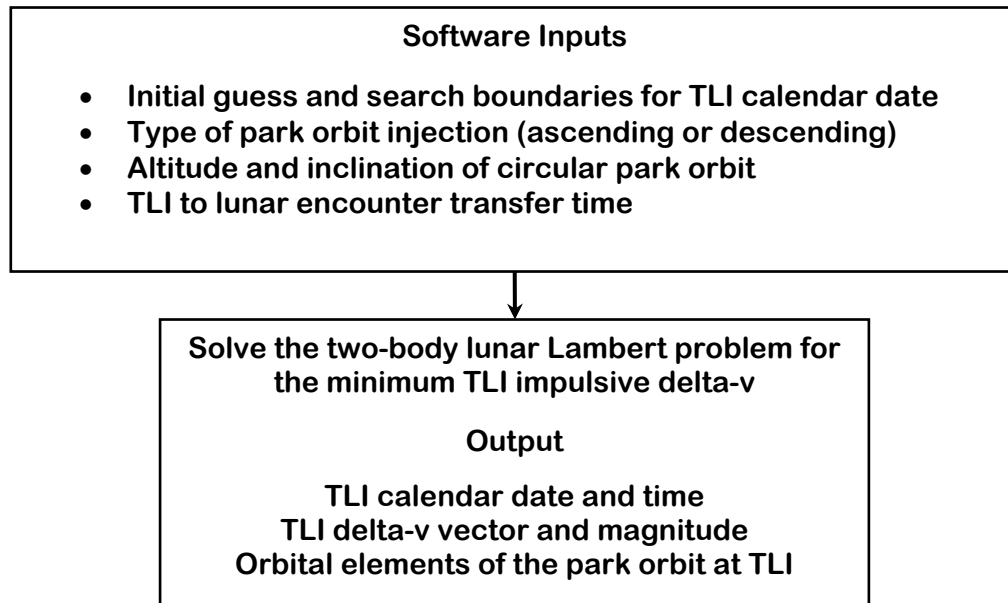


Program lguess

A Computer Program for Preliminary Trans-Lunar Mission Analysis

This document is the user's guide for a Fortran computer program called `lguess` that can be used to perform a preliminary performance assessment of trans-lunar trajectories. The software can also be used to create an initial guess for other lunar mission analysis computer programs. This algorithm assumes that trans-lunar injection (TLI) occurs *impulsively* from a circular Earth park orbit. The software solves for the minimum TLI delta-v using a *two-body* Lambert solution for the transfer trajectory from the Earth park orbit to the center of the moon.

The program inputs, major computational step and outputs for this implementation are as follows:



This computer program uses the BOBYQA algorithm written by M.J.D. Powell to solve this classic trajectory optimization problem. The lunar coordinates required by the software are computed using the JPL DE421 ephemeris. The source code for the `lguess` computer program was created using the Intel Visual Fortran compiler, version 11.1.

Input data file

The `lguess` computer program is “data-driven” by a simple text file created by the user. This section describes a typical input data file. In the following discussion the actual input file contents are in *courier* font and all explanations are in *times italic* font.

Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input.

The first four lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with four and only four initial text lines.

```
*****
* input file for program lguess
* lguess1.in - December 7, 2010
*****
```

The software allows the user to specify an initial guess for the TLI calendar date and lower and upper bounds on the actual date found during both the two-body TLI delta-v optimization process. For any guess for the TLI time t_{TLI} and user-defined lower and upper bounds Δt_l and Δt_u , the actual TLI time t is constrained as follows:

$$t_{TLI} - \Delta t_l \leq t \leq t_{TLI} + \Delta t_u$$

The first five inputs define the user-defined TLI calendar date and the lower and upper bounds, respectively. Be sure to include all four digits of the calendar year.

IMPORTANT: *The TLI calendar date is a control variable in the NLP formulation and must always have a lower and upper bound.*

```
TLI calendar date (month, day, year)
9,15,2008

lower bound for TLI calendar date search (hours)
0.0

upper bound for TLI calendar date search (hours)
+24.0
```

The next input is the user-defined number for the TLI-to-lunar encounter transfer time, in hours.

```
TLI-to-lunar encounter transfer time (hours)
110.0
```

The next two numbers define the fixed values for the park orbit altitude and orbital inclination.

```
*****
circular park orbit characteristics
*****

altitude (kilometers)
185.32

orbital inclination (degrees)
28.5
```

This next integer input defines the type of TLI maneuver to perform. The software uses this indicator to compute the park orbit RAAN. Please see the Technical Discussion later in this document for information about how the park orbit RAAN is determined.

```
type of TLI maneuver
(1 = ascending, 2 = descending)
2
```

Running the software

An input file created by the user can be run from the command line or a simple batch file with a statement similar to the following:

```
lguess lguess1.in
```

If the software is executed without an input file on the command line, the lguess computer program will display the following prompt:

```
please input the name of the simulation definition file
```

At this point the user should input the name of a valid input file, including the filename extension.

Program example

The following is the solution created by the computer program for this example. The results are presented in the Earth mean equator and equinox of J2000 coordinate system (EME2000). The trajectory characteristics are given before and after the impulsive TLI maneuver. The geocentric orbital elements and state vector of the spacecraft and the moon at encounter are also displayed.

```

program lguess

minimum TLI delta-v - two-body Lambert solution
-----

DE421 ephemeris

descending TLI maneuver

transfer time          110.000000000000          hours

time and conditions prior to TLI
(geocentric EME2000 coordinates)
-----

calendar date          September 15, 2008

TDB time               13:28:05.752

TDB Julian date        2454725.06117768

      sma (km)          eccentricity          inclination (deg)          argper (deg)
0.656345630000D+04    0.207703680281D-15    0.285000000000D+02    0.000000000000D+00

      raan (deg)        true anomaly (deg)          arglat (deg)          period (hrs)
0.357104409591D+03    0.242909717395D+03    0.242909717395D+03    0.146996629514D+01

      rx (km)           ry (km)           rz (km)           rmag (km)
-.324455523486D+04    -.497771531863D+04    -.278821988671D+04    0.656345630000D+04

      vx (kps)          vy (kps)          vz (kps)          vmag (kps)
0.677158909898D+01    -.346530416667D+01    -.169337334111D+01    0.779296254099D+01

impulsive TLI delta-v vector and magnitude
(geocentric EME2000 coordinates)
-----

delta-vx              2720.83248728905          meters/second
delta-vy              -1392.36667258734          meters/second
delta-vz              -680.401233802791          meters/second

deltav                3131.22343721745          meters/second

```

time and conditions after TLI
(geocentric EME2000 coordinates)

calendar date September 15, 2008
TDB time 13:28:05.752
TDB Julian date 2454725.06117768

sma (km)	eccentricity	inclination (deg)	argper (deg)
0.187780714768D+06	0.965047229115D+00	0.285000000000D+02	0.242909681798D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (hrs)
0.357104409591D+03	0.355961509164D-04	0.242909717395D+03	0.224949463452D+03
rx (km)	ry (km)	rz (km)	rmag (km)
-.324455523486D+04	-.497771531863D+04	-.278821988671D+04	0.656345630000D+04
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.949242158627D+01	-.485767083926D+01	-.237377457491D+01	0.109241859782D+02
energy	-2.12269104413893	(km/sec)**2	

time and conditions at lunar encounter
(geocentric EME2000 coordinates)

calendar date September 20, 2008
TDB time 03:28:05.752
TDB Julian date 2454729.64451101

sma (km)	eccentricity	inclination (deg)	argper (deg)
0.187780714768D+06	0.965047229115D+00	0.285000000000D+02	0.242909681798D+03
raan (deg)	true anomaly (deg)	arglat (deg)	period (hrs)
0.357104409591D+03	0.179731146959D+03	0.626408287569D+02	0.224949463452D+03
rx (km)	ry (km)	rz (km)	rmag (km)
0.183855964261D+06	0.278989583980D+06	0.156328383523D+06	0.368885845565D+06
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.155895536718D+00	0.106160944175D+00	0.532911953610D-01	0.195993663009D+00

coordinates of the moon at encounter
(geocentric EME2000 coordinates)

calendar date September 20, 2008
TDB time 03:28:05.752
TDB Julian date 2454729.64451101

sma (km)	eccentricity	inclination (deg)	argper (deg)
0.386687523939D+06	0.460363401223D-01	0.274675327593D+02	0.667488782745D+02
raan (deg)	true anomaly (deg)	arglat (deg)	period (hrs)
0.352453157500D+03	0.440281280327D-04	0.667489223026D+02	0.664735266590D+03

rx (km)	ry (km)	rz (km)	rmag (km)
0.183855964261D+06	0.278989583980D+06	0.156328383523D+06	0.368885845565D+06
vx (kps)	vy (kps)	vz (kps)	vmag (kps)
-.919440261341D+00	0.497446347203D+00	0.193581222756D+00	0.106315424672D+01
declination	25.0737984493215	degrees	
right ascension	56.6148560499457	degrees	

The final two items of the program output are the geocentric right ascension and declination of the moon at encounter, in the EME2000 system.

The specific orbital energy displayed by the software is calculated using the expression $E = v^2 - 2\mu/r$. TDB is the barycentric dynamic time which is the fundamental time argument for the JPL lunar ephemeris used by the code.

A brief guide to the other items displayed by the software is as follows;

sma (km) = semimajor axis in kilometers

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of perigee in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (min) = orbital period in minutes.

rx (km) = x-component of the eci position vector in kilometers

ry (km) = y-component of the eci position vector in kilometers

rz (km) = z-component of the eci position vector in kilometers

rmag (km) = geocentric position magnitude in kilometers

vx (kps) = x-component of the eci velocity vector in kilometers/second

vy (kps) = y-component of the eci velocity vector in kilometers/second

vz (kps) = z-component of the eci velocity vector in kilometers/second

vmag (kps) = velocity vector scalar magnitude in kilometers/seconds

delta-vx = x-component of the TLI impulsive velocity vector in meters/second

delta-vy = y-component of the TLI impulsive velocity vector in meters/second

delta-vz = z-component of the TLI impulsive velocity vector in meters/second

deltav = scalar magnitude of the TLI maneuver in meters/seconds

Technical Discussion

This section describes several of the algorithms implemented in the `lguess` computer program.

Nonlinear programming problem

A trajectory optimization problem can be described by a system of *dynamic variables*

$$\mathbf{z} = \begin{bmatrix} \mathbf{y}(t) \\ \mathbf{u}(t) \end{bmatrix}$$

consisting of the *state variables* \mathbf{y} and the *control variables* \mathbf{u} for any time t . In this discussion vectors are denoted in bold.

The system dynamics are defined by a vector system of ordinary differential equations called the *state equations* that can be represented as follows:

$$\dot{\mathbf{y}} = \frac{d\mathbf{y}}{dt} = \mathbf{f}[\mathbf{y}(t), \mathbf{u}(t), \mathbf{p}, t]$$

where \mathbf{p} is a vector of problem *parameters* that is not time dependent.

The initial dynamic variables at time t_0 are defined by $\boldsymbol{\psi}_0 \equiv \boldsymbol{\psi}[\mathbf{y}(t_0), \mathbf{u}(t_0), t_0]$ and the terminal conditions at the final time t_f are defined by $\boldsymbol{\psi}_f \equiv \boldsymbol{\psi}[\mathbf{y}(t_f), \mathbf{u}(t_f), t_f]$. These conditions are called the *boundary values* of the trajectory problem. The problem may also be subject to *path constraints* of the form $\mathbf{g}[\mathbf{y}(t), \mathbf{u}(t), t] = 0$.

The basic nonlinear programming problem (NLP) is to determine the control vector history and problem parameters that minimize the scalar performance index or objective function given by

$$J = \phi[\mathbf{y}(t_0), t_0, \mathbf{y}(t_f), t_f, \mathbf{p}]$$

while satisfying all the user-defined mission constraints.

During the two-body trajectory optimization, the control variables are the TLI calendar date and the true anomaly of the TLI maneuver. The objective function or performance index is the scalar magnitude of the TLI delta-v vector.

In addition to the bounds on the TLI calendar date mentioned earlier, the true anomaly during the two-body optimization is bounded according to

$$-180^\circ \leq \theta \leq +180^\circ$$

The final boundary conditions are the components of the moon's inertial position vector at encounter.

Solving the two body Lambert problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamics problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis a of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length c of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof(r_i + r_f, c, a)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}} (E - e \sin E)$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}} [E - E_0 - e(\sin E - \sin E_0)]$$

where E is the eccentric anomaly associated with radius r , E_0 is the eccentric anomaly at r_0 , and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sum and difference identities:

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t = \sqrt{\frac{a^3}{\mu}} \left\{ E - E_0 - 2 \sin \frac{E - E_0}{2} \left(e \cos \frac{E + E_0}{2} \right) \right\}$$

With the two substitutions given by

$$e \cos \frac{E + E_0}{2} = \cos \frac{\alpha + \beta}{2}$$

$$\sin \frac{E - E_0}{2} = \sin \frac{\alpha - \beta}{2}$$

the time equation becomes

$$t = \sqrt{\frac{a^3}{\mu}} \left\{ (\alpha - \beta) - 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2} \right\}$$

From the elliptic relationships given by

$$r = a(1 - e \cos E)$$

$$x = a(\cos E - e)$$

$$y = a \sin E \sqrt{1 - e^2}$$

and some more manipulation, we have the following equations:

$$\cos \alpha = \left(1 - \frac{r + r_0}{2a} \right) - \frac{c}{2a} = 1 - \frac{r + r_0 + c}{2a} = 1 - \frac{s}{a}$$

$$\sin \beta = \left(1 - \frac{r + r_0}{2a} \right) + \frac{c}{2a} = 1 - \frac{r + r_0 - c}{2a} = 1 - \frac{s - c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2} = 1 - \frac{r + r_0}{2}$$

$$\sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2} = \sin \frac{E - E_0}{2} \sqrt{1 - \left(e \cos \frac{E + E_0}{2} \right)^2}$$

$$\left(\sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2} \right)^2 = \left(\frac{x - x_0}{2a} \right)^2 + \left(\frac{y - y_0}{2a} \right)^2 = \left(\frac{c}{2a} \right)^2$$

With the use of the half angle formulas given by

$$\sin \frac{\alpha}{2} = \sqrt{\frac{s}{2a}} \quad \sin \frac{\beta}{2} = \sqrt{\frac{s - c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert's theorem

$$t = \sqrt{\frac{a^3}{\mu}} [(\alpha - \beta) - (\sin \alpha - \sin \beta)]$$

A discussion about the angles α and β can be found in “Geometrical Interpretation of the Angles α and β in Lambert’s Problem” by J. E. Prussing, *AIAA Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this computer program is based on the method described in “A Procedure for the Solution of Lambert’s Orbital Boundary-Value Problem” by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48**: 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

Park orbit RAAN

For a given TLI calendar date, there are two possible locations on the initial park orbit at which to perform the propulsive maneuver. One opportunity occurs during the ascending part of the park orbit and the other during the descending motion. The park orbit RAAN Ω_p at these two locations can be determined from spherical trigonometry relationships involving the park orbit inclination and the geocentric right ascension and declination of the moon at encounter. The equations implemented in this computer program are as follows:

ascending

$$\Omega_p = -180^\circ + \alpha_m + \sin^{-1} \left(\frac{\tan \delta_m}{\tan i_p} \right)$$

descending

$$\Omega_p = \alpha_m - \sin^{-1} \left(\frac{\tan \delta_m}{\tan i_p} \right)$$

where

α_m = right ascension of the moon at encounter

δ_m = declination of the moon at encounter

i_p = park orbit inclination

These opportunities are valid whenever $|\delta_m| \leq i_p$.

Additional information about these equations can be found in Appendix A of the journal reference, “Integrated Algorithm for Lunar Transfer Trajectories Using a Pseudostate Technique”, R. V. Ramanan, *AIAA Journal of Guidance, Control and Dynamics*, Vol. 25, No. 5, September-October 2002, pp. 946-952.

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