Program ca_sc2moon

Closest Approach Between a Spacecraft and the Moon

This document is the user's manual for a computer program called `ca_sc2moon` that can be used to estimate the time of closest approach between a spacecraft and the Moon. This information can be used to access an initial guess for other lunar trajectory analysis computer programs.

The software also includes the option to propagate a spacecraft's trajectory for a user-specified time duration. This allows the user to access the effects of such trajectory characteristics as TLI calendar date, initial park orbit orientation and so forth.

The important numerical methods used in this computer program are as follows:

- Equations of motion based on modified equinoctial orbital elements
- B-plane coordinates at closest approach
- JPL DE405 ephemeris and sun and moon point-mass gravity
- Tangential or gravity turn steering
- Brent's method for one-dimensional minimization
- Runge-Kutta-Fehlberg 7(8) numerical integration method

Running the computer program

An input file created by the user can be run from the command line or a simple batch file with a statement similar to the following:

```
ca_sc2moon lro1.in
```

If the software is executed without an input file on the command line, the computer program will display the following information screen and file name prompt:

```
*************************************
*         program ca_sc2moon        *
*                                   *
*      closest approach between     *
*     a spacecraft and the Moon     *
*                                   *
*           July 25, 2005           *
*************************************


please input the name of the simulation definition file
```

At this point the user should supply the name of a compatible input file. The next section of this document describes the proper format for an input file for this computer program.

Typical input file

The `ca_sc2moon` software is "data-driven" by a user-created text file. The following is a typical input file used by this computer program. This example searches for closest approach using a gravity-turn propulsive maneuver.

Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input. ASCII text input is <u>not</u> case sensitive but must be spelled correctly.

In the following discussion the actual input file contents are in `courier` font and all explanations are in *times italic* font.

The first six lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with six and only six initial text lines.

```
*****************************************************
** closest approach between a spacecraft and the moon
** n-body geocentric motion
** continuous, low-thrust TLI maneuver
** lro1.in - July 25, 2005
*****************************************************
```

The first program input is an integer that specifies the type of simulation. Option 1 will simply propagate the spacecraft for a user-defined duration and option 2 will search for close approach.

```
simulation type (1 = propagation, 2 = close approach)
2
```

The next inputs are the initial spacecraft mass, thrust magnitude and specific impulse.

```
initial spacecraft mass (kilograms)
1000.0

thrust magnitude (newtons)
5000.0

specific impulse (seconds)
450.0

thrust duration (seconds)
450.0

type of steering method (1 = gravity turn, 2 = tangential)
1
```

The next input is a number that represents either a guess for the transfer time to closest approach or the time duration for the orbit propagation option.

```
transfer time guess or propagation duration (hours)
120
```

These inputs define the trans-lunar injection (TLI) launch calendar date and universal time. Be sure to include all four digits of the calendar year.

```
******************
* TLI CONDITIONS *
******************

TLI calendar date (month, day, year)
10, 12, 2008

TLI universal time (hours, minutes, seconds)
4,23,5.376
```

The next six inputs are the classical orbital elements of the park orbit at the beginning of the TLI propulsive maneuver. These elements should be specified with respect to the Earth mean equator and equinox of J2000 (EME2000) coordinate system.

```
******************
initial park orbit
******************

semimajor axis (kilometers)
6563.34

orbital eccentricity (non-dimensional)
0.0d0

orbital inclination (degrees)
28.5d0

argument of perigee (degrees)
0.0d0

right ascension of the ascending node (degrees)
289.996

true anomaly (degrees)
280.5758
```

The next two inputs specify the third-body perturbations to include in the simulation.

```
*********************
orbital perturbations
*********************

include solar point-mass gravity (1 = yes, 0 = no)
1

include lunar point-mass gravity (1 = yes, 0 = no)
1
```

The next input is the name of the data file that will contain the trajectory information.

```
name of solution output file
lro1.csv
```

The final input specifies the time step size for the data in the output file.

```
output file step size (minutes)
10.0
```

Program solution and graphics

The software will display the characteristics of the finite-burn TLI maneuver and the trajectory characteristics at closest approach or at the end of the propagation interval.

```
program ca_sc2moon

closest approach between a spacecraft and the moon
==================================================


conditions at beginning of TLI propulsive maneuver
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date           October   12, 2008

TDB time                04:23:05.376

TDB Julian date         2454751.682701110839844

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 0.656334000000D+04    0.109294907647D-15    0.285000000000D+02    0.000000000000D+00

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.289996000000D+03    0.280575800000D+03    0.280575800000D+03    0.881956335064D+02

     rx (km)               ry (km)                rz (km)              rmag (km)
 -.491626555370D+04    -.307087258346D+04    -.307855591752D+04    0.656334000000D+04

     vx (kps)              vy (kps)               vz (kps)             vmag (kps)
 0.380079445708D+01    -.676901185656D+01    0.682481695207D+00    0.779303158492D+01


conditions at end of TLI propulsive maneuver
(Earth mean equator and equinox of J2000)
-----------------------------------------

calendar date           October   12, 2008

TDB time                04:30:35.376

TDB Julian date         2454751.687909444328398

     sma (km)             eccentricity       inclination (deg)       argper (deg)
 0.182182408149D+06    0.963689475858D+00    0.285122681403D+02    0.296686390532D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.289941643145D+03    0.193524224957D+02    0.316038813028D+03    0.128979113590D+05

     rx (km)               ry (km)                rz (km)              rmag (km)
 -.223099128979D+04    -.601926372743D+04    -.225450892411D+04    0.680379808962D+04

     vx (kps)              vy (kps)               vz (kps)             vmag (kps)
 0.821222424436D+01    -.618487272440D+01    0.304775038350D+01    0.107229688080D+02


spacecraft mass       490.141893511035732 kilograms

deltav               3146.729982208522415 meters/second

thrust duration       450.000000000000000 seconds
```

conditions at lunar closest approach
(Earth mean equator and equinox of J2000)
----

calendar date   October   17, 2008

TDB time   02:33:01.791

TDB Julian date   2454756.606270729564130

sma (km)	eccentricity	inclination (deg)	argper (deg)
0.179122336690D+06	0.963056247650D+00	0.285136918580D+02	0.296744891710D+03

raan (deg)	true anomaly (deg)	arglat (deg)	period (min)
0.289916718628D+03	0.181609059549D+03	0.118353951259D+03	0.125743153965D+05

rx (km)	ry (km)	rz (km)	rmag (km)
0.196735748666D+06	0.247092412043D+06	0.146215327323D+06	0.348049618354D+06

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.227770023258D+00	0.353250301545D-01	-.109801063175D+00	0.255310232315D+00

closest approach distance   16925.351468373439275 kilometers

conditions at lunar closest approach
(selenocentric - lunar equator and equinox of J2000)
----

calendar date   October   17, 2008

TDB time   02:33:01.791

TDB Julian date   2454756.606270729564130

sma (km)	eccentricity	inclination (deg)	argper (deg)
-.189171252628D+05	0.189471054635D+01	0.147060655084D+02	0.115217394887D+03

raan (deg)	true anomaly (deg)	arglat (deg)	period (min)
0.111866994762D+03	0.142733977216D-03	0.115217537621D+03	0.000000000000D+00

rx (km)	ry (km)	rz (km)	rmag (km)
-.110592598603D+05	-.122086061001D+05	0.388718807555D+04	0.169253514793D+05

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.658763135705D+00	-.628282202390D+00	-.990412465936D-01	0.915705500328D+00

----

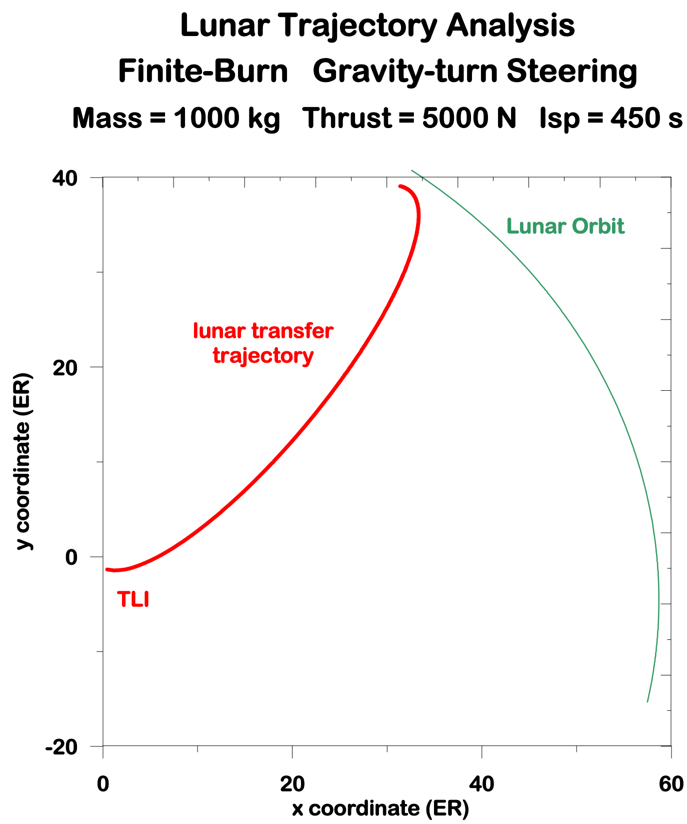
b-plane coordinates of incoming hyperbola
(selenocentric - lunar equator and equinox of J2000)

```
b-magnitude           30443.809071734696772 kilometers
b dot r               -7679.966257239961124
b dot t               29459.185818414796813
theta                   345.388301920592198 degrees
v-infinity              509.089956780863986 meters/second
r-periapsis           16925.351479275006568 kilometers
decl-asy                  1.681732863833258 degrees
rasc-asy                285.444089399385518 degrees
```

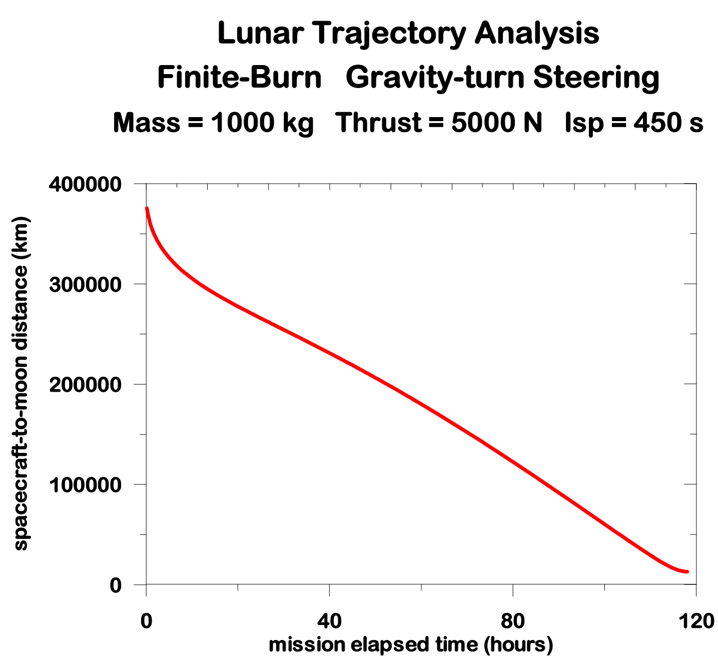
flight path angle   9.342542810087221E-005 degrees

lunar coast time   118.040670845657587 hours

The `ca_sc2moon` software will create a comma-separated-variable (csv) output file. This file contains the EME2000 position vectors of the spacecraft and moon. The following plot is a view of the trajectory and planetary orbits from the Earth's north pole looking down on the equatorial plane.



The following is a plot of the selenocentric distance of the spacecraft.



Technical Discussion

The closest approach encounter time can be computed by performing a one-dimensional minimization while numerically integrating the equations of motion. This process determines future close approach conditions between the spacecraft in its lunar transfer orbit and the Moon. This computational process is performed using Brent's minimization algorithm with the following objective function

$$f(t) = \Delta r(t) = \left|\mathbf{r}_m - \mathbf{r}_{sc}\right|$$

where $\mathbf{r}_p$ is the geocentric position vector of the Moon and $\mathbf{r}_{sc}$ is the geocentric position vector of the spacecraft at any simulation time *t*. This close approach algorithm uses the launch date, thrust and spacecraft mass provided by the user.

The software allows the user to select the type of steering method used to generate an initial guess for the trajectory. The two options available are *tangential* or *gravity turn* steering. For gravity turn steering the thrust vector is aligned with the instantaneous velocity vector. For tangential thrusting the unit thrust vector in the modified equinoctial frame at all times is simply $\mathbf{u}_T = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$.

For the gravity-turn steering option, the software creates the unit thrust vector in the Earth-centered-inertial (ECI) coordinate system and integrates the equations of motion in the modified equinoctial or radial frame system. The relationship between a unit thrust vector in the ECI coordinate system $\hat{\mathbf{u}}_{T_{ECI}}$ and the corresponding unit thrust vector in the modified equinoctial system $\hat{\mathbf{u}}_{T_{MEE}}$ is given by

$$\hat{\mathbf{u}}_{T_{ECI}} = \begin{bmatrix} \hat{\mathbf{i}}_r & \hat{\mathbf{i}}_t & \hat{\mathbf{i}}_n \end{bmatrix} \hat{\mathbf{u}}_{T_{MEE}}$$

where

$$\hat{\mathbf{i}}_r = \frac{\mathbf{r}}{|\mathbf{r}|} \qquad \hat{\mathbf{i}}_n = \frac{\mathbf{r}\times\mathbf{v}}{|\mathbf{r}\times\mathbf{v}|} \qquad \hat{\mathbf{i}}_t = \hat{\mathbf{i}}_n \times \hat{\mathbf{i}}_r = \frac{(\mathbf{r}\times\mathbf{v})\times\mathbf{r}}{|\mathbf{r}\times\mathbf{v}||\mathbf{r}|}$$

This relationship can also be expressed as

$$\hat{\mathbf{u}}_{T_{ECI}} = [Q]\hat{\mathbf{u}}_{T_{MEE}} = \begin{bmatrix} \hat{\mathbf{r}}_x & (\hat{\mathbf{h}}\times\hat{\mathbf{r}})_x & \hat{\mathbf{h}}_x \\ \hat{\mathbf{r}}_y & (\hat{\mathbf{h}}\times\hat{\mathbf{r}})_y & \hat{\mathbf{h}}_y \\ \hat{\mathbf{r}}_z & (\hat{\mathbf{h}}\times\hat{\mathbf{r}})_z & \hat{\mathbf{h}}_z \end{bmatrix} \hat{\mathbf{u}}_{T_{MEE}}$$

Finally, the transformation of the unit thrust vector in the ECI system to the modified equinoctial coordinate system is given by

$$\hat{\mathbf{u}}_{T_{MEE}} = [Q]^T \hat{\mathbf{u}}_{T_{ECI}}$$

For *tangential thrusting*, the unit thrust vector in the modified equinoctial frame at all times is $\mathbf{u}_T = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$.

Modified equinoctial orbital elements

The modified equinoctial orbital elements are a set of orbital elements that are useful for trajectory analysis and optimization. They are valid for circular, elliptic, and hyperbolic orbits. These equations exhibit no singularity for zero eccentricity and orbital inclinations equal to 0 and 90 degrees. However, two components of the orbital element set are singular for an orbital inclination of 180 degrees.

The relationship between direct modified equinoctial and classical orbital elements is defined by the following definitions

$$p = a\left(1 - e^2\right)$$

$$f = e\cos\left(\omega + \Omega\right)$$

$$g = e\sin\left(\omega + \Omega\right)$$

$$h = \tan\left(i/2\right)\cos\Omega$$

$$k = \tan\left(i/2\right)\sin\Omega$$

$$L = \Omega + \omega + \theta$$

where

$p =$ semiparameter
$a =$ semimajor axis
$e =$ orbital eccentricity
$i =$ orbital inclination
$\omega =$ argument of periapsis
$\Omega =$ right ascension of the ascending node
$\theta =$ true anomaly
$L =$ true longitude

The relationship between classical and modified equinoctial orbital elements is summarized as follows:

semimajor axis

$$a = \frac{p}{1 - f^2 - g^2}$$

orbital eccentricity

$$e = \sqrt{f^2 + g^2}$$

orbital inclination

$$i = 2\tan^{-1}\left(\sqrt{h^2 + k^2}\right)$$

argument of periapsis

$$\omega = \tan^{-1}\left(g/f\right) - \tan^{-1}\left(k/h\right)$$

right ascension of the ascending node

$$\Omega = \tan^{-1}\left(k/h\right)$$

true anomaly

$$\theta = L - \left(\Omega + \omega\right) = L - \tan^{-1}\left(g/f\right)$$

The mathematical relationships between an inertial state vector and the corresponding modified equinoctial elements are summarized as follows:

position vector

$$\mathbf{r} = \begin{bmatrix} \dfrac{r}{s^2}\left(\cos L + \alpha^2 \cos L + 2hk \sin L\right) \\ \dfrac{r}{s^2}\left(\sin L - \alpha^2 \sin L + 2hk \cos L\right) \\ \dfrac{2r}{s^2}\left(h \sin L - k \cos L\right) \end{bmatrix}$$

velocity vector

$$\mathbf{v} = \begin{bmatrix} -\dfrac{1}{s^2}\sqrt{\dfrac{\mu}{p}}\left(\sin L + \alpha^2 \sin L - 2hk \cos L + g - 2fhk + \alpha^2 g\right) \\ -\dfrac{1}{s^2}\sqrt{\dfrac{\mu}{p}}\left(-\cos L + \alpha^2 \cos L + 2hk \sin L - f + 2ghk + \alpha^2 f\right) \\ \dfrac{2}{s^2}\sqrt{\dfrac{\mu}{p}}\left(h \cos L + k \sin L + fh + gk\right) \end{bmatrix}$$

where

$$\alpha^2 = h^2 - k^2$$

$$s^2 = 1 + h^2 + k^2$$

$$r = \frac{p}{w}$$

$$w = 1 + f \cos L + g \sin L$$

The system of first-order modified equinoctial equations of orbital motion are given by

$$\dot{p} = \frac{dp}{dt} = \frac{2p}{w}\sqrt{\frac{p}{\mu}}\,\Delta_t$$

$$\dot{f} = \frac{df}{dt} = \sqrt{\frac{p}{\mu}}\left[\Delta_r \sin L + \left[\left(w+1\right)\cos L + f\right]\frac{\Delta_t}{w} - \left(h\sin L - k\cos L\right)\frac{g\Delta_n}{w}\right]$$

$$\dot{g} = \frac{dg}{dt} = \sqrt{\frac{p}{\mu}}\left[-\Delta_r \cos L + \left[\left(w+1\right)\sin L + g\right]\frac{\Delta_t}{w} + \left(h\sin L - k\cos L\right)\frac{g\Delta_n}{w}\right]$$

$$\dot{h} = \frac{dh}{dt} = \sqrt{\frac{p}{\mu}}\,\frac{s^2\Delta_n}{2w}\cos L$$

$$\dot{k} = \frac{dk}{dt} = \sqrt{\frac{p}{\mu}}\,\frac{s^2\Delta_n}{2w}\sin L$$

$$\dot{L} = \frac{dL}{dt} = \sqrt{\mu p}\left(\frac{w}{p}\right)^2 + \frac{1}{w}\sqrt{\frac{p}{\mu}}\left(h\sin L - k\cos L\right)\Delta_n$$

where $\Delta_r, \Delta_t, \Delta_n$ are *non-two-body* perturbations in the radial, tangential and normal directions, respectively. For a lunar spacecraft, the radial direction is along the geocentric radius vector of the spacecraft measured positive in a direction away from the gravitational center, the tangential direction is perpendicular to this radius vector measured positive in the direction of orbital motion, and the normal direction is positive along the angular momentum vector of the spacecraft's orbit.

The equations of orbital motion can also be expressed in vector form as follows:

$$\dot{\mathbf{y}} = \frac{d\mathbf{y}}{dt} = \mathbf{A}\left(\mathbf{y}\right)\mathbf{P} + \mathbf{b}$$

where

$$\mathbf{b} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \sqrt{\mu p}\left(\frac{w}{p}\right)^2 \end{bmatrix}^T$$

and

$$\mathbf{A} = \begin{Bmatrix} 0 & \dfrac{2p}{w}\sqrt{\dfrac{p}{\mu}} & 0 \\ \sqrt{\dfrac{p}{\mu}}\sin L & \sqrt{\dfrac{p}{\mu}}\dfrac{1}{w}\left[(w+1)\cos L + f\right] & -\sqrt{\dfrac{p}{\mu}}\dfrac{g}{w}\left[h\sin L - k\cos L\right] \\ -\sqrt{\dfrac{p}{\mu}}\cos L & \sqrt{\dfrac{p}{\mu}}\left[(w+1)\sin L + g\right] & \sqrt{\dfrac{p}{\mu}}\dfrac{f}{w}\left[h\sin L - k\cos L\right] \\ 0 & 0 & \sqrt{\dfrac{p}{\mu}}\dfrac{s^2\cos L}{2w} \\ 0 & 0 & \sqrt{\dfrac{p}{\mu}}\dfrac{s^2\sin L}{2w} \\ 0 & 0 & \sqrt{\dfrac{p}{\mu}}\left[h\sin L - k\cos L\right] \end{Bmatrix}$$

The <u>total</u> non-two-body acceleration vector is given by

$$\mathbf{P} = \Delta_r \hat{\mathbf{i}}_r + \Delta_t \hat{\mathbf{i}}_t + \Delta_n \hat{\mathbf{i}}_n$$

where $\hat{\mathbf{i}}_r, \hat{\mathbf{i}}_t$ and $\hat{\mathbf{i}}_n$ are unit vectors in the radial, tangential and normal directions. These unit vectors can be computed from the inertial position vector **r** and velocity vector **v** according to

$$\hat{\mathbf{i}}_r = \frac{\mathbf{r}}{|\mathbf{r}|}$$

$$\hat{\mathbf{i}}_n = \frac{\mathbf{r}\times\mathbf{v}}{|\mathbf{r}\times\mathbf{v}|}$$

$$\hat{\mathbf{i}}_t = \hat{\mathbf{i}}_n \times \hat{\mathbf{i}}_r = \frac{(\mathbf{r}\times\mathbf{v})\times\mathbf{r}}{|\mathbf{r}\times\mathbf{v}||\mathbf{r}|}$$

For *unperturbed* two-body motion, $\mathbf{P} = 0$ and the first five equations of motion are simply $\dot{p} = \dot{f} = \dot{g} = \dot{h} = \dot{k} = 0$. Therefore, for two-body motion these modified equinoctial orbital elements are constant. The true longitude is often called the *fast variable* of this orbital element set.

Non-spherical Earth Gravity

The non-spherical gravitational acceleration vector can be expressed as

$$\mathbf{g} = g_N \hat{\mathbf{i}}_N - g_r \hat{\mathbf{i}}_r$$

where

$$\hat{\mathbf{i}}_N = \frac{\hat{\mathbf{e}}_N - \left(\hat{\mathbf{e}}_N^T \hat{\mathbf{i}}_r\right)\hat{\mathbf{i}}_r}{\left\| \hat{\mathbf{e}}_N - \left(\hat{\mathbf{e}}_N^T \hat{\mathbf{i}}_r\right)\hat{\mathbf{i}}_r \right\|}$$

and

$$\hat{\mathbf{e}}_N = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

In these equations, the north direction component is indicated by subscript *N* and the radial direction component is subscript *r*.

The contributions due to the *zonal* gravity effects of J_2, J_3, J_4 are as follows:

$$g_N = -\frac{\mu \cos\phi}{r^2} \sum_{k=2}^{4} \left(\frac{R_e}{r}\right)^k P_k' J_k$$

$$g_r = -\frac{\mu}{r^2} \sum_{k=2}^{4} \left(k+1\right) \left(\frac{R_e}{r}\right)^k P_k J_k$$

where

$\mu =$ gravitational constant
$r =$ geocentric distance of the spacecraft
$R_e =$ equatorial radius of the Earth
$\phi =$ geocentric latitude
$J_k =$ zonal gravity coefficient
$P_k =$ k$^{\text{th}}$ order Legendre polynomial

For a zonal-only Earth gravity model, the east component is identically zero.

Finally, the zonal gravity perturbation contribution is

$$\mathbf{a}_g = \mathbf{Q}^T \mathbf{g}$$

where $\mathbf{Q} = \begin{bmatrix} \hat{\mathbf{i}}_r & \hat{\mathbf{i}}_t & \hat{\mathbf{i}}_n \end{bmatrix}$.

For J_2 effects only, the three components are as follows:

$$\Delta_{J_{2_r}} = -\frac{3\mu J_2 R_e^2}{2r^4} \left[1 - \frac{12\left(h\sin L - k\cos L\right)^2}{\left(1+h^2+k^2\right)^2} \right]$$

$$\Delta_{J_{2_t}} = -\frac{12\mu J_2 R_e^2}{r^4} \left[\frac{\left(h\sin L - k\cos L\right)\left(h\cos L + k\sin L\right)}{\left(1+h^2+k^2\right)^2} \right]$$

$$\Delta_{J_{2_n}} = -\frac{6\mu J_2 R_e^2}{r^4}\left[\frac{\left(1-h^2-k^2\right)\left(h\sin L - k\cos L\right)}{\left(1+h^2+k^2\right)^2}\right]$$

These are the equations modeled in this computer program,

Propulsive thrust

The acceleration due to propulsive thrust can be expressed as

$$\mathbf{a}_T = \frac{T}{m}\hat{\mathbf{u}}$$

where T is the thrust, m is the spacecraft mass and $\hat{\mathbf{u}} = \begin{bmatrix} u_r & u_t & u_n \end{bmatrix}$ is the unit pointing thrust vector expressed in the spacecraft-centered radial-tangential-normal coordinate system. The components of the unit thrust vector can also be defined in terms of the in-plane pitch angle θ and the out-of-plane yaw angle ψ as follows:

$$u_r = \sin\theta$$
$$u_t = \cos\theta\cos\psi$$
$$u_n = \cos\theta\sin\psi$$

Finally, the pitch and yaw angles can be determined from the components of the unit thrust vector according to

$$\theta = \sin^{-1}\left(u_r\right)$$

$$\psi = \tan^{-1}\left(u_n, u_t\right)$$

The pitch angle is positive above the "local horizontal" and the yaw angle is positive in the direction of the angular momentum vector.

During the TLI propulsive maneuver, the system of equations of motion also include the thrust acceleration and the propellant flow rate given by

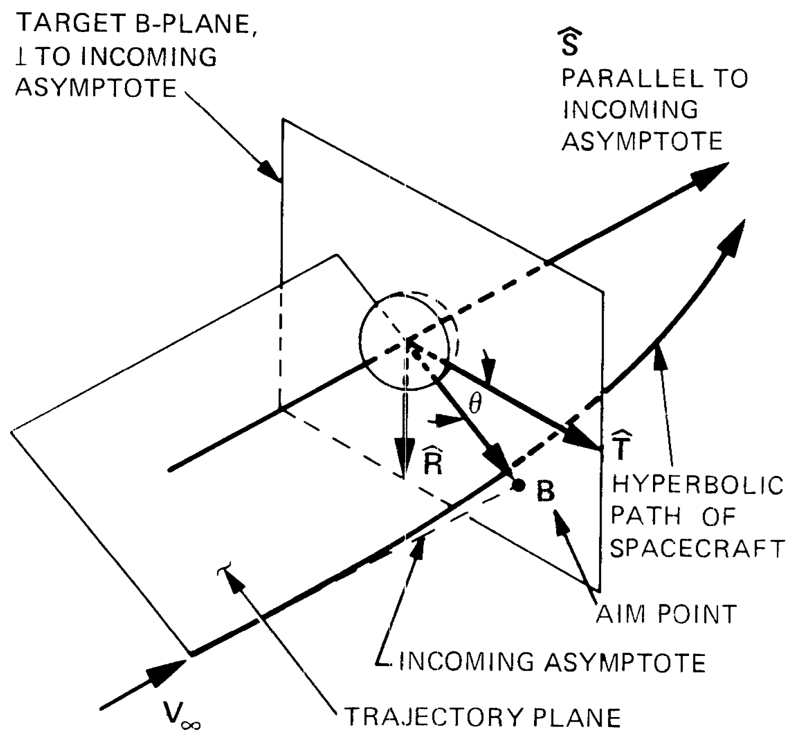
$$\dot{m} = \frac{dm}{dt} = \frac{F}{g\, I_{sp}}$$

where F is the thrust, g is the acceleration of gravity, and I_{sp} is the specific impulse. The thrust acceleration is used to predict the accumulated delta-v and the propellant flow rate is used to predict the propellant consumed during the TLI propulsive maneuver.

The B-plane

The derivation of B-plane coordinates is described in the classic JPL reports, "A Method of Describing Miss Distances for Lunar and Interplanetary Trajectories" and "Some Orbital Elements

Useful in Space Trajectory Calculations", both by William Kizner. The following diagram illustrates the fundamental geometry of the B-plane coordinate system.



The arrival asymptote unit vector $\hat{\mathbf{S}}$ is given by

$$\hat{\mathbf{S}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix}$$

where δ_∞ and α_∞ are the declination and right ascension of the asymptote of the incoming hyperbola.

The following computational steps summarize the calculation of the *predicted* B-plane vector from a moon-centered position vector **r** and velocity vector **v** at closest approach.

angular momentum vector

$$\mathbf{h} = \mathbf{r} \times \mathbf{v}$$

$$\hat{\mathbf{h}} = \frac{\mathbf{h}}{|\mathbf{h}|}$$

radius rate

$$\dot{r} = \frac{\mathbf{r} \bullet \mathbf{v}}{|\mathbf{r}|}$$

semiparameter

$$p = \frac{h^2}{\mu}$$

semimajor axis

$$a = \frac{r}{\left(2 - \frac{r\,v^2}{\mu}\right)}$$

orbital eccentricity

$$e = \sqrt{1 - p/a}$$

true anomaly

$$\cos\theta = \frac{p - r}{e\,r}$$

$$\sin\theta = \frac{\dot{r}\,h}{e\mu}$$

B-plane magnitude

$$B = \sqrt{p\,|a|}$$

fundamental vectors

$$\hat{\mathbf{z}} = \frac{r\,\mathbf{v} - \dot{r}\,\mathbf{r}}{h}$$

$$\hat{\mathbf{p}} = \cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\mathbf{z}}$$

$$\hat{\mathbf{q}} = \sin\theta\,\hat{\mathbf{r}} + \cos\theta\,\hat{\mathbf{z}}$$

S vector

$$\mathbf{S} = -\frac{a}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{b}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

B vector

$$\mathbf{B} = \frac{b^2}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{p}} + \frac{ab}{\sqrt{a^2 + b^2}}\,\hat{\mathbf{q}}$$

T vector

$$\mathbf{T} = \frac{\left(S_y^2, -S_x^2, 0\right)^T}{\sqrt{S_x^2 + S_y^2}}$$

R vector

$$\mathbf{R} = \mathbf{S} \times \mathbf{T} = \left(-S_z T_y,\ S_z T_x,\ S_x T_y - S_y T_x\right)^T$$

In this computer program the B-plane coordinates are based on the selenocentric flight conditions at closest approach.

If the encounter trajectory is not hyperbolic, these coordinates can not be computed and the software will display zeroes for this information.

Geocentric-to-selenocentric coordinate transformation

This section describes the transformation of coordinates between the Earth mean equator and equinox of J2000 (EME2000) and lunar mean equator and IAU node of epoch coordinate systems. This transformation is used to compute the B-plane coordinates at lunar encounter.

A unit vector in the direction of the pole of the moon can be determined from

$$\hat{\mathbf{p}}_{Moon} = \begin{bmatrix} \cos\alpha_p \cos\delta_p \\ \sin\alpha_p \cos\delta_p \\ \sin\delta_p \end{bmatrix}$$

The right ascension and declination of the lunar pole in the EME2000 coordinate system are given by the following expressions

$$\begin{aligned}\alpha_P &= 269.9949 + 0.0031T - 3.8787\sin E1 - 0.1204\sin E2 \\ &\quad + 0.0700\sin E3 - 0.0172\sin E4 + 0.0072\sin E6 \\ &\quad - 0.0052\sin E10 + 0.0043\sin E13\end{aligned}$$

$$\begin{aligned}\delta_P &= 66.5392 + 0.0130T + 1.5419\cos E1 + 0.0239\cos E2 \\ &\quad - 0.0278\cos E3 + 0.0068\cos E4 - 0.0029\cos E6 \\ &\quad + 0.0009\cos E7 + 0.0008\cos E10 - 0.0009\cos E13\end{aligned}$$

where T is the time in Julian centuries given by $T = \left(JD - 2451545.0\right)/36525$ and JD is the TDB Julian Date.

The trigonometric arguments, in degrees, for these equations are

$$E1 = 125.045 - 0.0529921d$$
$$E2 = 250.089 - 0.1059842d$$
$$E3 = 260.008 + 13.0120009d$$
$$E4 = 176.625 + 13.3407154d$$
$$E6 = 311.589 + 26.4057084d$$
$$E7 = 134.963 + 13.0649930d$$
$$E10 = 15.134 - 0.1589763d$$
$$E13 = 25.053 + 12.9590088d$$

where $d = JD - 2451545$ is the number of days since January 1.5, 2000. These equations are given in "Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 2000", *Celestial Mechanics and Dynamical Astronomy*, **82**: 83-110, 2002.

The unit vector in the x-axis direction of this selenocentric coordinate system is given by

$$\hat{\mathbf{x}} = \hat{\mathbf{z}} \times \hat{\mathbf{p}}_{Moon}$$

where $\hat{\mathbf{z}} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$.

The unit vector in the y-axis direction can be determined using

$$\hat{\mathbf{y}} = \hat{\mathbf{p}}_{Moon} \times \hat{\mathbf{x}}$$

Finally, the components of the matrix that transforms coordinates from the EME2000 system to the moon-centered (selenocentric) mean equator and IAU node of epoch system are as follows:

$$\mathbf{M} = \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{p}}_{Moon} \end{bmatrix}$$

Lunar and solar ephemeris

The software models the coordinates of the sun and moon using the DE405 algorithm from JPL. This numerical method and binary data file provide position and velocity vectors of the sun and moon relative to the Earth mean equator and equinox of J2000. The binary data file is named `de405.bin` and must be located in the same directory as the `ca_sc2moon` program.

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