Coverage Characteristics of Earth Satellites

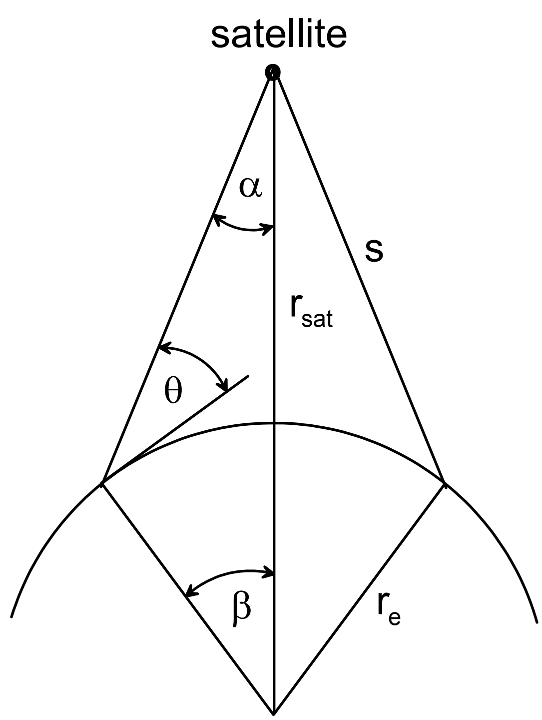
This ***Numerit*** program (`coverag1`) calculates a variety of coverage information for satellites in circular and elliptic Earth orbits. The user can elect to compute coverage characteristics from one of the following satellite orbital positions:

- perigee
- apogee
- extreme northern latitude
- extreme southern latitude
- user-defined true anomaly
- user-defined satellite latitude

The user can also select one of the following coverage constraints and perform calculations for one or two values:

- nadir angle
- Earth central angle
- elevation angle
- slant range

The following diagram illustrates the coverage geometry of a satellite in orbit above a spherical planet with a nadir-pointing conical sensor or field-of-view:



where

$\alpha =$ nadir angle
$\beta =$ central angle
$\theta =$ elevation angle
$s =$ topocentric slant range
$r_{sat} =$ geocentric radius of satellite $= r_e + h_{sat}$
$h_{sat} =$ altitude of satellite
$r_e =$ radius of the Earth

In the discussion that follows, all equations assume that the Earth is *spherical*. This implies that these equations do not account for flattening or the oblate shape. However, the equations are valid for any spherical planet provided the correct astrodynamic and *shape* constants are used.

The fundamental relationships between these angles and distances are given by

$$\theta + \alpha + \beta = 90^\circ$$

$$s\cos\theta = r_{sat}\sin\beta \tag{1}$$

$$s\sin\alpha = r_e\sin\beta$$

Furthermore, the nadir angle from the satellite's location to the Earth horizon is

$$\alpha_h = \sin^{-1}\left(\frac{r_e}{r_{sat}}\right) \tag{2}$$

The equations which define these angular relationships as a function of topocentric slant range s are as follows:

$$\alpha = \cos^{-1}\left(\frac{r_{sat}^2 - r_e^2}{2\,s\,r_{sat}} + \frac{s}{2r_{sat}}\right)$$

$$\beta = \cos^{-1}\left(\frac{r_{sat}^2 + r_e^2 - s^2}{2\,r_{sat}r_e}\right) \tag{3}$$

$$\theta = \sin^{-1}\left(\frac{r_{sat}^2 - r_e^2}{2\,s\,r_e} - \frac{s}{2r_e}\right)$$

The relationships between the nadir angle, central angle and the slant range as functions of the elevation angle θ are defined by the following expressions:

$$\alpha = \sin^{-1}\left(\frac{r_e}{r_{sat}}\cos\theta\right)$$

$$\beta = \cos^{-1}\left(\frac{r_e}{r_{sat}}\cos\theta\right) - \theta \qquad (4)$$

$$s = \sqrt{r_{sat}^2 - r_e^2\cos^2\theta} - r_e\sin\theta$$

The central angle, elevation angle and slant range as functions of the nadir angle α are

$$\beta = \sin^{-1}\left(\frac{r_{sat}}{r_e}\sin\alpha\right) - \alpha$$

$$\theta = \cos^{-1}\left(\frac{r_{sat}}{r_e}\sin\alpha\right) \qquad (5)$$

$$s = r_{sat}\cos\alpha - \sqrt{r_e^2 - r_{sat}^2\sin^2\alpha}$$

The relationship between the nadir angle, elevation angle and slant range and the central angle β is given by the following expressions:

$$\alpha = \tan^{-1}\left(\frac{r_e\sin\beta}{r_{sat} - r_{sat}\cos\beta}\right)$$

$$\theta = \tan^{-1}\left(\frac{r_{sat}\cos\beta - r_e}{r_{sat}\sin\beta}\right) \qquad (6)$$

$$s = \sqrt{r_{sat}^2 + r_e^2 - 2\,r_{sat}r_e\cos\beta}$$

The width of the swath on the Earth's surface is given by

$$w = 2\,r_e\beta \qquad (7)$$

Finally, the percentage of the Earth's surface viewed by a *conical* sensor is $50(1 - \cos\beta)$ and the coverage surface area is given by

$$A = 2\,\pi\,r_e^2(1 - \cos\beta) \qquad (8)$$

The following is a typical draft output created with this software.

```
program coverag1

< coverage characteristics of Earth satellites >

satellite altitude           1626.743 kilometers
true anomaly                       90 degrees
slant range                  4305.008 kilometers

nadir angle                  52.58293 degrees
earth central angle          32.41707 degrees
elevation angle                     5 degrees

earth coverage area      3.983124e+07 square kilometers
earth coverage area          7.791586 percent

arc distance                 3608.653 kilometers

view latitude  1            -3.917068 degrees
view latitude  2             60.91707 degrees
```