Composite Orbit Design

This ***Numerit*** program (`composit`) can be used to design sun-synchronous, repeating ground track, frozen orbits. These types of orbits have desirable features that are useful for remote sensing, ocean altimetry and other astrodynamic applications. The nonlinear system of orbit design equations is solved to find the mean classical orbital elements of the *composite* orbit. The orbital element perturbation equations used in this application are based on Kozai's method.

The frozen orbit design equations are

$$\frac{de}{dt} = 0 \quad \text{and} \quad \frac{dw}{dt} = 0 \tag{1}$$

The repeating ground track governing equation is

$$\frac{\frac{K}{N}}{w_e - \dot{l}} - \frac{1}{\dot{w} + \dot{M}} = 0 \tag{2}$$

The sun synchronous design equation is

$$\cos i + \frac{2\,\dot{l}\,a^{3/2}\left(1 - e^2\right)^2}{3 J_2 r_{eq}^2 \sqrt{m}} = 0 \tag{3}$$

where

$K =$ number of orbits in repeat cycle
$N =$ number of days in repeat cycle
$w_e =$ inertial rotation rate of the Earth
$\dot{l} =$ orbital rate of the Earth $\left(\approx 0.985 \text{ degrees/day}\right)$
$\dot{w} =$ argument of perigee perturbation
$\dot{M} =$ mean anomaly perturbation
$a =$ semimajor axis
$e =$ orbital eccentricity
$i =$ orbital inclination
$m =$ gravitational constant of the Earth
$r_{eq} =$ equatorial radius of the Earth

The gravity perturbations of orbital eccentricity and argument of perigee are given by

$$\frac{de}{dt} = \frac{3}{2}\frac{J_3 r_{eq}^3}{p^3}\left(1 - e^2\right) n \sin i \cos w \left(\frac{5}{4}\sin^2 i - 1\right) = 0 \tag{4}$$

and

$$\frac{d\omega}{dt} = \frac{3}{2}\frac{J_2 r_{eq}^2}{p^2} n\left(2 - \frac{5}{2}\sin^2 i\right) - \frac{3}{2}\frac{J_3 r_{eq}^3 \sin\omega}{p^3 e \sin i} n \left\{ \begin{aligned} &\left(\frac{5}{4}\sin^2 i - 1\right)\sin^2 i \\ &+ e^2\left(1 - \frac{35}{4}\sin^2 i \cos^2 i\right) \end{aligned} \right\} = 0 \qquad (5)$$

The software will prompt for an initial guess for the semimajor axis, orbital eccentricity and inclination. It will also ask you to input the integer number of orbits and days in the repeat cycle. The program "hardwires" the value of mean argument of perigee to 90°.

The following is a typical draft output created with this program:

```
program composit

< sun-synchronous, repeating ground track, frozen orbits >

mean semimajor axis                7176.61579448  kilometers
mean eccentricity                  0.00103332568498
mean orbital inclination           98.5964440098  degrees
mean argument of perigee           90  degrees

keplerian period                   100.841487498  minutes
nodal period                       100.959413519  minutes

number of orbits in repeat cycle   271
number of days in repeat cycle     19
ground trace repetition factor     14.2631578947
```

For this example the semimajor axis initial guess was 7176 kilometers, the orbital eccentricity initial guess was 0.001 and the inclination initial guess was 98°.