julian date        2452796.11624905

TDB time               14:48:28.050

TDB Julian date        2452796.11699132

     sma (au)            eccentricity       inclination (deg)      argper (deg)
 0.100035811608D+01   0.162374958178D-01   0.374268971909D-03   0.335346777183D+03

    raan (deg)        true anomaly (deg)      arglat (deg)         period (days)
 0.127105827228D+03   0.152049044490D+03   0.127395821673D+03   0.365453122444D+03

     rx (km)                ry (km)               rz (km)              rmag (km)
 -.405596783166D+08   -.146269821253D+09   0.787722736903D+03   0.151789156778D+09

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 0.282280819963D+02   -.806269209031D+01   -.115283324146D-03   0.293569687970D+02
```

The next section summarizes the time and conditions at the boundary of the Earth's sphere-of-influence.

```
time and conditions at Earth sphere-of-influence
(geocentric Earth mean equator and equinox of J2000)
----------------------------------------------------

calendar date          June  8, 2003

UTC time               18:21:52.360
```

```
UTC Julian date          2452799.26518936

TDB time                 18:22:56.492

TDB Julian date          2452799.26593162


     sma (km)              eccentricity       inclination (deg)       argper (deg)
 -.455952132484D+05    0.114329887029D+01    0.285067459380D+02    0.194701071343D+03

    raan (deg)          true anomaly (deg)       arglat (deg)
 0.268703911208D+01    0.149476424236D+03    0.344177495579D+03

     rx (km)                 ry (km)               rz (km)              rmag (km)
 0.899364334057D+06    -.179666139810D+06    -.120369733873D+06    0.925000000000D+06

     vx (kps)               vy (kps)              vz (kps)              vmag (kps)
 0.303091732413D+01    -.532486195991D+00    -.366050347590D+00    0.309903117629D+01


time and conditions at Earth sphere-of-influence
(heliocentric Earth mean equator and equinox of J2000)
------------------------------------------------------

calendar date            June  8, 2003

UTC time                 18:21:52.360

UTC Julian date          2452799.26518936

TDB time                 18:22:56.492

TDB Julian date          2452799.26593162


     sma (au)              eccentricity       inclination (deg)       argper (deg)
 0.127480209108D+01    0.203979869558D+00    0.234999959931D+02    0.253589169139D+03

    raan (deg)          true anomaly (deg)       arglat (deg)         period (days)
 0.526245473607D+00    0.379159057243D+01    0.257380759712D+03    0.525729779140D+03

     rx (km)                 ry (km)               rz (km)              rmag (km)
 -.319281368727D+08    -.136202685909D+09    -.590925967550D+08    0.151863466851D+09

     vx (kps)               vy (kps)              vz (kps)              vmag (kps)
 0.316293511557D+02    -.653259680468D+01    -.296664765047D+01    0.324328795179D+02
```

This section of the display summarizes the time and conditions at closest approach to Mars. It includes both the EME2000 heliocentric coordinates and the B-plane coordinates of the spacecraft with respect to a Mars-centered mean equator and IAU node of epoch coordinate system.

```
time and conditions at Mars closest approach
(heliocentric Earth mean equator and equinox of J2000)
------------------------------------------------------

calendar date            December 24, 2003

UTC time                 03:02:10.300

UTC Julian date          2452997.62650810

TDB time                 03:03:14.432

TDB Julian date          2452997.62725037
```

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.123003745794D+01	0.235788643938D+00	0.232764929876D+02	0.255337339519D+03

raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.359924922134D+03	0.151259368219D+03	0.465967077382D+02	0.498282841605D+03

rx (km)	ry (km)	rz (km)	rmag (km)
0.150722036270D+09	0.146011403916D+09	0.628963487375D+08	0.219071707382D+09

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.137529667372D+02	0.159460269856D+02	0.685193156143D+01	0.221442732286D+02

time and conditions at Mars closest approach
(areocentric mean equator and IAU node of epoch)
----

calendar date December 24, 2003

UTC time 03:02:10.300

UTC Julian date 2452997.62650810

TDB time 03:03:14.432

TDB Julian date 2452997.62725037

sma (km)	eccentricity	inclination (deg)	argper (deg)
-.314826168342D+03	0.730559747625D+02	0.171823640969D+02	0.308054488520D+03

raan (deg)	true anomaly (deg)	arglat (deg)
0.256050888869D+03	0.359999998911D+03	0.308054487431D+03

rx (km)	ry (km)	rz (km)	rmag (km)
-.199331519142D+05	-.945714313286D+04	-.527692644930D+04	0.226851064406D+05

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.127100518494D+01	-.301726266434D+01	0.606327736122D+00	0.332970891961D+01

B-plane coordinates at Mars closest approach
(areocentric mean equator and IAU node of epoch)
----

b-magnitude	27935.5322217447	kilometers
b dot r	7467.79135907311	
b dot t	26918.8791135420	
theta	15.5049913787983	degrees
v-infinity	2.70389698173721	kilometers/second
r-periapsis	22685.1064406095	kilometers
decl-asymptote	7.49728393140068	degrees
rasc-asymptote	281.240069686500	degrees

areocentric flight path angle -1.074605424729890E-006 degrees

time and conditions of Mars at closest approach
(heliocentric Earth mean equator and equinox of J2000)
----

calendar date December 24, 2003

UTC time 03:02:10.300

UTC Julian date 2452997.62650810

```
TDB time                03:03:14.432

TDB Julian date         2452997.62725037


     sma (au)            eccentricity       inclination (deg)       argper (deg)
 0.152368041832D+01    0.935420701947D-01    0.246772248997D+02    0.332979290565D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 0.337165820667D+01    0.704694083928D+02    0.434486989582D+02    0.686972348003D+03

     rx (km)                 ry (km)                rz (km)              rmag (km)
 0.150732235968D+09    0.146029075372D+09    0.629062633647D+08    0.219093349476D+09

     vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 -.166581771717D+02    0.168726246234D+02    0.818914158160D+01    0.250847038693D+02


time and conditions of Mars at closest approach
(heliocentric Earth mean ecliptic and equinox of J2000)
-------------------------------------------------------

calendar date           December 24, 2003

UTC time                03:02:10.300

UTC Julian date         2452997.62650810

TDB time                03:03:14.432

TDB Julian date         2452997.62725037


     sma (au)            eccentricity       inclination (deg)       argper (deg)
 0.152368041832D+01    0.935420701951D-01    0.184937158373D+01    0.286517486285D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (days)
 0.495409237949D+02    0.704694083925D+02    0.356986894678D+03    0.686972348004D+03

     rx (km)                 ry (km)                rz (km)              rmag (km)
 0.150732165879D+09    0.159001798173D+09    -.371660847201D+06    0.219093349476D+09

     vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 -.166581852700D+02    0.187377763931D+02    0.801852583212D+00    0.250847038693D+02


Earth to Mars transfer time      201.510259051807      days
```

Technical Discussion

The `pprop_e2m` computer program implements numerical methods that perform the following sequential trajectory calculations.

(1) determine the ephemeris time and conditions at the Earth's sphere-of-influence (SOI) based on the dynamical time and state vector at hyperbolic injection.

> These conditions are determined using a combination of one-dimensional root-finding and numerical integration. The SOI radius for these calculations is defined by the user in the constants/models data file. Arrival at the Earth's SOI boundary occurs whenever the geocentric distance of the spacecraft is equal to this value within a tolerance of `1.0d-8`.

(2) at the Earth's sphere-of-influence, convert the geocentric state vector of the spacecraft to sun-centered (heliocentric) position and velocity vectors

(3) determine closest approach, areocentric distance or user-defined epoch conditions between the spacecraft and Mars

The close approach and areocentric distance conditions are determined using a combination of one-dimensional root-finding and numerical integration. Closest approach occurs whenever the flight path angle of the spacecraft relative to Mars, and areocentric distance occurs when the Mars-to-spacecraft distance reaches the user-defined value.

In this computer program the heliocentric coordinates of the sun, Moon and planets are based on the JPL Development Ephemeris DE421. These coordinates are provided in the Earth mean equator and equinox of J2000 coordinate system (EME2000). The binary ephemeris file provided with this computer program was created for use on PC-compatible computers. For other platforms, you will need to create binary files specific to that system. Information and computer programs for creating these files can be found at the JPL solar system FTP site located at ftp://ssd.jpl.nasa.gov/pub/eph/planets/.

Propagating the spacecraft's geocentric trajectory

The spaceraft's orbital motion is modeled with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system. The following figure illustrates the geometry of the EME2000 coordinate system. The origin of this Earth-centered-inertial (ECI) inertial coordinate system is the geocenter and the fundamental plane is the Earth's mean equator. The z-axis of this system is normal to the Earth's mean equator at epoch J2000, the x-axis is parallel to the vernal equinox of the Earth's mean orbit at epoch J2000, and the y-axis completes the right-handed coordinate system. The epoch J2000 is the Julian Date 2451545.0 which corresponds to January 1, 2000, 12 hours Terrestrial Time (TT).

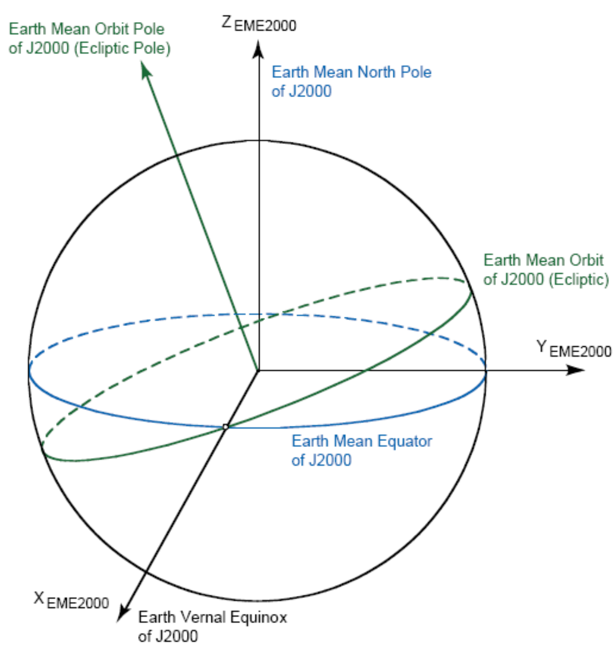


Figure 1. Earth mean equator and equinox of J2000 coordinate system

Program `pprop_e2m` implements a *special perturbation* technique which numerically integrates the vector system of second-order, nonlinear differential equations of motion of a spacecraft given by

$$\mathbf{a}(\mathbf{r},t) = \ddot{\mathbf{r}}(\mathbf{r},t) = \mathbf{a}_g(\mathbf{r},t) + \mathbf{a}_s(\mathbf{r},t) + \mathbf{a}_m(\mathbf{r},t) + \mathbf{a}_p(\mathbf{r},t) + \mathbf{a}_{srp}(\mathbf{r},t)$$

where

$t =$ dynamical time
$\mathbf{r} =$ inertial position vector of the spacecraft
$\mathbf{a}_g =$ acceleration due to Earth gravity
$\mathbf{a}_s =$ acceleration due to the sun
$\mathbf{a}_m =$ acceleration due to the moon
$\mathbf{a}_p =$ acceleration due to the planets
$\mathbf{a}_{srp} =$ acceleration due to solar radiation pressure

Geocentric acceleration due to non-spherical Earth gravity

The software uses a *spherical harmonic* representation of the Earth's geopotential function given by

$$\Phi(r,\phi,\lambda) = \frac{\mu}{r} + \frac{\mu}{r}\sum_{n=1}^{\infty} C_n^0 \left(\frac{R}{r}\right)^n P_n^0(u) + \frac{\mu}{r}\sum_{n=1}^{\infty}\sum_{m=1}^{n}\left(\frac{R}{r}\right)^n P_n^m(u)\left[S_n^m \sin m\lambda + C_n^m \cos m\lambda\right]$$

where ϕ is the geocentric latitude of the spacecraft, λ is the geocentric east longitude of the spacecraft and $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ is the geocentric distance of the spacecraft. In this expression the *S's* and *C's* are harmonic coefficients of the geopotential, and the *P's* are associated Legendre polynomials of degree *n* and order *m* with argument $u = \sin\phi$.

The software calculates the spacecraft's acceleration due to the Earth's gravity field with a vector equation derived from the gradient of the potential function expressed as

$$\mathbf{a}_g(\mathbf{r},t) = \nabla\Phi(\mathbf{r},t)$$

This acceleration vector is a combination of pure two-body or *point mass* gravity acceleration and the gravitational acceleration due to higher order nonspherical terms in the Earth's geopotential. In terms of the Earth's geopotential Φ, the inertial rectangular cartesian components of the spacecraft's acceleration vector are as follows:

$$\ddot{x} = \left(\frac{1}{r}\frac{\partial\Phi}{\partial r} - \frac{z}{r^2\sqrt{x^2+y^2}}\frac{\partial\Phi}{\partial\phi}\right)x - \left(\frac{1}{x^2+y^2}\frac{\partial\Phi}{\partial\lambda}\right)y$$

$$\ddot{y} = \left(\frac{1}{r}\frac{\partial\Phi}{\partial r} - \frac{z}{r^2\sqrt{x^2+y^2}}\frac{\partial\Phi}{\partial\phi}\right)y + \left(\frac{1}{x^2+y^2}\frac{\partial\Phi}{\partial\lambda}\right)x$$

$$\ddot{z} = \left(\frac{1}{r}\frac{\partial\Phi}{\partial r}\right)z + \left(\frac{\sqrt{x^2+y^2}}{r^2}\frac{\partial\Phi}{\partial\phi}\right)$$

The three partial derivatives of the geopotential with respect to r, ϕ, λ are given by

$$\frac{\partial \Phi}{\partial r} = -\frac{1}{r}\left(\frac{\mu}{r}\right)\sum_{n=2}^{N}\left(\frac{R}{r}\right)^{n}(n+1)\sum_{m=0}^{n}\left(C_n^m \cos m\lambda + S_n^m \sin m\lambda\right)P_n^m(\sin\phi)$$

$$\frac{\partial \Phi}{\partial \phi} = \left(\frac{\mu}{r}\right)\sum_{n=2}^{N}\left(\frac{R}{r}\right)^{n}\sum_{m=0}^{n}\left(C_n^m \cos m\lambda + S_n^m \sin m\lambda\right)\left[P_n^{m+1}(\sin\phi) - m\tan\phi\, P_n^m(\sin\phi)\right]$$

$$\frac{\partial \Phi}{\partial \lambda} = \left(\frac{\mu}{r}\right)\sum_{n=2}^{N}\left(\frac{R}{r}\right)^{n}\sum_{m=0}^{n} m\left(S_n^m \cos m\lambda - C_n^m \sin m\lambda\right)P_n^m(\sin\phi)$$

where

$R =$ radius of the Earth

$r =$ geocentric distance of the satellite

$S_n^m, C_n^m =$ harmonic coefficients

$\phi =$ geocentric declination of the satellite $= \sin^{-1}\left(\frac{z}{r}\right)$

$\lambda =$ longitude of the satellite $= \alpha - \alpha_g$

$\alpha =$ right ascension of the satellite $= \tan^{-1}\left(\frac{y}{x}\right)$

$\alpha_g =$ right ascension of Greenwich

The right ascension is measure positive east of the vernal equinox, longitude is measured positive east of Greenwich, and declination is positive above the Earth's equator and negative below.

For $m = 0$ the coefficients are called *zonal* terms, when $m = n$ the coefficients are *sectorial* terms, and for $n > m \neq 0$ the coefficients are called *tesseral* terms.

The Legendre polynomials with argument $\sin\phi$ are computed using recursion relationships given by:

$$\begin{aligned}
P_n^0(\sin\phi) &= \frac{1}{n}\left[(2n-1)\sin\phi\, P_{n-1}^0(\sin\phi) - (n-1)P_{n-2}^0(\sin\phi)\right] \\
P_n^n(\sin\phi) &= (2n-1)\cos\phi\, P_{n-1}^{n-1}(\sin\phi), \qquad m \neq 0,\ m < n \\
P_n^m(\sin\phi) &= P_{n-2}^m(\sin\phi) + (2n-1)\cos\phi\, P_{n-1}^{m-1}(\sin\phi), \qquad m \neq 0,\ m = n
\end{aligned}$$

where the first few associated Legendre functions are given by

$$P_0^0(\sin\phi) = 1, \quad P_1^0(\sin\phi) = \sin\phi, \quad P_1^1(\sin\phi) = \cos\phi$$

and $P_i^j = 0$ for $j > i$.

The trigonometric arguments are determined from expansions given by

$$\sin m\lambda = 2\cos\lambda\sin(m-1)\lambda - \sin(m-2)\lambda$$
$$\cos m\lambda = 2\cos\lambda\cos(m-1)\lambda - \cos(m-2)\lambda$$
$$m\tan\phi = (m-1)\tan\phi + \tan\phi$$

These gravity model data files are simple space delimited ASCII data files. The following is a portion of a typical gravity model data file. In this file, column one is the degree index, column two is the model order index, and columns three and four are the corresponding *un-normalized* gravity coefficients (zonals and tesserals, respectively).

```
 2   0  -0.10826300D-02   0.00000000D+00
 3   0   0.25321531D-05   0.00000000D+00
 4   0   0.16109876D-05   0.00000000D+00
 5   0   0.23578565D-06   0.00000000D+00
 6   0  -0.54316985D-06   0.00000000D+00
 7   0   0.33237640D-06   0.00000000D+00
 8   0   0.17721040D-06   0.00000000D+00
 9   0   0.14459876D-06   0.00000000D+00
10   0   0.23339780D-06   0.00000000D+00
11   0  -0.27870829D-06   0.00000000D+00
12   0   0.17036617D-06   0.00000000D+00
13   0   0.25024428D-06   0.00000000D+00
14   0  -0.13764093D-06   0.00000000D+00
15   0  -0.30920023D-07   0.00000000D+00
16   0   0.55350560D-07   0.00000000D+00
```

Gravity model coefficients are often published in *normalized* form. The relationship between normalized $\bar{C}_{l,m}, \bar{S}_{l,m}$ and un-normalized gravity coefficients $C_{l,m}, S_{l,m}$ is given by the following expression:

$$\begin{Bmatrix} \bar{C}_{l,m} \\ \bar{S}_{l,m} \end{Bmatrix} = \left[\frac{1}{(2-\delta_{m0})(2l+1)} \frac{(l+m)!}{(l-m)!} \right]^{1/2} \begin{Bmatrix} C_{l,m} \\ S_{l,m} \end{Bmatrix}$$

where δ_{m0} is equal to 1 if *m* is zero and equal to zero if *m* is greater than zero.

Geocentric acceleration due to the sun, Moon and planets

The acceleration contribution of the Moon represented by a *point mass* is given by

$$\mathbf{a}_m(\mathbf{r},t) = -\mu_m \left(\frac{\mathbf{r}_{m-sc}}{|\mathbf{r}_{m-sc}|^3} + \frac{\mathbf{r}_{e-m}}{|\mathbf{r}_{e-m}|^3} \right)$$

where

$\mu_m =$ gravitational constant of the moon

$\mathbf{r}_{m-sc} =$ position vector from the moon to the spacecraft

$\mathbf{r}_{e-m} =$ position vector from the Earth to the moon

The acceleration contribution of the sun represented by a *point mass* is given by

$$\mathbf{a}_s(\mathbf{r},t) = -\mu_s \left(\frac{\mathbf{r}_{s-sc}}{\left|\mathbf{r}_{s-sc}\right|^3} + \frac{\mathbf{r}_{e-s}}{\left|\mathbf{r}_{e-s}\right|^3} \right)$$

where

$\mu_s =$ gravitational constant of the sun

$\mathbf{r}_{s-sc} =$ position vector from the sun to the spacecraft

$\mathbf{r}_{e-s} =$ position vector from the Earth to the sun

The acceleration contribution of a planet represented by a *point mass* is given by

$$\mathbf{a}_p(\mathbf{r},t) = -\mu_p \left(\frac{\mathbf{r}_{s-sc}}{\left|\mathbf{r}_{s-sc}\right|^3} + \frac{\mathbf{r}_{e-p}}{\left|\mathbf{r}_{e-p}\right|^3} \right)$$

where

$\mu_s =$ gravitational constant of the sun

$\mathbf{r}_{s-sc} =$ position vector from the sun to the spacecraft

$\mathbf{r}_{e-p} =$ position vector from the Earth to the planet

The first-order system of equations required by this computer program can be created from the second-order system by the method of *order reduction*. With the following definitions,

$$y_1 = r_x \quad y_2 = r_y \quad y_3 = r_z$$
$$y_4 = v_x \quad y_5 = v_y \quad y_6 = v_z$$

where v_x, v_y, v_z are the velocity vector components of the spacecraft, the first-order system of differential equations is given by

$$\dot{y}_1 = v_x \qquad \dot{y}_2 = v_y \qquad \dot{y}_3 = v_z$$

$$\dot{y}_4 = -\mu_s \frac{r_x}{r^3} + a_{x-m} + a_{x-p} + a_{x-srp}$$

$$\dot{y}_5 = -\mu_s \frac{r_y}{r^3} + a_{y-m} + a_{y-p} + a_{y-srp}$$

$$\dot{y}_6 = -\mu_s \frac{r_z}{r^3} + a_{z-m} + a_{z-p} + a_{z-srp}$$

In these equations, μ_s is the gravitational constant of the sun, a_{x-p}, a_{y-p} and a_{z-p} are the *x*, *y* and *z* gravitational contributions of the planets, a_{x-m}, a_{y-m} and a_{z-m} are the *x*, *y* and *z* gravitational contributions of the Moon, and a_{x-srp}, a_{y-srp} and a_{z-srp} are the *x*, *y* and *z* gravitational contributions due to solar radiation pressure.

To avoid numerical problems, use is made of Richard Battin's $f\left(q\right)$ function given by

$$f\left(q_k\right) = q_k\left[\frac{3+3q_k+q_k^2}{1+\left(\sqrt{1+q_k}\right)^3}\right]$$

where

$$q_k = \frac{\mathbf{r}^T\left(\mathbf{r}-2\mathbf{s}_k\right)}{\mathbf{s}_k^T\mathbf{s}_k}$$

The point-mass acceleration due to *n* gravitational bodies can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^{n}\frac{\mu_k}{d_k^3}\left[\mathbf{r}+f\left(q_k\right)\mathbf{s}_k\right]$$

In these equations, $\mathbf{s}_k$ is the vector from the primary body to the secondary body, μ_k is the gravitational constant of the secondary body and $\mathbf{d}_k = \mathbf{r}-\mathbf{s}_k$, where **r** is the position vector of the spacecraft relative to the primary body. The derivation of the $f\left(q\right)$ functions is described in Section 8.4 of "An *Introduction to the Mathematics and Methods of Astrodynamics*, Revised Edition", by Richard H. Battin, AIAA Education Series, 1999.

Geocentric acceleration due to solar radiation pressure

We can define a *solar radiation constant* for any spacecraft as a function of its size, mass and surface reflective properties according to the equation:

$$C_{srp} = \gamma\, P_s\, a^2\,\frac{A}{m}$$

where

$\gamma =$ reflectivity constant

$P_s =$ solar radiation pressure constant

$a =$ astronomical unit

$A =$ surface area normal to the incident radiation

$m =$ mass of the spacecraft

The reflectivity constant is a dimensionless number between 0 and 2. For a perfectly absorbent body $\gamma = 1$, for a perfectly reflective body $\gamma = 2$, and for a translucent body $\gamma < 1$. For example, the reflectivity constant for an aluminum surface is approximately 1.96.

The value of the solar radiation pressure on a perfectly absorbing spacecraft surface at a distance of one Astronomical Unit from the Sun is

$$P_s = \frac{G_1}{c} \;\Rightarrow \frac{Newton}{meters^2}$$

where G_1 is the solar flux at a distance of one Astronomical Unit in watts per square meter, and c is the speed of light in meters per second. The values of the solar flux and speed of light used during a simulation are defined by the user in the constants and models data file.

The acceleration vector of the spacecraft due to solar radiation pressure is given by:

$$\mathbf{a}_{srp} = c_{srp} \frac{\mathbf{r}_{sc-s}}{\left|\mathbf{r}_{sc-s}\right|^3}$$

where

$$\mathbf{r}_{sc} = \text{geocentric, inertial position vector of the spacecraft}$$

$$\mathbf{r}_{e-s} = \text{geocentric, inertial position vector of the sun}$$

$$\mathbf{r}_{sc-s} = \mathbf{r}_{sc} - \mathbf{r}_{e-s}$$

During the geocentric integration process, the software must determine if the spacecraft is in Earth shadow or sunlight. Obviously, there can be no solar radiation perturbation during Earth eclipse of the spacecraft orbit. The software makes use of a *shadow parameter* to determine eclipse conditions. This parameter is defined by the following expression:

$$\varphi = -\frac{\left|\mathbf{r}_{sc} \times \mathbf{r}_{e-s}\right|}{\mathbf{r}_{e-s}} sign(\mathbf{r}_{sc} \bullet \mathbf{r}_{e-s})$$

where $\mathbf{r}_{sc}$ is the geocentric, inertial position vector of the spacecraft and $\mathbf{r}_{e-s}$ is the geocentric, inertial position vector of the sun relative to the spacecraft.

The *critical* values of the shadow parameter for the penumbra (subscript p) and umbra part (subscript u) of the shadow are given by:

$$\varphi_p = \left|\mathbf{r}_{sc}\right| \sin\psi_p$$

$$\varphi_u = \left|\mathbf{r}_{sc}\right| \sin\psi_u$$

The penumbra and umbra shadow angles are found from:

$$\psi_p = \eta + \theta_p$$

$$\psi_u = \eta - \theta_u$$

They are the angles between the geocentric anti-sun vector and the vector to a spacecraft at the time of shadow entrance or exit.

If we represent the shadow as a cylinder, the shadow angle is given by:

$$\eta = \sin^{-1}\left(\frac{r_e}{r_{sc}}\right)$$

The corresponding penumbra and umbra *cone* angles are as follows:

$$\theta_p = \sin^{-1}\left(\frac{r_s + r_e}{r_{e-s}}\right)$$

$$\theta_u = \sin^{-1}\left(\frac{r_s - r_e}{r_{e-s}}\right)$$

where

$r_e =$ radius of the Earth
$r_s =$ radius of the sun
$r_{e-s} =$ distance from the Earth to the sun

If the condition $\varphi_u < \varphi \le \varphi_p$ is true, the spacecraft is in the penumbra part of the Earth's shadow, and if the inequality $0 \le \varphi \le \varphi_u$ is true, the spacecraft is in the umbra part of the shadow. If the absolute value of the shadow parameter is larger than the penumbra value, the spacecraft is in full sunlight. The shadow calculations used in this computer program also assume the Earth's atmosphere increases the radius of the Earth by two percent.

Heliocentric trajectory propagation

The second-order, vector system of heliocentric equations of motion for *point-mass* gravity perturbations such as the Moon or planets are given by

$$\ddot{\mathbf{r}} = -\sum_{j=1}^{n} \mu_j \left[\frac{\mathbf{d}_j}{d_j^3} + \frac{\mathbf{s}_j}{s_j^3}\right]$$

In this equation, $\mathbf{s}_j$ is the vector from the primary body to the secondary body *j*, μ_j is the gravitational constant of the secondary body, and $\mathbf{d}_j = \mathbf{r} - \mathbf{s}_j$, where $\mathbf{r}$ is the position vector of the spacecraft relative to the primary body.

Following the geocentric formulation, use is again made of Battin's $F(q)$ function given by

$$F(q_k) = q_k \left[\frac{3 + 3q_k + q_k^2}{1 + \left(\sqrt{1 + q_k}\right)^3}\right]$$

where

$$q_k = \frac{\mathbf{r}^T(\mathbf{r} - 2\mathbf{s}_k)}{\mathbf{s}_k^T \mathbf{s}_k}$$

The third-body acceleration can now be expressed as

$$\ddot{\mathbf{r}} = -\sum_{k=1}^{n} \frac{\mu_k}{d_k^3} \left[\mathbf{r} + F\left(q_k\right) \mathbf{s}_k \right]$$

Heliocentric acceleration due to solar radiation pressure

The heliocentric acceleration vector of the spacecraft due to solar radiation pressure is given by:

$$\mathbf{a}_{srp} = c_{srp} \frac{\mathbf{r}_{sc}}{\left|\mathbf{r}_{sc}\right|^3}$$

where $\mathbf{r}_{sc}$ is the heliocentric position vector of the spacecraft. The equation for c_{srp} is defined in the previous geocentric trajectory propagation discussion.

EME2000-to-areocentric coordinate transformation

This section describes the transformation of coordinates between the Earth mean equator and equinox of J2000 (EME2000) and the areocentric mean equator and IAU node of epoch coordinate systems. This transformation is used to compute the Mars-centered flight path angle and the coordinates of the spacecraft at encounter.

A unit vector in the direction of the pole of Mars can be determined from

$$\hat{\mathbf{p}}_{Mars} = \begin{bmatrix} \cos\alpha_p \cos\delta_p \\ \sin\alpha_p \cos\delta_p \\ \sin\delta_p \end{bmatrix}$$

The IAU 2000 right ascension and declination of the pole of Mars in the EME2000 coordinate system are given by the following expressions

$$\alpha_p = 317.68143 - 0.1061T$$

$$\delta_p = 52.88650 - 0.0609T$$

where T is the time in Julian centuries given by $T = \left(JD - 2451545.0\right)/36525$ and JD is the TDB Julian Date.

The unit vector in the direction of the *IAU-defined* x-axis is computed from

$$\hat{\mathbf{x}} = \hat{\mathbf{p}}_{J2000} \times \hat{\mathbf{p}}_{Mars}$$

where $\hat{\mathbf{p}}_{J2000} = \left[0\;0\;1\right]^T$ is unit vector in the direction of the pole of the J2000 coordinate system.

The unit vector in the y-axis direction of this coordinate system is

$$\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Mars} \times \hat{\mathbf{x}}$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the Mars-centered mean equator and IAU node of epoch system are as follows:

$$\mathbf{M} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{p}}_{Mars} \end{bmatrix}$$

Predicting the conditions at the Earth's sphere of influence

The trajectory conditions at the boundary of the Earth's sphere of influence is determined during the numerical integration of the spacecraft's geocentric equations of motion by finding the time at which the difference between the geocentric distance and the user-defined value is essentially zero. This mission constraint is computed as follows

$$\Delta r = |\mathbf{r}_{sc}| - r_{soi} \approx 0$$

where $\mathbf{r}_{sc}$ is the geocentric position vector of the spacecraft and r_{soi} is the user-defined value of the geocentric distance of the SOI boundary.

Predicting closest approach to Mars

Closest approach is determined during the numerical integration of the spacecraft's heliocentric equations of motion by finding the time at which the spacecraft's flight path angle relative to Mars is essentially zero. This mission constraint is computed as follows

$$\gamma = \sin^{-1}\left(\frac{\mathbf{r} \bullet \mathbf{v}}{|\mathbf{r} \bullet \mathbf{v}|}\right) \approx 0$$

where **r** and **v** are the Mars-centered position and velocity vectors, respectively. Both orbital events are predicted using a Runge-Kutta-Fehlberg (RKF7(8)) integrator embedded with a one-dimensional form of Brent's root-finding method.

Predicting the conditions at a user-defined areocentric distance

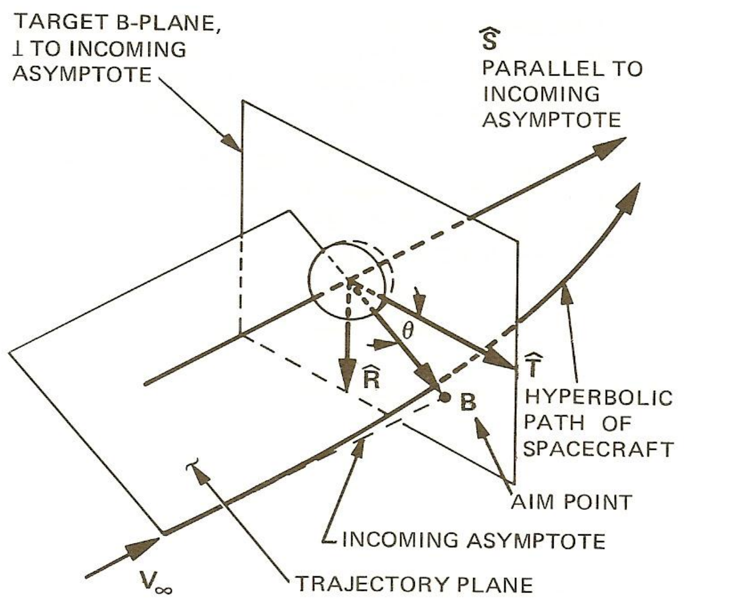
The trajectory conditions at the boundary of a user-defined areocentric distance are determined during the numerical integration of the spacecraft's geocentric heliocentric of motion by finding the time at which the difference between the areocentric distance and the user-defined value is essentially zero. This mission constraint is computed as follows

$$\Delta r = |\mathbf{r}_{sc}| - r_{soi} \approx 0$$

where $\mathbf{r}_{sc}$ is the areocentric position vector of the spacecraft and r_{user} is the user-specified value of the areocentric distance.

The B-plane

The derivation of B-plane coordinates is described in the classic JPL reports, "A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories" and "Some Orbital Elements Useful in Space Trajectory Calculations", both by William Kizner. The following diagram illustrates the fundamental geometry of the B-plane coordinate system.



The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola at Mars encounter.

The following computational steps summarize the calculation of the B-plane vector from a Mars-centered position vector **r** and velocity vector **v** evaluated at closest approach to Mars.

angular momentum vector

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

radius rate

$$\dot{r} = \mathbf{r} \bullet \mathbf{v} / |\mathbf{r}|$$

semiparameter

$$p = h^2 / \mu$$

semimajor axis

$$a = \frac{r}{\left(2 - \frac{r v^2}{\mu}\right)}$$

orbital eccentricity

$$e = \sqrt{1 - p/a}$$

true anomaly

$$\cos\theta = \frac{p - r}{e r} \qquad \sin\theta = \frac{\dot{r} h}{e\mu}$$

B-plane magnitude

$$B = \sqrt{p|a|}$$

fundamental vectors

$$\hat{\mathbf{z}} = \frac{r\,\mathbf{v} - \dot{r}\,\mathbf{r}}{h}$$

$$\hat{\mathbf{p}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\mathbf{z}} \qquad \hat{\mathbf{q}} = \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\mathbf{z}}$$

S vector

$$\mathbf{S} = -\frac{a}{\sqrt{a^2 + b^2}}\hat{\mathbf{p}} + \frac{b}{\sqrt{a^2 + b^2}}\hat{\mathbf{q}}$$

B vector

$$\mathbf{B} = \frac{b^2}{\sqrt{a^2 + b^2}}\hat{\mathbf{p}} + \frac{ab}{\sqrt{a^2 + b^2}}\hat{\mathbf{q}}$$

T vector

$$\mathbf{T} = \frac{\left(S_y^2, -S_x^2, 0\right)^T}{\sqrt{S_x^2 + S_y^2}}$$

R vector

$$\mathbf{R} = \mathbf{S} \times \mathbf{T} = \left(-S_z T_y,\ S_z T_x,\ S_x T_y - S_y T_x\right)^T$$

Equatorial to ecliptic transformation

In this computer program the heliocentric coordinates of the Earth and Mars are computed in the Earth mean equator and equinox of J2000 coordinate system using the JPL DE421 algorithm. The `pprop_e2m` computer program also provides these coordinates in the Earth mean ecliptic and equinox of J2000 coordinate system.

The transformation of vectors from the equatorial to the ecliptic system involves the following matrix multiplication

$$\mathbf{S}_{ec} = \begin{bmatrix} 1 & -0.000000479966 & 0 \\ 0.000000440360 & 0.917482137087 & 0.397776982902 \\ -0.000000190919 & -0.397776982902 & 0.917482137087 \end{bmatrix}^{T} \mathbf{S}_{eq}$$

where $\mathbf{S}_{eq}$ is the state vector (position and velocity vectors) in the equatorial system and $\mathbf{S}_{ec}$ is the state vector in the ecliptic system.

Terrestrial Time, TT

Terrestrial Time is the time scale that would be kept by an ideal clock on the geoid - approximately, sea level on the surface of the Earth. Since its unit of time is the SI (atomic) second, TT is independent of the variable rotation of the Earth. TT is meant to be a smooth and continuous "coordinate" time scale independent of Earth rotation. In practice TT is derived from International Atomic Time (TAI), a time scale kept by real clocks on the Earth's surface, by the relation **TT = TAI + $32^{s}.184$.** It is the time scale now used for the precise calculation of future astronomical events observable from Earth.

TT = TAI + 32.184 seconds

TT = UTC + (number of leap seconds) + 32.184 seconds

Barycentric Dynamical Time, TDB

Barycentric Dynamical Time is the time scale that would be kept by an ideal clock, free of gravitational fields, co-moving with the solar system barycenter. It is always within 2 milliseconds of TT, the difference caused by relativistic effects. TDB is the time scale now used for investigations of the dynamics of solar system bodies.

TDB = TT + periodic corrections

where typical periodic corrections (USNO Circular 179) are

$$\begin{aligned} TDB = TT &+ 0.001657\sin\left(628.3076T + 6.2401\right) \\ &+ 0.000022\sin\left(575.3385T + 4.2970\right) \\ &+ 0.000014\sin\left(1256.6152T + 6.1969\right) \\ &+ 0.000005\sin\left(606.9777T + 4.0212\right) \\ &+ 0.000005\sin\left(52.9691T + 0.4444\right) \\ &+ 0.000002\sin\left(21.3299T + 5.5431\right) \\ &+ 0.000010T\sin\left(628.3076T + 4.2490\right) + \cdots \end{aligned}$$

In this equation, the coefficients are in seconds, the angular arguments are in radians, and T is the number of Julian centuries of TT from J2000; T = (Julian Date(TT) – 2451545.0) / 36525.

Algorithm and Modeling Resources

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(4) "The Planetary and Lunar Ephemeris DE 421", W. M. Folkner, J. G. Williams, D. H. Boggs, JPL IOM 343R-08-003, 31-March-2008.

(5) "Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 2009", *Celestial Mechanics and Dynamical Astronomy*, **109**: 101-135, 2011.

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(7) "Planetary Constants and Models", R. Vaughan, JPL D-12947, December 1995.

(8) R. P. Brent, *Algorithms for Minimization Without Derivatives*, Prentice-Hall, 1972.

(9) W. Kizner, "A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories", Publication 674, Jet Propulsion Laboratory, August 1, 1959.

(10) F. M. Sturms, Jr., "Error Analysis of Multiple Planet Trajectories", JPL Space Programs Summary, No. 37-27, Vol. IV.

(11) R. H. Battin, *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA, 1987.

(12) "Interplanetary Mission Design Handbook, Volume 1, Part 2", JPL Publication 82-43, September 15, 1983.

APPENDIX A

Contents of the Simulation Summary

This appendix is a brief summary of the information contained in the simulation summary screen display produced by the `pprop_e2m` software. It is possible to "redirect" the screen output to a simple text file with a DOS command similar to

```
pprop_e2m pprop_e2m.in >pprop_e2m.txt
```

The simulation summary screen display contains the following information:

calendar date = calendar date of trajectory event

UTC time = UTC time of trajectory event

UTC Julian Date = Julian Date of trajectory event on UTC time scale

TDB time = TDB time of trajectory event

TDB Julian Date = Julian Date of trajectory event on TDB time scale

sma (km) = semimajor axis in kilometers or astronomical units

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of periapsis in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (days) = orbital period in days

rx (km) = x-component of the spacecraft's position vector in kilometers

ry (km) = y-component of the spacecraft's position vector in kilometers

rz (km) = z-component of the spacecraft's position vector in kilometers

rmag (km) = scalar magnitude of the spacecraft's position vector in kilometers

vx (kps) = x-component of the spacecraft's velocity vector in kilometers per second

vy (kps) = y-component of the spacecraft's velocity vector in kilometers per second

vz (ksp) = z-component of the spacecraft's velocity vector in kilometers per second

vmag (kps) = scalar magnitude of the spacecraft's velocity vector in kilometers per second

right ascension = right ascension of Earth departure v-infinity vector in degrees

declination = declination of Earth departure v-infinity vector in degrees

orbital energy = specific orbital energy in kilometers squared per seconds squared

v-infinity = departure speed at "infinity" in kilometers per second

ta-infinity = departure true anomaly at "infinity" in degrees

b-magnitude = magnitude of the b-plane vector

b dot r = dot product of the B-plane b-vector and r-vector

b dot t = dot product of the B-plane b-vector and t-vector

theta = orientation of the b-plane vector in degrees

v-infinity = magnitude of incoming v-infinity vector at Mars in kilometers/second

r-periapsis = periapsis radius of incoming hyperbola at Mars in kilometers

decl-asymptote = declination of incoming asymptote at Mars in degrees

rasc-asymptote = right ascension of incoming asymptote at Mars in degrees

areocentric flight path angle = flight path angle relative to Mars in degrees.

propagation duration = trajectory time from the initial epoch to the final user-defined trajectory event or epoch in days

APPENDIX B

Additional Program Examples

This appendix summarizes typical output data created by the `pprop_e2m` software for the other two program options. The initial epoch and state vector for both examples are the same as the close approach example given earlier in this document.

The first output summary is for the user-defined areocentric distance program option. The user-defined Mars-to-spacecraft distance is 500,000 kilometers.

```
program pprop_e2m
-----------------

precision propagation from Earth to Mars

input file ==> pprop_e2m.in

models/constants file ==> pprop_e2m_cm.dat

DE421 ephemeris


initial time and conditions
(geocentric Earth mean equator and equinox of J2000)
----------------------------------------------------

calendar date           June  5, 2003

UTC time                14:47:23.918

UTC Julian date         2452796.11624905

TDB time                14:48:28.050

TDB Julian date         2452796.11699132


     sma (km)              eccentricity        inclination (deg)        argper (deg)
 -.453500501333D+05    0.114472883670D+01    0.286442848562D+02    0.194742119763D+03

    raan (deg)          true anomaly (deg)       arglat (deg)
 0.267494586008D+01    0.763333123551D-13    0.194742119763D+03

     rx (km)                 ry (km)                rz (km)               rmag (km)
 -.627207312608D+04    -.176041957307D+04    -.800642741502D+03    0.656346000000D+04

     vx (kps)                vy (kps)               vz (kps)              vmag (kps)
 0.335306759387D+01    -.954013022081D+01    -.529081568531D+01    0.114127068452D+02


asymptote coordinates and specific orbital energy
(geocentric Earth mean equator and equinox of J2000)
----------------------------------------------------

right ascension      349.992647760703      degrees

declination         -6.83825103161977      degrees

orbital energy       8.78941567492588      (km/sec)**2
```

v-infinity 2.96469487045899 km/sec

ta-infinity 150.876105569553 degrees

time and conditions of the Earth
(heliocentric Earth mean equator and equinox of J2000)
----

calendar date	June 5, 2003
UTC time	14:47:23.918
UTC Julian date	2452796.11624905
TDB time	14:48:28.050
TDB Julian date	2452796.11699132

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.100035811608D+01	0.162374958182D-01	0.234390545150D+02	0.102451915932D+03
raan (deg)	**true anomaly (deg)**	**arglat (deg)**	**period (days)**
0.722897068242D-03	0.152049044492D+03	0.254500960424D+03	0.365453122443D+03
rx (km)	**ry (km)**	**rz (km)**	**rmag (km)**
-.405597427281D+08	-.134200242066D+09	-.581820454662D+08	0.151789156778D+09
vx (kps)	**vy (kps)**	**vz (kps)**	**vmag (kps)**
0.282280784458D+02	-.739734366114D+01	-.320725910415D+01	0.293569687969D+02

time and conditions of the Earth
(heliocentric Earth mean ecliptic and equinox of J2000)
----

calendar date	June 5, 2003
UTC time	14:47:23.918
UTC Julian date	2452796.11624905
TDB time	14:48:28.050
TDB Julian date	2452796.11699132

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.100035811608D+01	0.162374958178D-01	0.374268971909D-03	0.335346777183D+03
raan (deg)	**true anomaly (deg)**	**arglat (deg)**	**period (days)**
0.127105827228D+03	0.152049044490D+03	0.127395821673D+03	0.365453122444D+03
rx (km)	**ry (km)**	**rz (km)**	**rmag (km)**
-.405596783166D+08	-.146269821253D+09	0.787722736903D+03	0.151789156778D+09
vx (kps)	**vy (kps)**	**vz (kps)**	**vmag (kps)**
0.282280819963D+02	-.806269209031D+01	-.115283324146D-03	0.293569687970D+02

time and conditions at Earth sphere-of-influence
(geocentric Earth mean equator and equinox of J2000)
----

calendar date June 8, 2003

```
UTC time               18:21:52.360

UTC Julian date        2452799.26518936

TDB time               18:22:56.492

TDB Julian date        2452799.26593162

     sma (km)            eccentricity       inclination (deg)       argper (deg)
 -.455952132484D+05    0.114329887029D+01    0.285067459380D+02    0.194701071343D+03

    raan (deg)         true anomaly (deg)      arglat (deg)
 0.268703911208D+01    0.149476424236D+03    0.344177495579D+03

     rx (km)               ry (km)               rz (km)              rmag (km)
 0.899364334057D+06    -.179666139810D+06    -.120369733873D+06    0.925000000000D+06

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 0.303091732413D+01    -.532486195991D+00    -.366050347590D+00    0.309903117629D+01

time and conditions at Earth sphere-of-influence
(heliocentric Earth mean equator and equinox of J2000)
------------------------------------------------------

calendar date          June  8, 2003

UTC time               18:21:52.360

UTC Julian date        2452799.26518936

TDB time               18:22:56.492

TDB Julian date        2452799.26593162

     sma (au)            eccentricity       inclination (deg)       argper (deg)
 0.127480209108D+01    0.203979869558D+00    0.234999959931D+02    0.253589169139D+03

    raan (deg)         true anomaly (deg)      arglat (deg)         period (days)
 0.526245473607D+00    0.379159057243D+01    0.257380759712D+03    0.525729779140D+03

     rx (km)               ry (km)               rz (km)              rmag (km)
 -.319281368727D+08    -.136202685909D+09    -.590925967550D+08    0.151863466851D+09

     vx (kps)              vy (kps)              vz (kps)             vmag (kps)
 0.316293511557D+02    -.653259680468D+01    -.296664765047D+01    0.324328795179D+02

time and conditions at user-defined distance
(heliocentric Earth mean equator and equinox of J2000)
------------------------------------------------------

calendar date          December 22, 2003

UTC time               01:17:40.875

UTC Julian date        2452995.55394532

TDB time               01:18:45.007

TDB Julian date        2452995.55468759
```

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.125685412431D+01	0.195896017755D+00	0.234837903129D+02	0.253038671066D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.469048138440D+00	0.152012622443D+03	0.450512935098D+02	0.514666308164D+03
rx (km)	ry (km)	rz (km)	rmag (km)
0.153287005797D+09	0.143174490809D+09	0.616585913500D+08	0.218626674612D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.142295507045D+02	0.160187706860D+02	0.701014969026D+01	0.225438090443D+02

time and conditions at user-defined distance
(areocentric mean equator and IAU node of epoch)
----

calendar date	December 22, 2003
UTC time	01:17:40.875
UTC Julian date	2452995.55394532
TDB time	01:18:45.007
TDB Julian date	2452995.55468759

sma (km)	eccentricity	inclination (deg)	argper (deg)
-.586036979643D+04	0.486480976847D+01	0.171829646439D+02	0.308078666336D+03
raan (deg)	true anomaly (deg)	arglat (deg)	
0.256052156145D+03	0.261318131144D+03	0.209396797480D+03	
rx (km)	ry (km)	rz (km)	rmag (km)
-.122558502794D+06	0.479293659708D+06	-.725051848699D+05	0.500000000677D+06
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.529631979759D+00	-.265920689748D+01	0.357152052964D+00	0.273485812183D+01

B-plane coordinates at user-defined distance
(areocentric mean equator and IAU node of epoch)
----

b-magnitude	27900.7609028832	kilometers
b dot r	7458.44235333134	
b dot t	26885.3881620089	
theta	15.5048766055420	degrees
v-infinity	2.70335632906689	kilometers/second
r-periapsis	22649.2144360795	kilometers
decl-asymptote	7.49893674313970	degrees
rasc-asymptote	281.246345294421	degrees

areocentric flight path angle -86.8380307914380 degrees

time and conditions of Mars at user-defined distance
(heliocentric Earth mean equator and equinox of J2000)
----

calendar date	December 22, 2003
UTC time	01:17:40.875
UTC Julian date	2452995.55394532

```
TDB time              01:18:45.007

TDB Julian date       2452995.55468759

     sma (au)            eccentricity       inclination (deg)       argper (deg)
 0.152368143894D+01   0.935427878711D-01   0.246772246494D+02   0.332979489863D+03

    raan (deg)         true anomaly (deg)      arglat (deg)         period (days)
 0.337165765088D+01   0.692967443283D+02   0.422762341911D+02   0.686973038245D+03

     rx (km)               ry (km)              rz (km)              rmag (km)
 0.153684455870D+09   0.142978312256D+09   0.614271882053D+08   0.218712160618D+09

     vx (kps)              vy (kps)             vz (kps)             vmag (kps)
 -.163133165027D+02   0.172000317767D+02   0.832999486316D+01   0.251267626816D+02

time and conditions of Mars at user-defined distance
(heliocentric Earth mean ecliptic and equinox of J2000)
-------------------------------------------------------

calendar date         December 22, 2003

UTC time              01:17:40.875

UTC Julian date       2452995.55394532

TDB time              01:18:45.007

TDB Julian date       2452995.55468759

     sma (au)            eccentricity       inclination (deg)       argper (deg)
 0.152368143894D+01   0.935427878715D-01   0.184937124315D+01   0.286517684410D+03

    raan (deg)         true anomaly (deg)      arglat (deg)         period (days)
 0.495409244629D+02   0.692967443279D+02   0.355814428738D+03   0.686973038245D+03

     rx (km)               ry (km)              rz (km)              rmag (km)
 0.153684387245D+09   0.155614436754D+09   -.515163100923D+06   0.218712160618D+09

     vx (kps)              vy (kps)             vz (kps)             vmag (kps)
 -.163133247582D+02   0.190941949530D+02   0.800847857543D+00   0.251267626816D+02

propagation duration    199.437696265522      days
```

The second example exercises the user-defined epoch program option. The user-defined epoch for this case is 15 days past the initial epoch. Please note that the software will only display the areocentric and B-plane coordinates if the spacecraft is within 600,000 kilometers of Mars.

```
program pprop_e2m
-----------------

precision propagation from Earth to Mars

input file ==> pprop_e2m.in

models/constants file ==> pprop_e2m_cm.dat

DE421 ephemeris
```

```
initial time and conditions
(geocentric Earth mean equator and equinox of J2000)
----------------------------------------------------

calendar date          June  5, 2003

UTC time               14:47:23.918

UTC Julian date        2452796.11624905

TDB time               14:48:28.050

TDB Julian date        2452796.11699132

     sma (km)              eccentricity       inclination (deg)       argper (deg)
 -.453500501333D+05    0.114472883670D+01    0.286442848562D+02    0.194742119763D+03

    raan (deg)          true anomaly (deg)       arglat (deg)
 0.267494586008D+01    0.763333123551D-13    0.194742119763D+03

     rx (km)                ry (km)                rz (km)               rmag (km)
 -.627207312608D+04    -.176041957307D+04    -.800642741502D+03    0.656346000000D+04

     vx (kps)               vy (kps)               vz (kps)              vmag (kps)
 0.335306759387D+01    -.954013022081D+01    -.529081568531D+01    0.114127068452D+02

asymptote coordinates and specific orbital energy
(geocentric Earth mean equator and equinox of J2000)
----------------------------------------------------

right ascension      349.992647760703      degrees

declination         -6.83825103161977      degrees

orbital energy       8.78941567492588      (km/sec)**2

v-infinity           2.96469487045899      km/sec

ta-infinity          150.876105569553      degrees

time and conditions of the Earth
(heliocentric Earth mean equator and equinox of J2000)
------------------------------------------------------

calendar date          June  5, 2003

UTC time               14:47:23.918

UTC Julian date        2452796.11624905

TDB time               14:48:28.050

TDB Julian date        2452796.11699132

     sma (au)              eccentricity       inclination (deg)       argper (deg)
 0.100035811608D+01    0.162374958182D-01    0.234390545150D+02    0.102451915932D+03

    raan (deg)          true anomaly (deg)       arglat (deg)         period (days)
 0.722897068242D-03    0.152049044492D+03    0.254500960424D+03    0.365453122443D+03
```

rx (km)	ry (km)	rz (km)	rmag (km)
-.405597427281D+08	-.134200242066D+09	-.581820454662D+08	0.151789156778D+09

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.282280784458D+02	-.739734366114D+01	-.320725910415D+01	0.293569687969D+02

time and conditions of the Earth
(heliocentric Earth mean ecliptic and equinox of J2000)
----

calendar date June 5, 2003

UTC time 14:47:23.918

UTC Julian date 2452796.11624905

TDB time 14:48:28.050

TDB Julian date 2452796.11699132

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.100035811608D+01	0.162374958178D-01	0.374268971909D-03	0.335346777183D+03

raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.127105827228D+03	0.152049044490D+03	0.127395821673D+03	0.365453122444D+03

rx (km)	ry (km)	rz (km)	rmag (km)
-.405596783166D+08	-.146269821253D+09	0.787722736903D+03	0.151789156778D+09

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.282280819963D+02	-.806269209031D+01	-.115283324146D-03	0.293569687970D+02

time and conditions at Earth sphere-of-influence
(geocentric Earth mean equator and equinox of J2000)
----

calendar date June 8, 2003

UTC time 18:21:52.360

UTC Julian date 2452799.26518936

TDB time 18:22:56.492

TDB Julian date 2452799.26593162

sma (km)	eccentricity	inclination (deg)	argper (deg)
-.455952132484D+05	0.114329887029D+01	0.285067459380D+02	0.194701071343D+03

raan (deg)	true anomaly (deg)	arglat (deg)
0.268703911208D+01	0.149476424236D+03	0.344177495579D+03

rx (km)	ry (km)	rz (km)	rmag (km)
0.899364334057D+06	-.179666139810D+06	-.120369733873D+06	0.925000000000D+06

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.303091732413D+01	-.532486195991D+00	-.366050347590D+00	0.309903117629D+01

time and conditions at Earth sphere-of-influence
(heliocentric Earth mean equator and equinox of J2000)
----

calendar date	June 8, 2003
UTC time	18:21:52.360
UTC Julian date	2452799.26518936
TDB time	18:22:56.492
TDB Julian date	2452799.26593162

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.127480209108D+01	0.203979869558D+00	0.234999959931D+02	0.253589169139D+03
raan (deg)	**true anomaly (deg)**	**arglat (deg)**	**period (days)**
0.526245473607D+00	0.379159057243D+01	0.257380759712D+03	0.525729779140D+03
rx (km)	**ry (km)**	**rz (km)**	**rmag (km)**
-.319281368727D+08	-.136202685909D+09	-.590925967550D+08	0.151863466851D+09
vx (kps)	**vy (kps)**	**vz (kps)**	**vmag (kps)**
0.316293511557D+02	-.653259680468D+01	-.296664765047D+01	0.324328795179D+02

time and conditions at user-defined final epoch
(heliocentric Earth mean equator and equinox of J2000)
----

calendar date	June 20, 2003
UTC time	14:47:23.918
UTC Julian date	2452811.11624905
TDB time	14:48:28.050
TDB Julian date	2452811.11699132

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.126191115220D+01	0.195869474816D+00	0.234988199178D+02	0.253413635983D+03
raan (deg)	**true anomaly (deg)**	**arglat (deg)**	**period (days)**
0.508102779472D+00	0.164186400061D+02	0.269832275989D+03	0.517775616660D+03
rx (km)	**ry (km)**	**rz (km)**	**rmag (km)**
0.795488435268D+06	-.140147750414D+09	-.609352151105D+08	0.152823837108D+09
vx (kps)	**vy (kps)**	**vz (kps)**	**vmag (kps)**
0.321250437201D+02	-.117414518432D+01	-.634347829944D+00	0.321527517967D+02

time and conditions of Mars at user-defined final epoch
(heliocentric Earth mean equator and equinox of J2000)
----

calendar date	June 20, 2003
UTC time	14:47:23.918
UTC Julian date	2452811.11624905
TDB time	14:48:28.050
TDB Julian date	2452811.11699132

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.152371117457D+01	0.935630362122D-01	0.246770854982D+02	0.332972596328D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.337184653992D+01	0.315765207839D+03	0.288737804167D+03	0.686993148428D+03
rx (km)	ry (km)	rz (km)	rmag (km)
0.786226485290D+08	-.177900391228D+09	-.837220324807D+08	0.211753273371D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.234157913557D+02	0.103048441382D+02	0.409372685844D+01	0.259084483734D+02

time and conditions of Mars at user-defined final epoch
(heliocentric Earth mean ecliptic and equinox of J2000)
----

calendar date June 20, 2003

UTC time 14:47:23.918

UTC Julian date 2452811.11624905

TDB time 14:48:28.050

TDB Julian date 2452811.11699132

sma (au)	eccentricity	inclination (deg)	argper (deg)
0.152371117457D+01	0.935630362125D-01	0.184933256511D+01	0.286506155921D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (days)
0.495457335599D+02	0.315765207840D+03	0.242271363760D+03	0.686993148429D+03
rx (km)	ry (km)	rz (km)	rmag (km)
0.786227339151D+08	-.196523093993D+09	-.604880341246D+07	0.211753273371D+09
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.234157864097D+02	0.110829110522D+02	-.343113014363D+00	0.259084483734D+02

propagation duration 15.0000000000000 days