Interplanetary Trajectory Optimization

This ***Numerit*** application (`ipto`) can be used to determine the characteristics of *optimum* patched-conic transfer trajectories between any two planets of our solar system.. This type of trajectory ignores the gravitational effect of both the launch and arrivals planets on the trajectory. The solution is based on solving Lambert's problem relative to the Sun (heliocentric). Patched-conic trajectories are suitable for preliminary mission design.

The user can calculate the minimum launch, arrival or total ΔV required for this type of mission. This program uses a quasi-Newton algorithm with numerical derivatives to solve this unconstrained, multi-dimensional optimization problem.

The $\Delta V's$ required at launch and arrival are simply the differences between the velocity on the transfer trajectory determined by the solution of Lambert's problem and the velocities of the two planets. If we treat each planet as a point mass and assume impulsive $\Delta V's$, the magnitude and direction of the required maneuvers are given by the two vector equations

$$\begin{aligned} \Delta \mathbf{v}_1 &= \mathbf{v}_{to_1} - \mathbf{v}_{lp} \\ \Delta \mathbf{v}_2 &= \mathbf{v}_{to_2} - \mathbf{v}_{ap} \end{aligned} \tag{1}$$

where

$\mathbf{v}_{to_1} =$ heliocentric velocity vector of the transfer trajectory at launch
$\mathbf{v}_{to_2} =$ heliocentric velocity vector of the transfer trajectory at arrival
$\mathbf{v}_{lp} =$ heliocentric velocity vector of the launch planet
$\mathbf{v}_{ap} =$ heliocentric velocity vector of the arrival planet

These two $\Delta V's$ are also called the "hyperbolic excess velocity" or V_∞ at launch and arrival. They are the speed of a spacecraft relative to each planet at an infinite distance from the planet. Furthermore, the "specific" orbital energy at launch or arrival is equal to the square of the V_∞ magnitude required for the respective maneuver.

The planetary ephemeris used in this computer program is based on the algorithm described in Chapter 30 of *Astronomical Algorithms* by Jean Meeus. Each planetary orbital element is represented by a cubic polynomial of the form

$$a_0 + a_1 T + a_2 T^2 + a_3 T^3 \tag{2}$$

where the fundamental time argument T is given by

$$T = \frac{JED - 2451545}{36525} \tag{3}$$

In this expression JED is the Julian date.

The software will prompt you for a departure calendar date guess and planet. It will also ask you for an arrival calendar date guess and planet.

The following is a typical draft output created with this software. This example determines the minimum launch delta-v for a typical Earth-to-Mars trajectory.

```
program ipto

< interplanetary trajectory optimization >

optimum launch delta-v

departure planet             Earth
departure calendar date      December 27, 2013
departure universal time     16 h 1 m 11.6099 s
departure julian date        2456654.1675

arrival planet               Mars
arrival calendar date        July 23, 2014
arrival universal time       12 h 4 m 50.3438 s
arrival julian date          2456862.00336

transfer time                207.835864976  days

heliocentric ecliptic orbital elements of the departure planet

semimajor axis (au)                      eccentricity
1.0000010180000003e+00                   1.6702737387977507e-02

inclination (deg)                        argument of perigee (deg)
0.0000000000000000e+00                   1.0317788679466118e+02

raan (deg)                               true anomaly (deg)
0.0000000000000000e+00                   3.5289209966354309e+02

argument of latitude (deg)               period (days)
9.6069986458204269e+01                   3.6525745343845932e+02

heliocentric ecliptic orbital elements of the arrival planet

semimajor axis (au)                      eccentricity
1.5236793419999992e+00                   9.3413790047661649e-02

inclination (deg)                        argument of perigee (deg)
1.8496387818372448e+00                   2.8665775095985157e+02

raan (deg)                               true anomaly (deg)
4.9670488072920676e+01                   2.7500938466056078e+02

argument of latitude (deg)               period (days)
2.0166713562041235e+02                   6.8697161513746346e+02

heliocentric ecliptic orbital elements of the transfer orbit

semimajor axis (au)                      eccentricity
```

```
1.2552637948328016e+00                    2.1676799417023607e-01

inclination (deg)                         argument of perigee (deg)
1.6310348747715806e+00                    1.7688026979693439e+02

raan (deg)                                true anomaly (deg)
2.7606998645820431e+02                    3.1197302030655520e+00

argument of latitude (deg)                period (days)
1.7999999999999994e+02                    5.1368978298110824e+02


departure delta-v and energy requirements

x-component of delta-v          -2.85467590e+00 km/sec
y-component of delta-v           8.14167089e-02 km/sec
z-component of delta-v          -9.42869457e-01 km/sec

total delta-v                    3.00745839e+00 km/sec

energy                           9.04480599e+00 km^2/sec^2


arrival delta-v and energy requirements

x-component of delta-v           3.96702288e+00 km/sec
y-component of delta-v           3.35286669e+00 km/sec
z-component of delta-v          -1.24214793e+00 km/sec

total delta-v                    5.34059146e+00 km/sec

energy                           2.85219171e+01 km^2/sec^2


mission total delta-v            8.34804985e+00 km/sec

mission total energy             3.75667231e+01 km^2/sec^2
```

The software will also allow you to create a two-dimensional plot of the planet orbits and transfer trajectory. If you elect this program option, you will be asked for a plot step size in days. A value between 5 and 10 days should be adequate for planets of the inner solar system. Larger step sizes can be input for the outer planets. The graphics display is a view from the north ecliptic pole looking down upon the ecliptic plane. For best results and proper perspective the user can manually adjust the width and height of the plot to be identical. The following is the companion graphics display for this example.

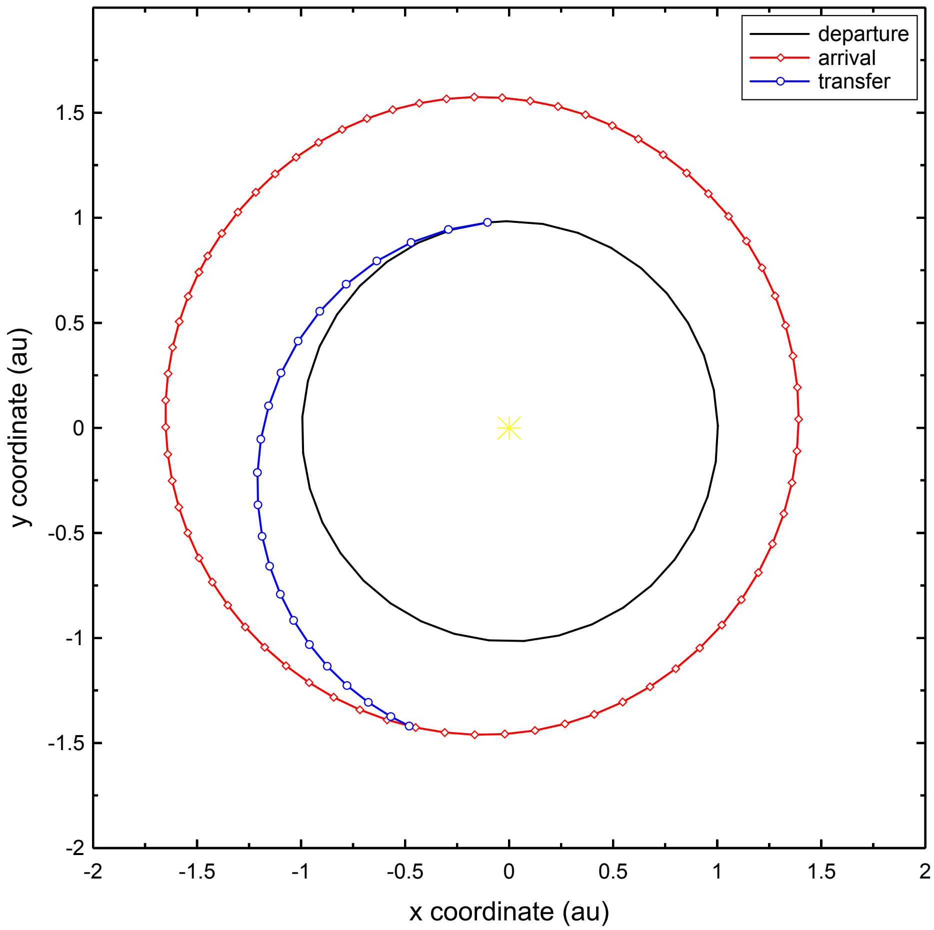

Figure 1. Patched-Conic Interplanetary Trajectory