

Gravity Assist Interplanetary Trajectories

This *Numerit* application (flyby) can be used to determine the characteristics of patched-conic, gravity assist transfer trajectories between any two planets of our solar system which includes a gravity assist flyby of a third planet.

A patched-conic heliocentric transfer trajectory is represented in this software by the following sequential trajectory segments:

- (1) a heliocentric elliptical orbit from the departure planet to the flyby planet
- (2) a planetocentric hyperbola relative to the flyby planet
- (3) a heliocentric ellipse from the flyby planet to the destination planet

Each trajectory segment is modeled as two-body motion governed by the respective central body. During the heliocentric cruise segments, the Sun is the central body and during the gravity-assist encounter the flyby planet is the central body.

During a flyby the vector relationships between the incoming v -infinity vector \mathbf{v}_∞^- , the outgoing v -infinity vector \mathbf{v}_∞^+ and the two legs of the heliocentric transfer orbit that "connect" at the flyby planet are as follows:

$$\begin{aligned}\mathbf{v}_\infty^- &= \mathbf{v}_{fb} - \mathbf{v}_{to_1} \\ \mathbf{v}_\infty^+ &= \mathbf{v}_{to_2} - \mathbf{v}_{fb}\end{aligned}\tag{1}$$

where

- \mathbf{v}_{fb} = heliocentric velocity vector of the flyby planet at the flyby date
- \mathbf{v}_{to_1} = heliocentric velocity vector of the first transfer orbit at the flyby date
- \mathbf{v}_{to_2} = heliocentric velocity vector of the second transfer orbit at the flyby date

The turn angle of the planet-centered trajectory during the flyby is determined from

$$\mathbf{f} = 2 \sin^{-1} \frac{\mathbf{a}}{\mathbf{c}} \frac{1}{\mathbf{e}} \frac{\mathbf{\ddot{o}}}{1 + r_p v_\infty^2 / \mathbf{m}_{\mathbf{\ddot{o}}}}\tag{2}$$

where r_p is the periapsis radius of the flyby hyperbola, v_∞ is the magnitude of the incoming v -infinity vector and \mathbf{m} is the gravitational constant of the flyby planet.

The maximum turn angle possible during a gravity assist flyby occurs when the spacecraft just gazes the planet's surface. It is given by

$$\mathbf{f}_{\max} = 2 \sin^{-1} \frac{\mathbf{a}}{\mathbf{c}} \frac{1}{\mathbf{e}} \frac{\mathbf{\ddot{o}}}{1 + r_e v_\infty^2 / \mathbf{m}_{\mathbf{\ddot{o}}}}\tag{3}$$

where r_e is the radius of the flyby planet.

The semimajor axis and orbital eccentricity of the flyby hyperbola are given by

$$\begin{aligned} a &= -\mathbf{m}/|\mathbf{v}_\infty^-|^2 = -\mathbf{m}/|\mathbf{v}_\infty^+|^2 \\ e &= -1/\cos\mathbf{q}_\infty = 1 - \frac{r_p}{a} = 1 + \frac{r_p v_\infty^2}{\mathbf{m}} \end{aligned} \quad (4)$$

where \mathbf{q}_∞ is true anomaly at infinity which is determined from the following expression:

$$\mathbf{q}_\infty = \frac{\mathbf{p}}{2} - \frac{1}{2} \sin^{-1} \frac{\mathbf{a} |\mathbf{v}_\infty^- \times \mathbf{v}_\infty^+| \mathbf{\ddot{o}}}{\mathbf{c} |\mathbf{v}_\infty^-| |\mathbf{v}_\infty^+| \mathbf{\ddot{\theta}}} \quad (5)$$

The periapsis radius of the flyby hyperbola is determined from the expression $r = a(1 - e)$ and the flyby altitude is $h = r - r_p$.

The heliocentric speed gained during the flyby and the heliocentric delta-v vector caused by the close encounter can be determined from the following two equations:

$$\begin{aligned} \Delta v &= 2v_\infty/e \\ \Delta \mathbf{v} &= \mathbf{v}_h^- - \mathbf{v}_h^+ \end{aligned} \quad (6)$$

In these equations \mathbf{v}_h^- is the heliocentric velocity vector of the spacecraft prior to the flyby and \mathbf{v}_h^+ is the heliocentric velocity vector after the flyby.

Numerical Solution

This section describes the solution steps and numerical method used in this computer program. The flyby software solves the following system of two nonlinear equations:

$$\begin{aligned} |\mathbf{v}_\infty^-| - |\mathbf{v}_\infty^+| &= 0 \\ h_{fb} - h_t &= 0 \end{aligned} \quad (7)$$

The first equation is the *v-infinity matching constraint* and the second equation is the (positive) *flyby altitude constraint*. In the second expression h_{fb} is the actual flyby altitude and h_t is the desired or "targeted" flyby altitude.

The control variables for this problem are the flyby and arrival calendar dates. The computational steps required to compute the system of nonlinear equations for any combination of departure, flyby and arrival calendar dates are as follows:

- (1) compute the state vector of the departure planet at the departure date
- (2) compute the state vector of the flyby planet at the flyby date
- (3) compute the state vector of the arrival planet at the arrival date

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- (4) solve Lambert's problem for the departure-to-flyby leg and determine the initial velocity vector of the first leg
- (5) compute the departure launch energy
- (6) propagate the first leg of the heliocentric transfer orbit to the flyby calendar date
- (7) calculate the incoming v-infinity speed relative to the flyby planet
- (8) solve Lambert's problem for the second heliocentric leg and determine the initial velocity vector of the second leg
- (9) calculate the outgoing v-infinity speed relative to the flyby planet
- (10) determine the actual flyby angle
- (11) calculate the planet-centered hyperbolic orbital elements
- (12) compute the flyby altitude (kilometers)
- (13) form the system of nonlinear equations

This computer program examines the launch energy of each valid patch-conic solution during the grid search and saves the smallest value as the global optimum solution. After the search is complete, the script provides a complete mission summary and heliocentric trajectory graphics.

The inputs required by this computer program are minimal. The user provides the following information:

- (1) departure calendar date and universal time
- (2) departure, flyby and arrival planets
- (3) desired flyby altitude
- (4) size and number of time steps for each leg of the heliocentric trajectory

The following is a typical draft output created with this software.

```
program flyby
< gravity assist interplanetary trajectories >

departure planet           Earth

departure calendar date    February 1, 2009
departure universal time   00 h 00 m 00 s
departure julian date      2454863.5

heliocentric ecliptic orbital elements of the first leg

      sma (au)      eccentricity      inclination (deg)      argper (deg)
0.8529275052      0.1675686617      12.11270024      161.271537

      raan (deg)      true anomaly (deg)      arglat (deg)      period (years)
132.2599929      198.728463      0      0.787728174

flyby planet           Venus

flyby calendar date      June 23, 2009
flyby universal time     19 h 55 m 36.4692 s
flyby julian date        2455006.33

v-infinity             7.73399874 kilometers/second
eccentricity           3.034936108
semimajor axis         -5431.079662 kilometers
```

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```
flyby altitude           5000.000108 kilometers
perigee velocity        10.89048043 kilometers/second

maximum turn angle      56.4542323 degrees
actual turn angle       38.4764029 degrees

arrival planet           Mars

arrival calendar date    February 13, 2010
arrival universal time    15 h 00 m 39.1484 s
arrival julian date      2455241.12545
```

heliocentric ecliptic orbital elements of the second leg

sma (au)	eccentricity	inclination (deg)	argper (deg)
1.18963405	0.3922600192	7.477476693	192.6802122
raan (deg)	true anomaly (deg)	arglat (deg)	period (years)
122.1395383	12.74195197	205.4221641	1.297562477

The software will also allow you to create a two-dimensional plot of the planet orbits and transfer trajectory. If you elect this program option, you will be asked for a plot step size in days. A value between 5 and 10 days should be adequate for planets of the inner solar system. Larger step sizes can be input for the outer planets. The graphics display is a view from the north ecliptic pole looking down on the ecliptic plane. For best results and proper perspective the user can manually adjust the width and height of the plot to be identical. The following is the companion graphics display for this example.

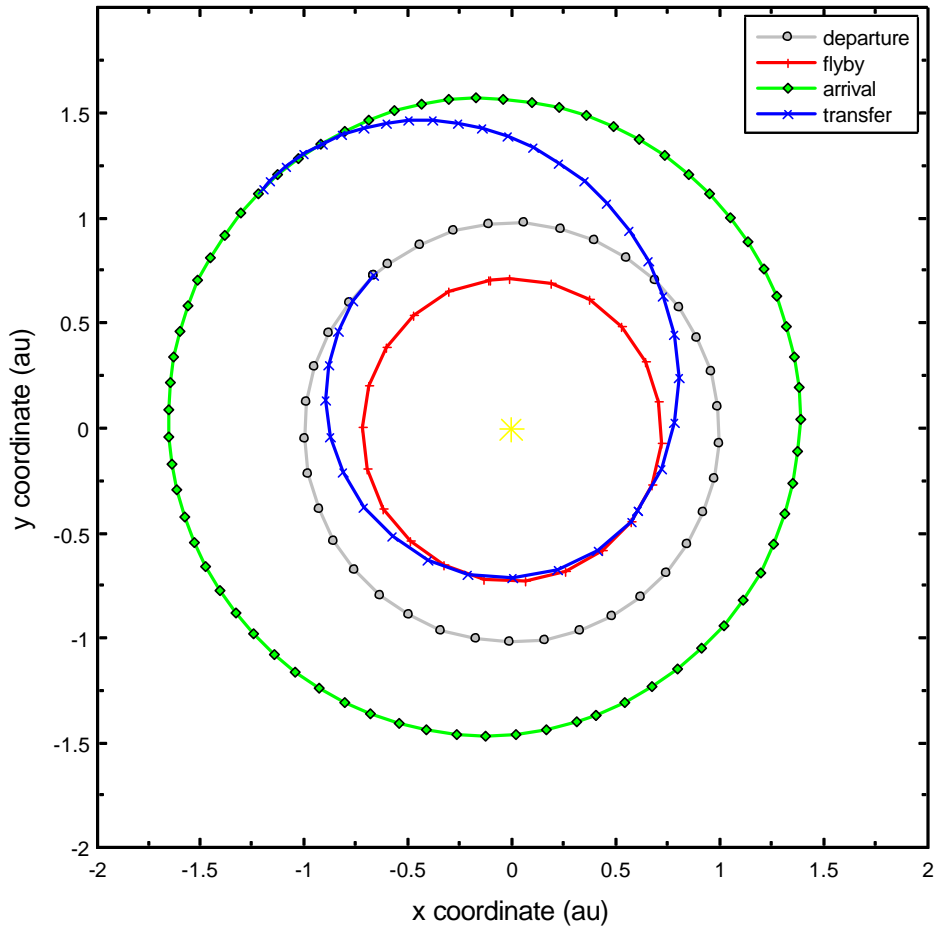


Figure 1. Gravity Assist Interplanetary Trajectory