Long-Term Evolution of Frozen Orbits

This ***Numerit*** application (`frozen2`) allows the user to analyze the long-term behavior of frozen orbits by numerically integrating the equations of orbital motion subject to zonal gravity perturbations. The software can model up to 18 zonal coefficients and the program will plot the long term behavior of eccentricity versus time, argument of perigee versus time, and argument of perigee versus orbital eccentricity. This computer program uses a fourth-order Nystrom method to solve the equations of motion.

The steps involved in this algorithm are as follows:

(1) input the initial mean orbital elements

(2) convert to osculating orbital elements and then to state vector

(3) propagate the second-order "zonal" orbital equations of motion

(4) convert the state vector at each step to mean orbital elements

(5) plot the evolution of the mean orbital elements

The Earth-centered inertial (ECI) second-order, nonlinear differential equations of orbital motion subject to both the spherical and zonal components of the Earth's gravitation attraction are given by

$$\frac{d^2 r}{dt^2} = -m \frac{\boldsymbol{r}}{|\boldsymbol{r}|^3} + \sum_{n=2}^{N} J_n \boldsymbol{U}_n \tag{1}$$

where $\boldsymbol{r}$ is the inertial position vector of the satellite, J_n are the zonal coefficients of the Earth gravity and the "perturbed" or non-spherical contributions are

$$\begin{aligned}
U_{2_x} &= \frac{3}{2} m \frac{r_{eq}^2}{r^5} \left(5 \frac{z^2}{r^2} - 1 \right) x \\
U_{2_y} &= \frac{3}{2} m \frac{r_{eq}^2}{r^5} \left(5 \frac{z^2}{r^2} - 1 \right) y \\
U_{2_z} &= \frac{3}{2} m \frac{r_{eq}^2}{r^5} \left(5 \frac{z^2}{r^2} - 1 \right) z \\
U_{3_x} &= \frac{1}{2} m \frac{r_{eq}^3}{r^6} \left(35 \frac{z^3}{r^3} - 15 \frac{z}{r} \right) x \\
U_{3_x} &= \frac{1}{2} m \frac{r_{eq}^3}{r^6} \left(35 \frac{z^3}{r^3} - 15 \frac{z}{r} \right) y \\
U_{3_x} &= \frac{1}{2} m \frac{r_{eq}^3}{r^6} \left(35 \frac{z^3}{r^3} - 30 \frac{z}{r} + 3 \frac{z}{r} \right) z
\end{aligned} \tag{2}$$

In these equations, m is the gravitational constant of the Earth, r_{eq} is the equatorial radius of the Earth, r is the geocentric radius of the satellite and the inertial rectangular components of the position vector are x, y and z. The recurrence relationships for the U's are as follows:

$$\begin{aligned} U_{n_x} &= \left(\frac{2n+1}{n}\right)\frac{r_{eq}}{r}\frac{z}{r}U_{n-1_x} - \left(\frac{n+1}{n}\right)\frac{r_{eq}^2}{r^2}U_{n-2_x} \\ U_{n_y} &= \left(\frac{2n+1}{n}\right)\frac{r_{eq}}{r}\frac{z}{r}U_{n-1_y} - \left(\frac{n+1}{n}\right)\frac{r_{eq}^2}{r^2}U_{n-2_y} \\ U_{n_z} &= \left(\frac{2n+1}{n}\right)\frac{r_{eq}}{r}\frac{z}{r}U_{n-1_z} - \left(\frac{n+1}{n}\right)\frac{r_{eq}^2}{r^2}U_{n-2_z} \end{aligned} \tag{3}$$

The software will prompt you for the initial mean orbital elements, a simulation step size, graphics step size and simulation duration. The default values use a semimajor axis of 8000 kilometers, an orbital eccentricity of 0.001, an orbital inclination of 60° and an initial argument of perigee of 90°.

The following is a graphics display of argument of perigee versus orbital eccentricity for this example. The orbit evolution for this example is clockwise in this display starting from the top $(e = 0.001,\ w = 90°)$. We can see that the frozen orbit eccentricity for this example is about 0.0008 for an argument of perigee of 90°. The integration step size for this example was 2 minutes, the graphics step size was 10 days and the simulation duration was 650 days.

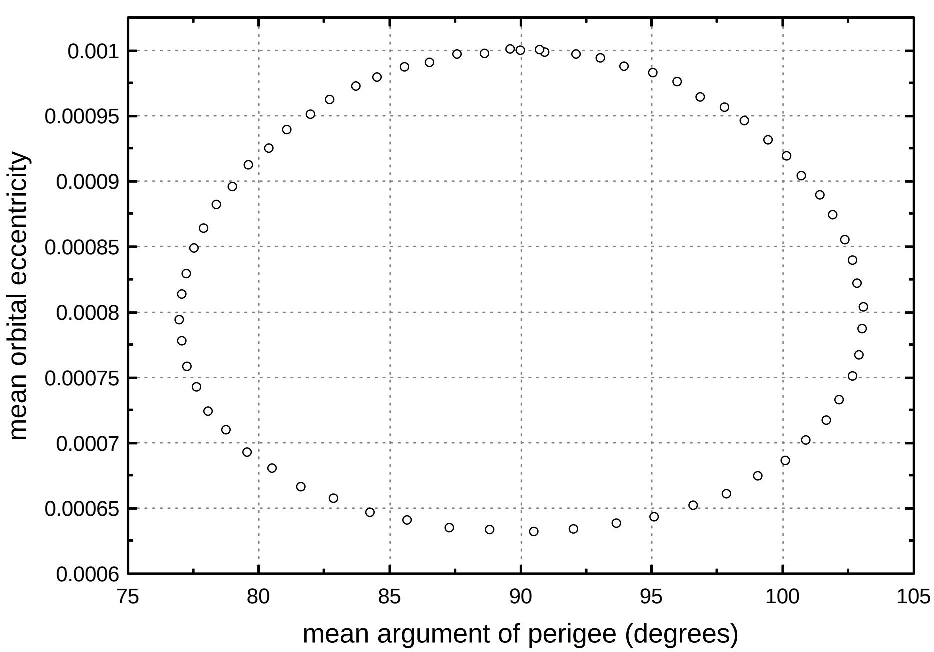

Figure 1. Evolution of Mean Eccentricity and Argument of Perigee

Figure 2 illustrates the long-term evolution of the mean orbital eccentricity.

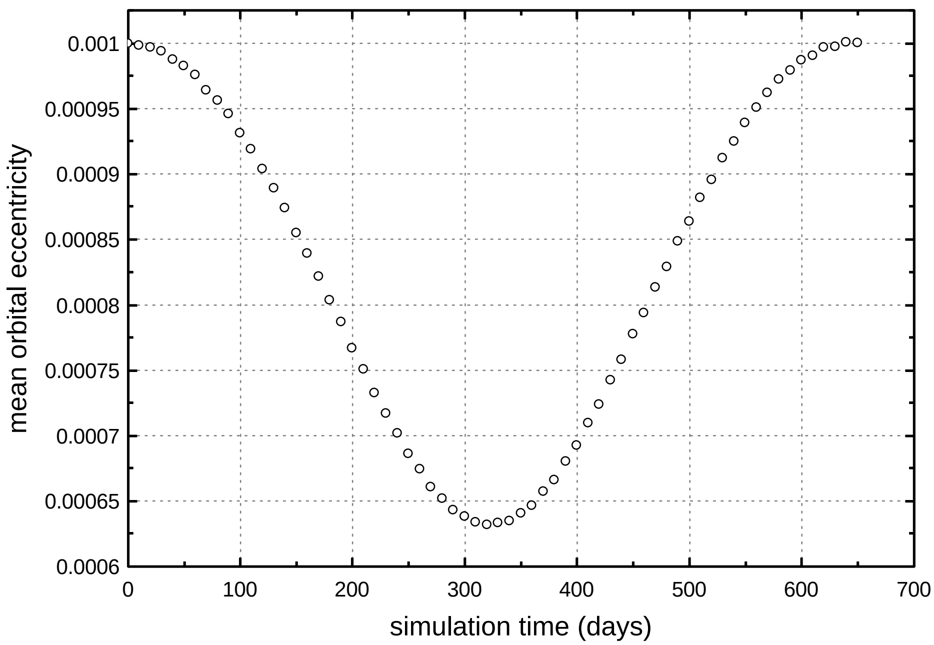

Figure 2. Evolution of Mean Orbital Eccentricity

Figure 3 shows the long-term behavior of the mean argument of perigee.

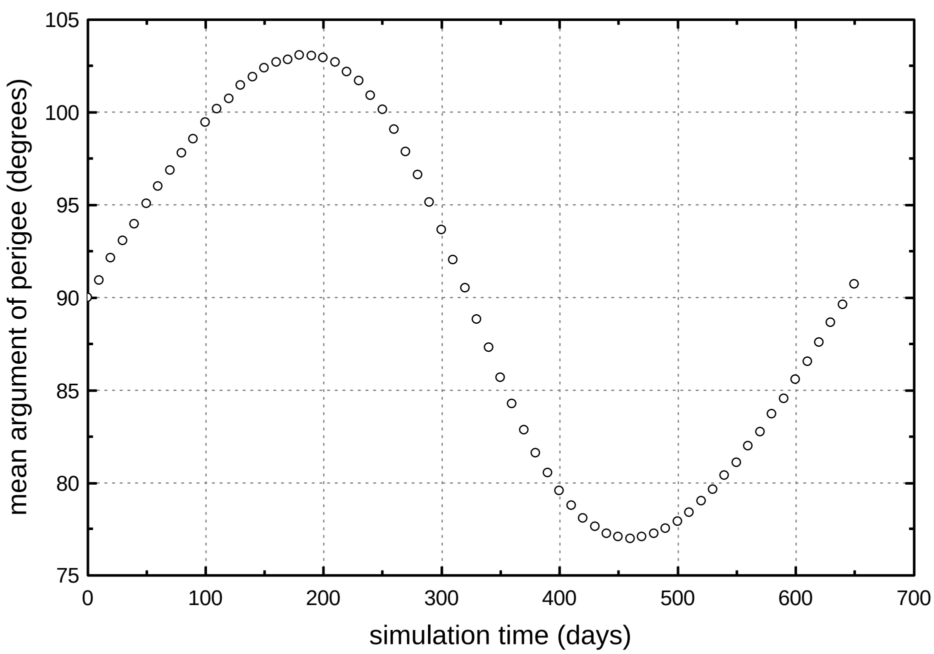

Figure 3. Evolution of Mean Argument of Perigee