Impulsive Hyperbolic Injection from a Circular Park Orbit

This ***Numerit*** computer program (`hyper1`) can be used to determine the characteristics of a single impulsive maneuver from a circular park orbit to a departure hyperbola. The algorithm implemented in this program is based on the equations derived in Chapter 4 of Richard Battin's classic text, *Astronautical Guidance*, and Chapter 11 of *An Introduction to the Mathematics and Methods of Astrodynamics*, also written by Professor Battin and published by the American Institute of Aeronautics and Astronautics.

The Earth departure trajectory for interplanetary missions is usually defined by a "targeting specification" which consists of twice the specific (per unit mass) orbital energy C_3 , and the right ascension RLA and declination DLA of the outgoing asymptote. These numbers may be supplied by a spacecraft customer or determined with a patched-conic or more sophisticated trajectory analysis computer program that solves Lambert's problem for a lunar or interplanetary space mission.

The `hyper1` software determines the orbital elements and state vectors of the park orbit and departure hyperbola at injection, and the delta-v vector and magnitude of the injection propulsive maneuver. This information can be used as initial guesses for other trajectory simulations.

This computer program assumes that the hyperbolic targets and orbital characteristics are in the same Earth-centered-inertial (ECI) coordinate system. For example, targeting specs are often provided or computed in an Earth mean equator and equinox of J2000 coordinate system (EME2000). For this situation, the state vectors and orbital elements computed by this code will also be with respect to the EME2000 coordinate system. With a change of the gravitational constant, this program can also be used for hyperbolic departure trajectories relative to the Moon and other planets.

The `hyper1` program will interactively prompt the user for the park orbit altitude and orbital inclination, and the departure hyperbola characteristics.

Program Output

The following is a typical output created by this program.

```
---------------------------------------------------
Interplanetary Injection from a Circular Park Orbit
---------------------------------------------------

departure hyperbola characteristics
-----------------------------------

c3                            9.28 km**2/sec**2

asymptote right ascension   352.59 degrees

asymptote declination         2.27 degrees

opportunity #1
```

```
orbital elements and state vector of park orbit at injection
------------------------------------------------------------

semimajor axis (km)                      eccentricity
6.5633400000000000e+03                   0.0000000000000000e+00

inclination (deg)                        argument of perigee (deg)
2.8500000000000004e+01                   0.0000000000000000e+00

raan (deg)                               true anomaly (deg)
1.7677673366930848e+02                   2.5074779914719508e+01

argument of latitude (deg)               period (min)
2.5074779914719508e+01                   8.8195627034412224e+01

rx                  -6.0728215132587832e+03 kilometers
ry                  -2.1063485210179452e+03 kilometers
rz                  1.3272402690194524e+03 kilometers

vx                  2.9486811758231704e+00 km/sec
vy                  -6.3790889122749224e+00 km/sec
vz                  3.3680638606380168e+00 km/sec

opportunity #1
orbital elements and state vector of hyperbola at injection
-----------------------------------------------------------

semimajor axis (km)                      eccentricity
-4.2952640086207168e+04                   1.1528041113847060e+00

inclination (deg)                        argument of perigee (deg)
2.8499999999999992e+01                   2.5074779914719544e+01

raan (deg)                               true anomaly (deg)
1.7677673366930848e+02                   3.5999999999999996e+02

argument of latitude (deg)               period (min)
2.5074779914719496e+01                   0.0000000000000000e+00

rx                  -6.0728215132587832e+03 kilometers
ry                  -2.1063485210179452e+03 kilometers
rz                  1.3272402690194524e+03 kilometers

vx                  4.3264339149052304e+00 km/sec
vy                  -9.3596780969571936e+00 km/sec
vz                  4.9417705222617088e+00 km/sec


injection delta-v vector and magnitude - opportunity #1
-------------------------------------------------------

x-component of delta-v       1377.7527 meters/second

y-component of delta-v      -2980.5892 meters/second

z-component of delta-v       1573.7067 meters/second
```

```
delta-v magnitude            3641.2453 meters/second


opportunity #2
orbital elements and state vector of park orbit at injection
------------------------------------------------------------

semimajor axis (km)                      eccentricity
6.5633400000000000e+03                   0.0000000000000000e+00

inclination (deg)                        argument of perigee (deg)
2.8500000000000004e+01                   0.0000000000000000e+00

raan (deg)                               true anomaly (deg)
3.4840326633069152e+02                   2.1459790192119576e+02

argument of latitude (deg)               period (min)
2.1459790192119576e+02                   8.8195627034412224e+01

rx                   -5.9507486891590640e+03 kilometers
ry                   -2.1222246783165360e+03 kilometers
rz                   -1.7782531903010758e+03 kilometers

vx                   3.2013960363628608e+00 km/sec
vy                   -6.4119556941095320e+00 km/sec
vz                   -3.0609210691939012e+00 km/sec

opportunity #2
orbital elements and state vector of hyperbola at injection
-----------------------------------------------------------

semimajor axis (km)                      eccentricity
-4.2952640086207464e+04                   1.1528041113847046e+00

inclination (deg)                        argument of perigee (deg)
2.8500000000000000e+01                   2.1459790192119580e+02

raan (deg)                               true anomaly (deg)
3.4840326633069152e+02                   0.0000000000000000e+00

argument of latitude (deg)               period (min)
2.1459790192119580e+02                   0.0000000000000000e+00

rx                   -5.9507486891590640e+03 kilometers
ry                   -2.1222246783165360e+03 kilometers
rz                   -1.7782531903010758e+03 kilometers

vx                   4.6972282050455440e+00 km/sec
vy                   -9.4079016759486864e+00 km/sec
vz                   -4.4911171927265056e+00 km/sec


injection delta-v vector and magnitude - opportunity #2
-------------------------------------------------------

x-component of delta-v       1495.8322 meters/second
```

```
y-component of delta-v      -2995.9460 meters/second

z-component of delta-v      -1430.1961 meters/second

delta-v magnitude            3641.2453 meters/second
```

The following is a graphics display of the park orbit and departure hyperbola for both opportunities. This display is labeled with an Earth centered, inertial coordinate system. The x-axis of this system is red, the y-axis is green and the z-axis is blue. The outgoing asymptote is colored magneta. The park orbit traces are red, and the hyperbolic trajectories are black. Please note the units for each coordinate axis are Earth radii (ER).

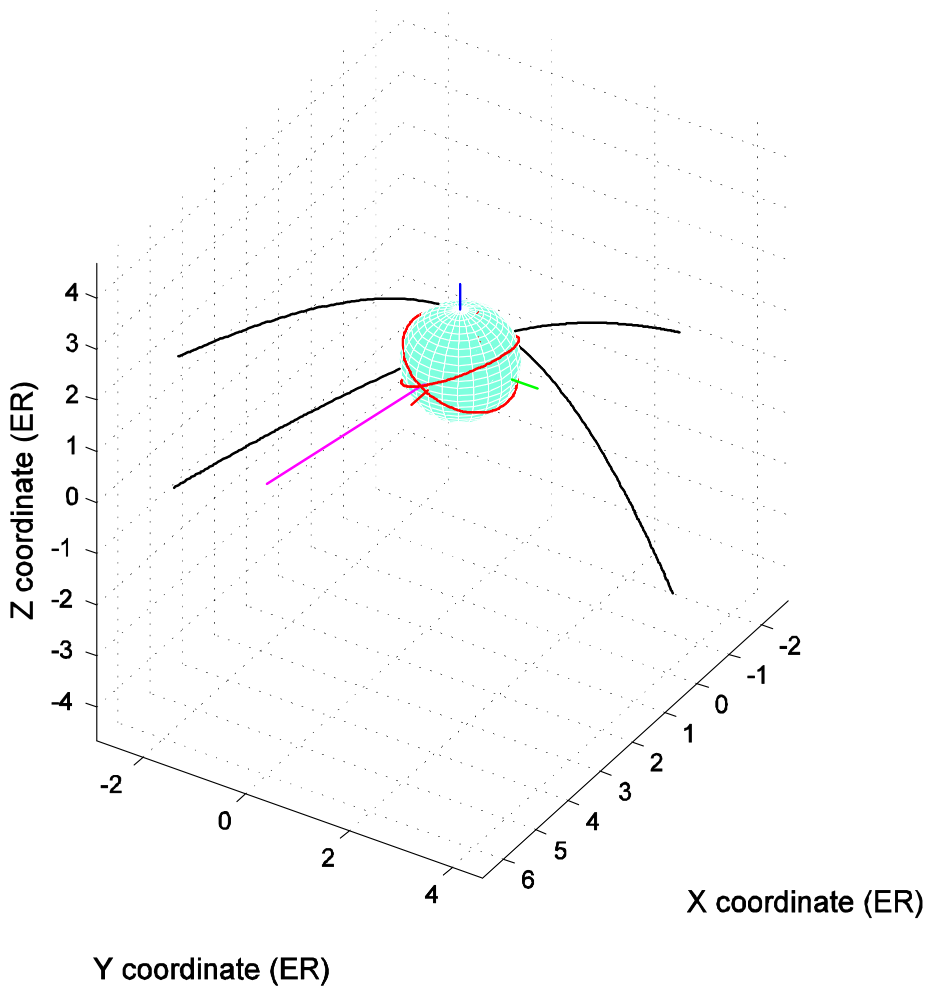

Figure 1 Park Orbits and Departure Hyperbolas

Technical Discussion

This section describes the numerical algorithms implemented in this Matlab script. The discussion describes computations for both tangential and non-tangential injection. The first part explains calculations that are common to both types of interplanetary injection. The script assumes that injection occurs impulsively at perigee of the departure hyperbola.

A unit vector is the direction of the departure asymptote is given by

$$\hat{\mathbf{s}} = \begin{Bmatrix} \cos\delta_\infty \cos\alpha_\infty \\ \cos\delta_\infty \sin\alpha_\infty \\ \sin\delta_\infty \end{Bmatrix} \tag{1}$$

where

$$\begin{aligned} \alpha_\infty &= \text{ right ascension of departure asymptote} \\ \delta_\infty &= \text{ declination of departure asymptote} \end{aligned} \tag{2}$$

The angle between the outgoing asymptote and the spin axis of the Earth is given by

$$\beta = \cos^{-1}\left(\hat{\mathbf{s}} \cdot \hat{\mathbf{z}}\right) \tag{3}$$

where $\hat{\mathbf{z}} = [0 \;\; 0 \;\; 1]^T$. Note that $\beta = 90° - \delta_\infty$.

Departure delta-V

The velocity vector at any geocentric position vector $\mathbf{r}$ required to achieve a launch hyperbola defined by V_∞, α_∞ and δ_∞ is given by

$$\mathbf{v}_h = \left(d + \tfrac{1}{2} V_\infty\right)\hat{\mathbf{s}} + \left(d - \tfrac{1}{2} V_\infty\right)\hat{\mathbf{r}} \tag{4}$$

where

$$d = \sqrt{\frac{\mu}{\left(1 + \cos\psi\right) r_p} + \frac{V_\infty^2}{4}} \tag{5}$$

and ψ is the angle between the spacecraft's position vector and the departure asymptote unit vector which can be computed using

$$\cos\psi = \hat{\mathbf{s}} \cdot \hat{\mathbf{r}} \tag{6}$$

The injection $\Delta\mathbf{v}$ vector can be determined from the following expression

$$\Delta\mathbf{v} = \mathbf{v}_h - \mathbf{v}_p \tag{7}$$

where $\mathbf{v}_p$ is the inertial velocity vector in the park orbit prior to injection and $\hat{\mathbf{r}} = \mathbf{r}/\left|\mathbf{r}\right|$.

Finally, the scalar injection delta-v is $\Delta v = \left|\Delta\mathbf{v}\right|$.

Tangential injection $\left(i \geq |\delta_\infty|\right)$

This part of the numerical algorithm is valid for geocentric orbit inclinations that satisfy the following geometric constraint

$$i \geq |\delta_\infty| \tag{8}$$

where i is the orbital inclination of the park orbit.

Whenever $i > |\delta_\infty|$, there will be two opportunities to establish a departure hyperbola that will satisfy the energy and orientation of the outgoing asymptote. One injection opportunity will occur while the spacecraft is ascending and the other while the spacecraft is descending along the park orbit. For the case where $i = |\delta_\infty|$, there will be a single injection opportunity.

Orientation of the park orbit and departure hyperbola

This section summarizes the equations used to determine the right ascension of the ascending node (RAAN) of the park orbit and the injection true anomaly on the park orbit.

The park orbit right ascension of the ascending node for each opportunity can be determined from these next two equations;

$$\begin{aligned} \Omega_1 &= 180° + \alpha_\infty + \sin^{-1}\left(\frac{\cot\beta}{\tan i}\right) \\ \Omega_2 &= 360° + \alpha_\infty - \sin^{-1}\left(\frac{\cot\beta}{\tan i}\right) \end{aligned} \tag{9}$$

The true anomaly on the park orbit for each injection opportunity can be determined from

$$\begin{aligned} \theta_1 &= \cos^{-1}\left(\frac{\cos\beta}{\sin i}\right) - \eta \\ \theta_2 &= -\cos^{-1}\left(\frac{\cos\beta}{\sin i}\right) - \eta \end{aligned} \tag{10}$$

where

$$\eta = \sin^{-1}\left(\frac{1}{1 + r_p V_\infty^2 / \mu}\right) \tag{11}$$

In the last equation, r_p is the geocentric radius of the park orbit and μ is the gravitational constant of the Earth. The velocity vector at infinity V_∞ is determined from $V_\infty = \sqrt{C_3}$.

For a tangential impulsive injection maneuver that occurs at perigee of the hyperbola, the true anomaly on the hyperbola is zero. Furthermore, since the orbit transfer modeled by this software is coplanar, the right ascension of the ascending node computed above should be the same for both the park orbit and the launch hyperbola. This can be verified by examining the hyperbola's RAAN which is computed using the state vector at injection.

Non-tangential injection $(\delta_\infty \ge i)$

This case involves high declination trajectories that require a non-tangential impulsive because the angle between the park orbit and departure hyperbola orbit planes is nonzero.

A unit vector normal to the park orbit plane can be computed from

$$\hat{\mathbf{h}} = \frac{\sin(i+\beta)}{\sin\beta}\hat{\mathbf{z}} - \frac{\sin i}{\sin\beta}\hat{\mathbf{s}} \tag{12}$$

A unit vector in the direction of the ascending node of the park orbit is given by

$$\hat{\mathbf{n}} = \frac{\hat{\mathbf{z}} \times \hat{\mathbf{h}}}{\sin i} \tag{13}$$

The right ascension of the ascending node of the park orbit can be computed from the x and y components of the node vector using a four quadrant inverse tangent according to

$$\Omega = \tan^{-1}(n_y, n_x) \tag{14}$$

Finally, the true anomaly of the injection impulse on the park orbit is given by

$$\theta = 360^\circ - \sin^{-1}\left\{\frac{\sin\eta}{\sin(i+\beta)}\right\} \tag{15}$$

The following is the `hyper1` output for a typical non-tangential maneuver.

```
---------------------------------------------------
Interplanetary Injection from a Circular Park Orbit
---------------------------------------------------

departure hyperbola characteristics
-----------------------------------

c3                            9.28 km**2/sec**2

asymptote right ascension   352.59 degrees

asymptote declination       35 degrees

opportunity #1
orbital elements and state vector of park orbit at injection
------------------------------------------------------------

semimajor axis (km)                        eccentricity
6.5633400000000000e+03                     0.0000000000000000e+00
```

```
inclination (deg)                         argument of perigee (deg)
2.8500000000000004e+01                    0.0000000000000000e+00

raan (deg)                                true anomaly (deg)
2.6258999999999996e+02                    2.9918353533314824e+02

argument of latitude (deg)                period (min)
2.9918353533314824e+02                    8.8195627034412224e+01

rx                   -5.4064898347472264e+03 kilometers
ry                   -2.5241545283119496e+03 kilometers
rz                   -2.7342171712534432e+03 kilometers

vx                   2.4340956904326572e+00 km/sec
vy                   -7.1776672373169144e+00 km/sec
vz                   1.8131799129879992e+00 km/sec

opportunity #1
orbital elements and state vector of hyperbola at injection
-----------------------------------------------------------

semimajor axis (km)                       eccentricity
-4.2952640086207568e+04                    1.1528041113847054e+00

inclination (deg)                         argument of perigee (deg)
3.6599116896096192e+01                    3.1567510430140368e+02

raan (deg)                                true anomaly (deg)
2.4312764530449668e+02                    3.5999999999999996e+02

argument of latitude (deg)                period (min)
3.1567510430140368e+02                    0.0000000000000000e+00

rx                   -5.4064898347472264e+03 kilometers
ry                   -2.5241545283119496e+03 kilometers
rz                   -2.7342171712534432e+03 kilometers

vx                   2.2466896700102232e+00 km/sec
vy                   -1.0095049277618806e+01 km/sec
vz                   4.8769935413648592e+00 km/sec


injection delta-v vector and magnitude - opportunity #1
-------------------------------------------------------

x-component of delta-v    -1.8740602042243416e+02 meters/second

y-component of delta-v    -2.9173820403018916e+03 meters/second

z-component of delta-v    3.0638136283768600e+03 meters/second

delta-v magnitude         4.2347600799802560e+03 meters/second
```

The following is the graphics display for this example.

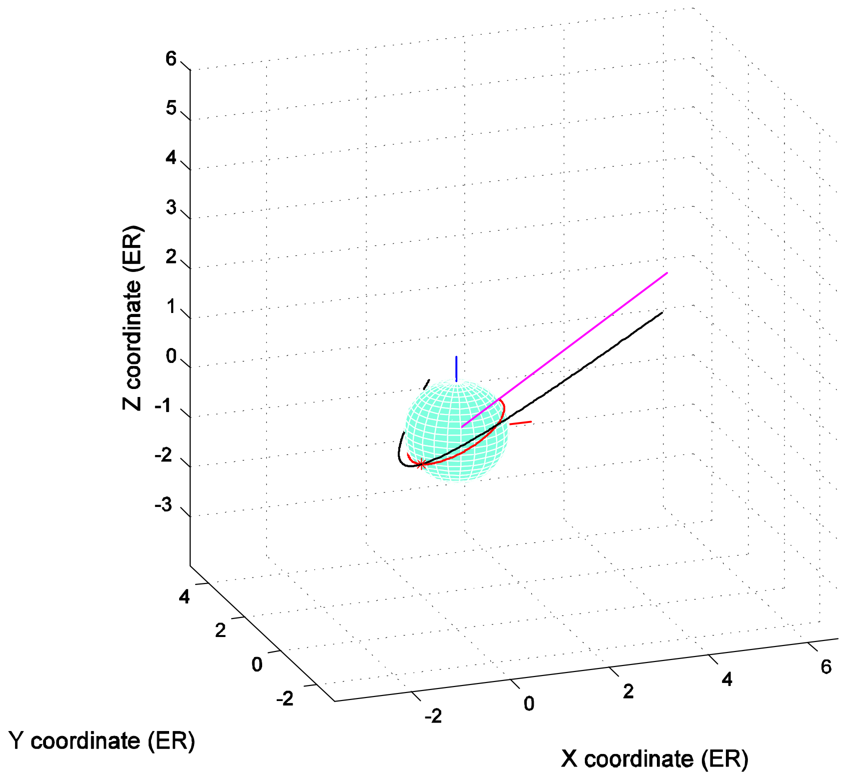

Figure 2 Non-tangential Injection - Park Orbit and Departure Hyperbola

Delta-v penalty for off-nominal injection

The velocity-required equation given above can also be used to access the delta-v penalty for off-nominal injection. Such things as ignition timing errors and other spacecraft contingencies may result in an injection maneuver that does not occur at the optimal true anomaly on the park orbit. The following plot illustrates how the injection delta-v penalty changes as the true anomaly at injection is displaced from the optimal.

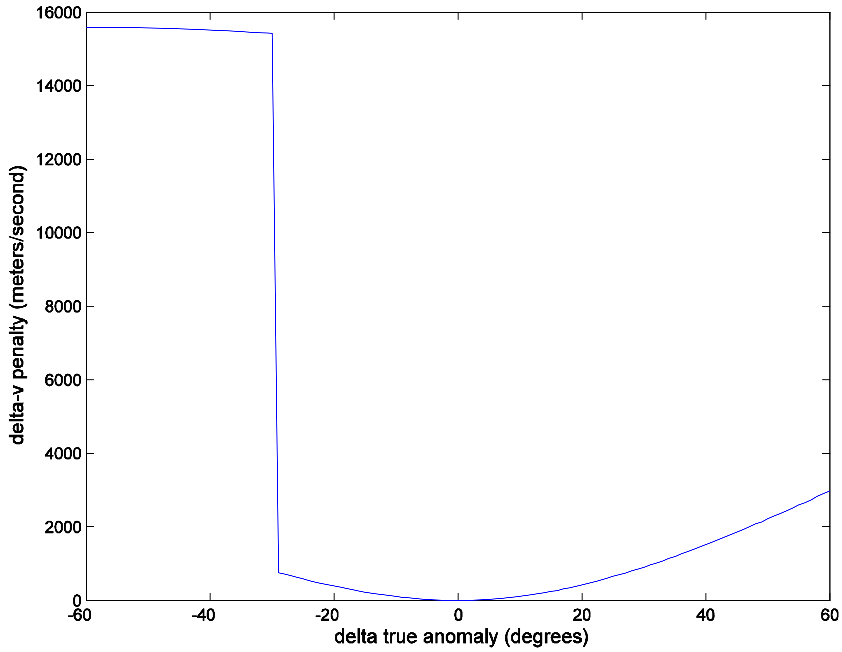

Figure 3 Delta-V Penalty for Off-nominal Injection