Program Lguess

An Initial Guess Generator for Lunar Mission Analysis

This document is the user's guide for a Fortran computer program called `lguess` that can be used to create an initial guess for the `fb_lsocs` and other lunar mission analysis computer programs. The software assumes that trans-lunar injection (TLI) occurs *impulsively* from a circular Earth park orbit. The software solves for the minimum TLI delta-v using a two-body Lambert solution for the transfer trajectory from the Earth park orbit to the center of the moon. This computer program also estimates the effects of the J_2 Earth gravity perturbation and the point-mass gravity of the sun on the two-body Lambert solution.

The program inputs, major computational step and outputs for this software are as follows:

Software Inputs

- **Initial guess and search boundaries for TLI calendar date**
- **Type of park orbit injection (ascending or descending)**
- **Altitude and inclination of circular park orbit**
- **TLI to lunar encounter transfer time**

↓

Solve the two-body lunar Lambert problem for the minimum TLI impulsive delta-v

Output

TLI calendar date and time
TLI delta-v vector and magnitude
Orbital elements of the park orbit at TLI

This computer program uses a multi-dimensional, constrained nonlinear programming (NLP) method to solve this classic trajectory optimization problem. The lunar coordinates required by the software are computed using the JPL DE421 ephemeris. The perturbed solution solves the two-point boundary value (TPBVP) trajectory problem using a multi-dimensional algorithm that solves a system of nonlinear equations (SNLE). The NLP and SNLE algorithms implemented in this software are part of the IMSL subroutine library (www.vni.com).

Input data file

The `lguess` computer program is "data-driven" by a simple text file created by the user. This section describes a typical input data file. In the following discussion the actual input file contents are in `courier` font and all explanations are in *times italic* font.

Each data item within an input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. However, you may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input.

The first four lines of any input file are reserved for user comments. These lines are ignored by the software. However the input file must begin with four and only four initial text lines.

```
*************************
* input file for lguess.f
* lg1.in – May 3, 2005
*************************
```

The software allows the user to specify an initial guess for the TLI calendar date and lower and upper bounds on the actual date found during both the two-body TLI delta-v optimization process. For any guess for the TLI time t_{TLI} *and user-defined lower and upper bounds* Δt_l *and* Δt_u*, the actual TLI time* t *is constrained as follows:*

$$t_{TLI} - \Delta t_l \le t \le t_{TLI} + \Delta t_u$$

The first five inputs define the initial guesses for the TLI calendar date and the lower and upper bounds, respectively. Be sure to include all four digits of the calendar year.

IMPORTANT: The TLI calendar date is a control variable in the NLP formulation and must always have a lower and upper bound. For a fixed TLI calendar date, input small values (e.g., plus and minus 1.0e-8) for the bounds.

```
initial guess for TLI calendar date (month, day, year)
10,12,2008

lower bound for TLI calendar date search (two-body optimization; hours)
-48.0

upper bound for TLI calendar date search (two-body optimization; hours)
+48.0
```

The next input is the user's initial guess for the TLI-to-lunar encounter transfer time, in hours.

```
initial guess for transfer time (hours)
96.0
```

The next two numbers define the fixed values for the park orbit altitude and orbital inclination.

```
***********************************
circular park orbit characteristics
***********************************

altitude (kilometers)
185.2

orbital inclination (degrees)
28.5
```

This next integer input defines the type of TLI maneuver to perform. The software uses this indicator to compute the park orbit RAAN. Please see the Technical Discussion later in this document for information about how RAAN is computed.

```
type of TLI maneuver
(1 = ascending, 2 = descending)
2
```

The next two inputs specify the type of perturbations included in the equations of motion for the perturbed solution.

```
******************
perturbed solution
******************

include earth j2 perturbation (1 = yes, 0 = no)
1

include solar point mass gravity perturbation (1 = yes, 0 = no)
1
```

Running the software

An input file created by the user can be run from the command line or a simple batch file with a statement similar to the following:

```
lguess lg1.in
```

If the software is executed without an input file on the command line, the `lguess` computer program will display the following file name prompt:

```
please input the name of the simulation definition file
```

At this point the user should input the name of a valid input file, including the filename extension.

Program example

The following is the solution created by the computer program for this example. The solution is presented in the Earth mean equator and equinox of J2000 coordinate system (EME2000). The trajectory characteristics are given before and after the impulsive TLI maneuver. The geocentric orbital elements and state vector of the spacecraft and the moon at encounter is also displayed.

```
--------------
program lguess
--------------

descending TLI maneuver

-----------------------------------------------
minimum TLI delta-v - two-body Lambert solution
-----------------------------------------------

transfer time            96.0000000000000000 hours


time and conditions prior to TLI
(geocentric - EME2000 coordinates)
----------------------------------

calendar date           October   13, 2008

TDB time                04:15:36.023

TDB Julian date         2454752.677500265184790
```

sma (km)	eccentricity	inclination (deg)	argper (deg)
0.656333630000D+04	0.187665932362D-15	0.285000000000D+02	0.000000000000D+00

raan (deg)	true anomaly (deg)	arglat (deg)	period (min)
0.357058844491D+03	0.240586964095D+03	0.240586964095D+03	0.881955589276D+02

rx (km)	ry (km)	rz (km)	rmag (km)
-.347682975455D+04	-.485248743389D+04	-.272807696897D+04	0.656333630000D+04

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.660700957748D+01	-.370727720543D+01	-.182616967591D+01	0.779303378153D+01

time and conditions after TLI
(geocentric - EME2000 coordinates)
----

calendar date October 13, 2008

TDB time 04:15:36.023

TDB Julian date 2454752.677500265184790

sma (km)	eccentricity	inclination (deg)	argper (deg)
0.187661569467D+06	0.965025677241D+00	0.285000000000D+02	0.240586706271D+03

raan (deg)	true anomaly (deg)	arglat (deg)	period (min)
0.357058844491D+03	0.257824034568D-03	0.240586964095D+03	0.134841242743D+05

rx (km)	ry (km)	rz (km)	rmag (km)
-.347682975455D+04	-.485248743389D+04	-.272807696897D+04	0.656333630000D+04

vx (kps)	vy (kps)	vz (kps)	vmag (kps)
0.926165184838D+01	-.519685581130D+01	-.255992324634D+01	0.109242259365D+02

energy -2.124038729041956 (km/sec)**2

TLI impulsive ECI delta-v vector and magnitude
(geocentric - EME2000 coordinates)
----

delta-vx 2654.642270896005812 meters/second
delta-vy -1489.578605865280451 meters/second
delta-vz -733.753570426599822 meters/second

deltav 3131.192155009532144 meters/second

time and conditions at lunar encounter
(geocentric - EME2000 coordinates)
----

calendar date October 17, 2008

TDB time 04:15:36.023

TDB Julian date 2454756.677500265184790

sma (km)	eccentricity	inclination (deg)	argper (deg)
0.187661569467D+06	0.965025677241D+00	0.285000000000D+02	0.240586706271D+03

raan (deg)	true anomaly (deg)	arglat (deg)	period (min)
0.357058844491D+03	0.178203875978D+03	0.587905822492D+02	0.134841242743D+05

```
     rx (km)              ry (km)              rz (km)              rmag (km)
 0.202306886003D+06   0.263431684726D+06   0.148479440070D+06   0.363827531621D+06

     vx (kps)             vy (kps)             vz (kps)             vmag (kps)
 -.702213444399D-01   0.220022688976D+00   0.117348899882D+00   0.259059424037D+00


lunar coordinates at encounter
(geocentric - EME2000 coordinates)
----------------------------------

calendar date           October   17, 2008

TDB time                04:15:36.023

TDB Julian date         2454756.677500265184790


     sma (km)             eccentricity         inclination (deg)      argper (deg)
 0.390018571783D+06   0.671631281224D-01   0.272659330574D+02   0.640243975654D+02

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.352323888145D+03   0.358952198589D+03   0.629765961547D+02   0.404005857410D+05

     rx (km)              ry (km)              rz (km)              rmag (km)
 0.202306885298D+06   0.263431686933D+06   0.148479441248D+06   0.363827533307D+06

     vx (kps)             vy (kps)             vz (kps)             vmag (kps)
 -.896948253996D+00   0.560539354198D+00   0.224557831720D+00   0.108127089933D+01
```

The following is the solution with the user-defined perturbations.

```
------------------
PERTURBED SOLUTION
------------------


time and conditions after TLI
(geocentric - EME2000 coordinates)
----------------------------------

calendar date           October   13, 2008

TDB time                04:15:36.023

TDB Julian date         2454752.677500265184790


     sma (km)             eccentricity         inclination (deg)      argper (deg)
 0.189770886716D+06   0.965414420284D+00   0.287777400147D+02   0.239719538578D+03

    raan (deg)         true anomaly (deg)       arglat (deg)          period (min)
 0.358068364656D+03   0.359981404176D+03   0.239700942753D+03   0.137121043807D+05

     rx (km)              ry (km)              rz (km)              rmag (km)
 -.347682975455D+04   -.485248743389D+04   -.272807696897D+04   0.656333630000D+04

     vx (kps)             vy (kps)             vz (kps)             vmag (kps)
 0.926566864797D+01   -.514512540369D+01   -.265280799209D+01   0.109253064560D+02

TLI impulsive ECI delta-v vector and magnitude
(geocentric - EME2000 coordinates)
----------------------------------
```

```
delta-vx              2658.659070489257374 meters/second
delta-vy             -1437.848198252748489 meters/second
delta-vz              -826.638316178115247 meters/second

deltav                3133.561296685811612 meters/second


time and conditions at lunar encounter
(geocentric - EME2000 coordinates)
----------------------------------

calendar date           October   17, 2008

TDB time                04:15:36.023

TDB Julian date         2454756.677500265184790

     sma (km)              eccentricity          inclination (deg)         argper (deg)
 0.187591287003D+06    0.966291896573D+00    0.287157710187D+02    0.239925620719D+03

    raan (deg)          true anomaly (deg)         arglat (deg)           period (min)
 0.357793292158D+03    0.178220174518D+03    0.581457952374D+02    0.134765499312D+05

     rx (km)                 ry (km)                 rz (km)                rmag (km)
 0.202306885298D+06    0.263431686933D+06    0.148479441248D+06    0.363827533307D+06

     vx (kps)                vy (kps)                vz (kps)               vmag (kps)
 -.662948063035D-01    0.218859261145D+00    0.118412882164D+00    0.257518908421D+00
```

The specific orbital energy displayed by the software is calculated using the expression $E = v^2 - 2\mu/r$. TDB stands for barycentric dynamic time which is the fundamental time argument for the JPL lunar ephemeris.

A guide to the other items displayed by the software is as follows;

sma (km) = semimajor axis in kilometers

eccentricity = orbital eccentricity (non-dimensional)

inclination (deg) = orbital inclination in degrees

argper (deg) = argument of perigee in degrees

raan (deg) = right ascension of the ascending node in degrees

true anomaly (deg) = true anomaly in degrees

arglat (deg) = argument of latitude in degrees. The argument of latitude is the sum of true anomaly and argument of perigee.

period (min) = orbital period in minutes.

rx (km) = x-component of the eci position vector in kilometers

ry (km) = y-component of the eci position vector in kilometers

rz (km) = z-component of the eci position vector in kilometers

rmag (km) = geocentric position magnitude in kilometers

vx (kps) = x-component of the eci velocity vector in kilometers/second

vy (kps) = y-component of the eci velocity vector in kilometers/second

vz (kps) = z-component of the eci velocity vector in kilometers/second

vmag (kps) = velocity vector scalar magnitude in kilometers/seconds

delta-vx = x-component of the TLI impulsive velocity vector in meters/second

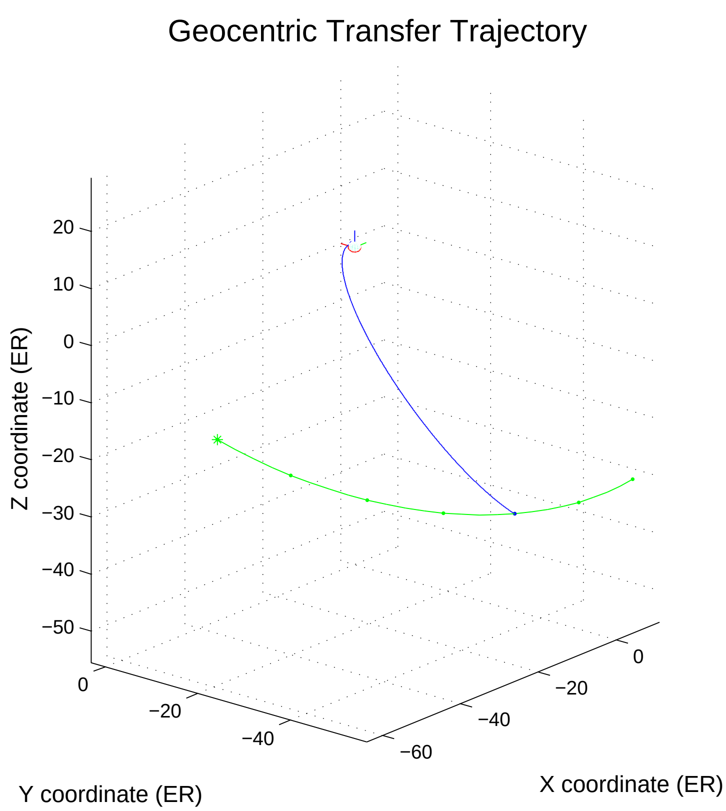
delta-vy = y-component of the TLI impulsive velocity vector in meters/second

delta-vz = z-component of the TLI impulsive velocity vector in meters/second

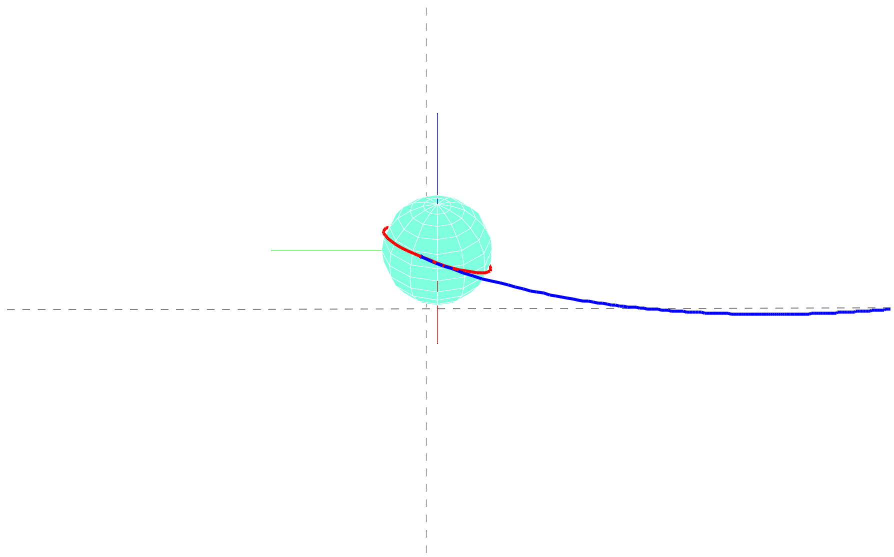
deltav = scalar magnitude of the TLI maneuver in meters/seconds

Trajectory graphics

The following is a plot of the geocentric transfer trajectory for this example. Please note that the coordinates are displayed in the units of Earth radii (ER). The asterisk symbol is the position of the moon at the moment of trans-lunar injection and the moon's orbit is marked with a small green dot symbol at 24 hour intervals. The park orbit trace is red and the transfer trajectory is blue.

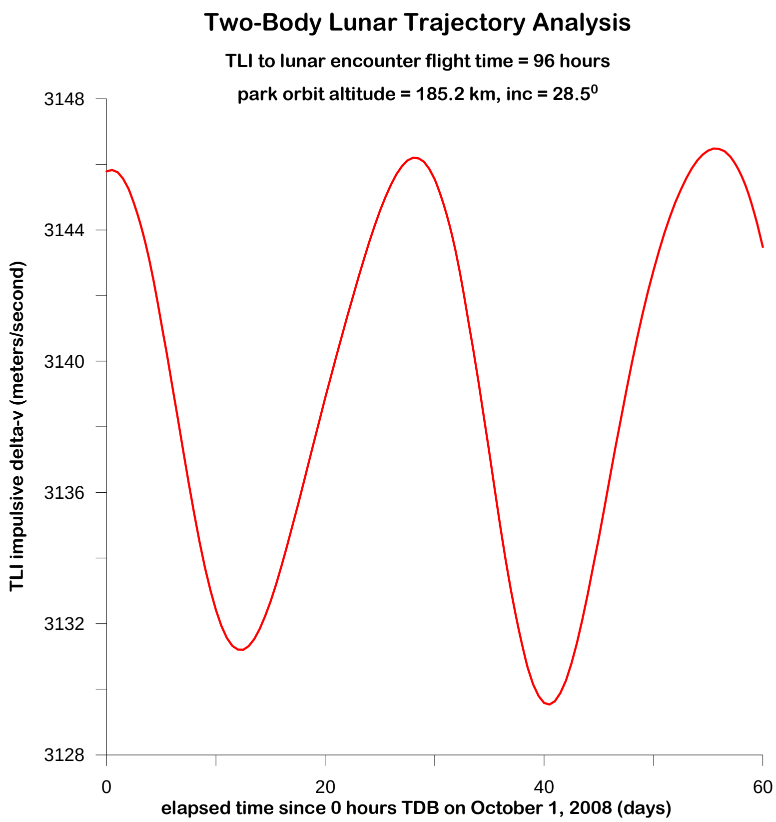


This next plot is a "zoomed" display of the first plot closer to the Earth. The initial park orbit is displayed in red, the transfer trajectory is blue, and an inertial, Earth-centered coordinate system is on the plot. The x-axis of this system is red, the y-axis green and the z-axis blue. The location on the park orbit at which TLI occurs is marked with a small dot symbol.

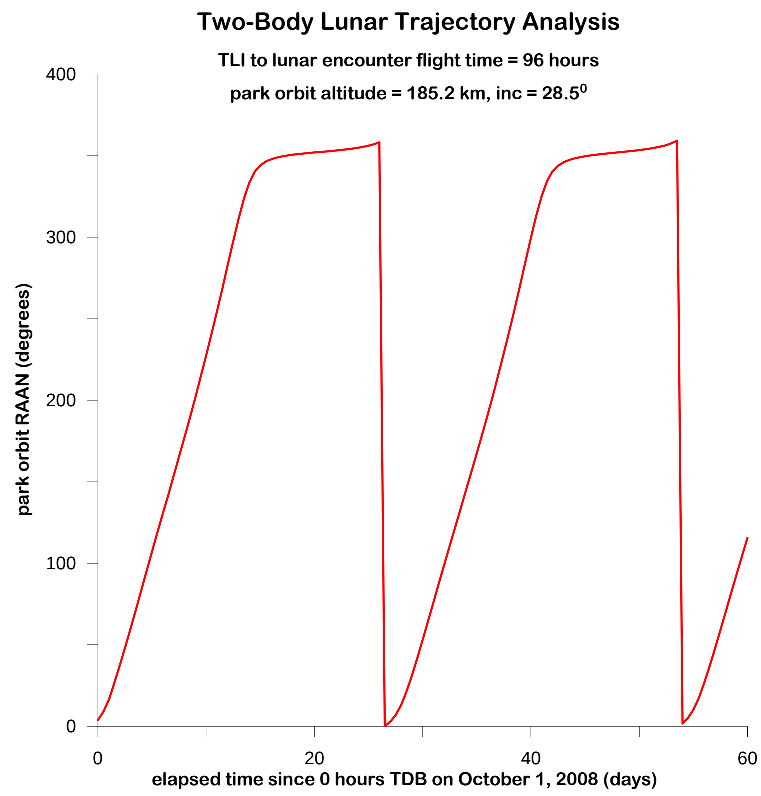


TLI characteristics

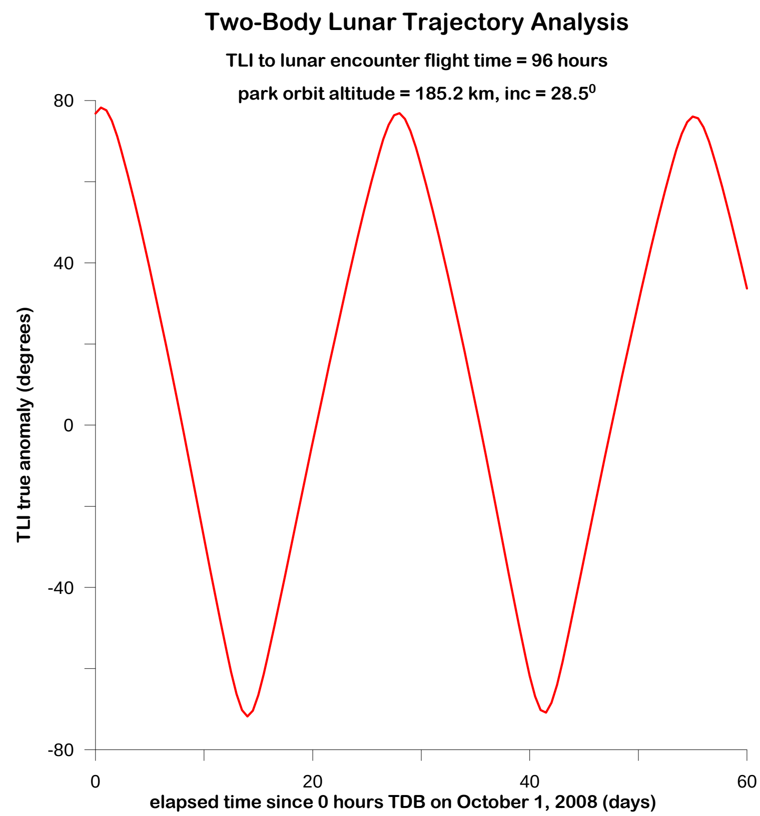
This section contains plots of the behavior of the TLI characteristics for a typical ascending node transfer for a period of two months. The first plot shows the variation of the TLI impulsive delta-v as a function of time. The initial calendar date and time is 0 hours TDB on October 1, 2008 and the transfer time from TLI until lunar encounter is 96 hours.



This next plot illustrates the behavior of the parking orbit RAAN.



The final plot summarizes the true anomaly of the TLI maneuver on the circular park orbit.



Technical Discussion

This section describes several of the algorithms implemented in the `lguess` computer program.

Nonlinear programming problem

A trajectory optimization problem can be described by a system of *dynamic variables*

$$\mathbf{z} = \begin{bmatrix} \mathbf{y}(t) \\ \mathbf{u}(t) \end{bmatrix}$$

consisting of the *state variables* $\mathbf{y}$ and the *control variables* $\mathbf{u}$ for any time t. In this discussion vectors are denoted in bold.

The system dynamics are defined by a vector system of ordinary differential equations called the *state equations* that can be represented as follows:

$$\dot{\mathbf{y}} = \frac{d\mathbf{y}}{dt} = \mathbf{f}\left[\mathbf{y}(t), \mathbf{u}(t), \mathbf{p}, t\right]$$

where $\mathbf{p}$ is a vector of problem *parameters* that is not time dependent.

The initial dynamic variables at time t_0 are defined by $\boldsymbol{\psi}_0 \equiv \boldsymbol{\psi}\left[\mathbf{y}(t_0), \mathbf{u}(t_0), t_0\right]$ and the terminal conditions at the final time t_f are defined by $\boldsymbol{\psi}_f \equiv \boldsymbol{\psi}\left[\mathbf{y}(t_f), \mathbf{u}(t_f), t_f\right]$. These conditions are called the *boundary values* of the trajectory problem. The problem may also be subject to *path constraints* of the form $\mathbf{g}\left[\mathbf{y}(t), \mathbf{u}(t), t\right] = 0$.

The basic nonlinear programming problem (NLP) is to determine the control vector history and problem parameters that minimize the scalar performance index or objective function given by

$$J = \phi\left[\mathbf{y}(t_0), t_0, \mathbf{y}(t_f), t_f, \mathbf{p}\right]$$

while satisfying all the user-defined mission constraints.

During the two-body trajectory optimization, the control variables are the TLI calendar date and the true anomaly of the TLI maneuver. The objective function or performance index is the scalar magnitude of the TLI delta-v vector.

In addition to the bounds on the TLI calendar date mentioned earlier, the true anomaly during the two-body optimization is bounded according to

$$-180^\circ \le \theta \le +180^\circ$$

The final boundary conditions are the components of the moon's inertial position vector at encounter.

Solving the two body Lambert problem

Lambert's problem is concerned with the determination of an orbit that passes between two positions within a specified time-of-flight. This classic astrodynamic problem is also known as the orbital two-point boundary value problem (TPBVP).

The time to traverse a trajectory depends only upon the length of the semimajor axis *a* of the transfer trajectory, the sum $r_i + r_f$ of the distances of the initial and final positions relative to a central body, and the length *c* of the chord joining these two positions. This relationship can be stated as follows:

$$tof = tof\left(r_i + r_f, c, a\right)$$

From the following form of Kepler's equation

$$t - t_0 = \sqrt{\frac{a^3}{\mu}\left(E - e\sin E\right)}$$

we can write

$$t = \sqrt{\frac{a^3}{\mu}}\left[E - E_0 - e\left(\sin E - \sin E_0\right)\right]$$

where *E* is the eccentric anomaly associated with radius *r*, E_0 is the eccentric anomaly at r_0, and $t = 0$ when $r = r_0$.

At this point we need to introduce the following trigonometric sun and difference identities:

$$\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha-\beta}{2}\sin\frac{a+\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha-\beta}{2}\cos\frac{a+\beta}{2}$$

If we let $E = \alpha$ and $E_0 = \beta$ and substitute the first trig identity into the second equation above, we have the following equation:

$$t = \sqrt{\frac{a^3}{\mu}}\left\{E - E_0 - 2\sin\frac{E-E_0}{2}\left(e\cos\frac{E+E_0}{2}\right)\right\}$$

With the two substitutions given by

$$e\cos\frac{E+E_0}{2}=\cos\frac{\alpha+\beta}{2}$$

$$\sin\frac{E-E_0}{2}=\sin\frac{\alpha-\beta}{2}$$

the time equation becomes

$$t=\sqrt{\frac{a^3}{\mu}\left\{(\alpha-\beta)-2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}\right\}}$$

From the elliptic relationships given by

$$r=a(1-e\cos E)$$

$$x=a(\cos E-e)$$

$$y=a\sin E\sqrt{1-e^2}$$

and some more manipulation, we have the following equations:

$$\cos\alpha=\left(1-\frac{r+r_0}{2a}\right)-\frac{c}{2a}=1-\frac{r+r_0+c}{2a}=1-\frac{s}{a}$$

$$\sin\beta=\left(1-\frac{r+r_0}{2a}\right)+\frac{c}{2a}=1-\frac{r+r_0-c}{2a}=1-\frac{s-c}{a}$$

This part of the derivation makes use of the following three relationships:

$$\cos\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}=1-\frac{r+r_0}{2}$$

$$\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}=\sin\frac{E-E_0}{2}\sqrt{1-\left(e\cos\frac{E+E_0}{2}\right)^2}$$

$$\left(\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}\right)^2=\left(\frac{x-x_0}{2a}\right)^2+\left(\frac{y-y_0}{2a}\right)^2=\left(\frac{c}{2a}\right)^2$$

With the use of the half angle formulas given by

$$\sin\frac{\alpha}{2}=\sqrt{\frac{s}{2a}}\qquad\qquad\sin\frac{\beta}{2}=\sqrt{\frac{s-c}{2a}}$$

and several additional substitutions, we have the time-of-flight form of Lambert's theorem

$$t = \sqrt{\frac{a^3}{\mu}}\left[(\alpha - \beta) - (\sin\alpha - \sin\beta)\right]$$

A discussion about the angles α and β can be found in "Geometrical Interpretation of the Angles α and β in Lambert's Problem" by J. E. Prussing, AIAA *Journal of Guidance and Control*, Volume 2, Number 5, Sept.-Oct. 1979, pages 442-443.

The algorithm used in this computer program is based on the method described in "A Procedure for the Solution of Lambert's Orbital Boundary-Value Problem" by R. H. Gooding, *Celestial Mechanics and Dynamical Astronomy* **48:** 145-165, 1990. This iterative solution is valid for elliptic, parabolic and hyperbolic transfer orbits which may be either posigrade or retrograde, and involve one or more revolutions about the central body.

Park orbit RAAN

For a given TLI calendar date, there are two possible locations on the initial park orbit at which to perform the propulsive maneuver. One opportunity occurs during the ascending part of the park orbit and the other during the descending motion. The park orbit RAAN Ω_p at these two locations can be determined from spherical trigonometry relationships involving the park orbit inclination and the geocentric right ascension and declination of the moon at encounter. The equations implemented in this computer program are as follows:

ascending

$$\Omega_p = -180^\circ + \alpha_m + \sin^{-1}\left(\frac{\tan\delta_m}{\tan i_p}\right)$$

descending

$$\Omega_p = \alpha_m - \sin^{-1}\left(\frac{\tan\delta_m}{\tan i_p}\right)$$

where

$\alpha_m =$ right ascension of the moon at encounter
$\delta_m =$ declination of the moon at encounter
$i_p =$ park orbit inclination

These opportunities are valid whenever $|\delta_m| \le i_p$.

Trajectory solution with perturbations

The perturbed solution algorithm uses the optimized two-body TLI delta-v vector as an initial guess for the solution of the following system of nonlinear equations

$$\mathbf{r}_{sc} - \mathbf{r}_{moon} = 0$$

where $\mathbf{r}_{sc}$ and $\mathbf{r}_{moon}$ are the geocentric position vectors of the spacecraft and moon, both evaluated at the final time. The spacecraft's position vector is determined by numerically integrating the geocentric equations of motion. The initial conditions for this integration are given by

$$\mathbf{r}_{to} = \mathbf{r}_{po}$$

$$\mathbf{v}_{to} = \mathbf{v}_{po} + \Delta\mathbf{v}_{TLI}$$

where

$\mathbf{r}_{po} =$ park orbit position vector

$\mathbf{r}_{to} =$ transfer orbit position vector

$\mathbf{v}_{po} =$ park orbit velocity vector

$\mathbf{v}_{to} =$ transfer orbit velocity vector

$\Delta\mathbf{v}_{TLI} =$ TLI delta-v vector

Propagating the spacecraft's trajectory

This part of the trajectory analysis implements a *special perturbation* technique which numerically integrates the vector system of second-order, nonlinear differential equations of motion of a spacecraft given by

$$\vec{a}\left(\vec{r},\vec{v},t\right) = \ddot{\vec{r}}\left(\vec{r},\dot{\vec{r}},t\right) = \vec{a}_g\left(\vec{r}\right) + \vec{a}_s\left(\vec{r},t\right)$$

where

$t =$ dynamical time

$\vec{r} =$ inertial position vector of the spacecraft

$\vec{v} =$ inertial velocity vector of the spacecraft

$\vec{a}_g =$ acceleration due to the Earth's gravity

$\vec{a}_s =$ acceleration due to the Sun

The system of six <u>first-order</u> differential equations of motion perturbed by the Earth's J_2 oblateness is defined by

$$\dot{y}_1 = v_x = y_4 \qquad \dot{y}_2 = v_y = y_5 \qquad \dot{y}_2 = v_z = y_6$$

$$\dot{y}_4 = -\mu\frac{r_x}{r^3}\left\{1+\frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(1-\frac{5r_z^2}{r^2}\right)\right\}$$

$$\dot{y}_5 = -\mu\frac{r_y}{r^3}\left\{1+\frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(1-\frac{5r_z^2}{r^2}\right)\right\}$$

$$\dot{y}_6 = -\mu\frac{r_z}{r^3}\left\{1+\frac{3}{2}\frac{J_2 r_{eq}^2}{r^2}\left(3-\frac{5r_z^2}{r^2}\right)\right\}$$

where $r = \sqrt{r_x^2 + r_y^2 + r_z^2} = \sqrt{y_1^2 + y_2^2 + y_3^2}$. In these equations μ and r_{eq} are the gravitational constant and equatorial radius of the Earth, respectively and J_2 is the Earth oblateness gravity coefficient.

The acceleration contribution of the sun represented by a *point mass* is given by

$$\vec{a}_s(\vec{r}, t) = -\mu_s \left(\frac{\vec{r}_{s-b}}{\left|\vec{r}_{s-b}\right|^3} + \frac{\vec{r}_{e-s}}{\left|\vec{r}_{e-s}\right|^3} \right)$$

where

$\mu_s =$ gravitational constant of the sun
$\vec{r}_{s-b} =$ position vector from the sun to the spacecraft
$\vec{r}_{e-s} =$ position vector from the Earth to the sun

References and Bibliography

"Lunar Trajectories", NASA TN D-866, August 1961.

"Earth-Moon Trajectories", JPL Technical Report No. 32-503, May 1, 1964.

"Three-Dimensional Lunar Trajectories", V. A. Egorov, Mechanics of Space Flight Series, Israel Program for Scientific Translations, Jerusalem 1969.

"Circumlunar Trajectory Calculations", MIT Instrumentation Laboratory Report R-353, April 1962.

"Optimal Low Thrust Trajectories to the Moon", John T. Betts and Sven O. Erb, *SIAM Journal on Applied Dynamical Systems,* Vol. 2, No. 2, pp. 144-170, 2003.

"Integrated Algorithm for Lunar Transfer Trajectories Using a Pseudostate Technique", R. V. Ramanan, AIAA *Journal of Guidance, Control and Dynamics,* Vol. 25, No. 5, September-October 2002, pp. 946-952.

"Nonimpact Lunar Transfer Trajectories Using the Pseudostate Technique", R. V. Ramanan and V. Adimurthy, AIAA *Journal of Guidance, Control and Dynamics*, Vol. 28, No. 2, March-April 2005, pp. 217-225.

"Injection Conditions for Lunar Trajectories", R. Kolenkiewicz and W. Putney, NASA TM X-55390, November 1965.

"Coplanar Three-Body Trans-Earth Lunar Trajectory Simulation Methodology", H. Ikawa, AIAA 88-0381, AIAA 26th Aerospace Sciences Meeting, Reno, Nevada, January 11-14, 1988.

"Earth-Moon Trajectories, 1964-69", R. J. Richard, V. C. Clarke, Jr., R. Y. Roth and W. E. Kirhofer, JPL Technical Report No. 32-503, May 1, 1964.