Patched-Conic Interplanetary Trajectories

This ***Numerit*** application (`lambert2`) can be used to determine the characteristics of patched-conic transfer trajectories between any two planets of our solar system.. This type of trajectory ignores the gravitational effect of both the launch and arrivals planets on the trajectory. The solution is based on solving Lambert's problem relative to the Sun (heliocentric). Patched-conic trajectories are suitable for preliminary mission design.

The ΔV'*s* required at launch and arrival are simply the differences between the velocity on the transfer trajectory determined by the solution of Lambert's problem and the velocities of the two planets. If we treat each planet as a point mass and assume impulsive ΔV'*s*, the magnitude and direction of the required maneuvers are given by the two vector equations

$$\begin{aligned} \Delta\mathbf{v}_1 &= \mathbf{v}_{to_1} - \mathbf{v}_{lp} \\ \Delta\mathbf{v}_2 &= \mathbf{v}_{to_2} - \mathbf{v}_{ap} \end{aligned} \qquad (1)$$

where

$\mathbf{v}_{to_1} =$ heliocentric velocity vector of the transfer trajectory at launch
$\mathbf{v}_{to_2} =$ heliocentric velocity vector of the transfer trajectory at arrival
$\mathbf{v}_{lp} =$ heliocentric velocity vector of the launch planet
$\mathbf{v}_{ap} =$ heliocentric velocity vector of the arrival planet

These two ΔV'*s* are also called the "hyperbolic excess velocity" or V_∞ at launch and arrival. They are the speed of a spacecraft relative to each planet at an infinite distance from the planet. Furthermore, the "specific" orbital energy at launch or arrival is equal to the square of the V_∞ magnitude required for the respective maneuver.

The planetary ephemeris used in this computer program is based on the algorithm described in Chapter 30 of *Astronomical Algorithms* by Jean Meeus. Each planetary orbital element is represented by a cubic polynomial of the form

$$a_0 + a_1 T + a_2 T^2 + a_3 T^3 \qquad (2)$$

where the fundamental time argument T is given by

$$T = \frac{JED - 2451545}{36525} \qquad (3)$$

In this expression *JED* is the Julian date.

The software will prompt you for the departure calendar date, universal time and planet. It will also ask you for the arrival calendar date, universal time and planet. Each planet is defined by a *body number* with 1 = Mercury, 2 = Venus, 3 = Earth and so forth.

The following is a typical draft output created with this software.

```
program lambert2

< patched-conic interplanetary trajectories >

departure planet              Earth
departure calendar date       September 1, 1998
departure universal time      00 h 00 m 00 s
departure julian date         2451057.5

arrival planet                Mars
arrival calendar date         August 15, 1999
arrival universal time        00 h 00 m 00 s
arrival julian date           2451405.5

transfer time                 348  days

heliocentric ecliptic orbital elements of the departure planet

     sma (au)       eccentricity     inclination (deg)    argper (deg)
   1.000001018     0.0167091810468             0       102.914397518

    raan (deg)    true anomaly (deg)   arglat (deg)      period (years)
            0     235.460503804     338.374901322       1.0000204064

heliocentric ecliptic orbital elements of the arrival planet

     sma (au)       eccentricity     inclination (deg)    argper (deg)
   1.523679342     0.0934002744169    1.84972829559       286.498058394

    raan (deg)    true anomaly (deg)   arglat (deg)      period (years)
  49.5551441467     297.045526664     223.543585058       1.88082577724

solution            1

heliocentric ecliptic orbital elements of the transfer orbit

     sma (au)       eccentricity     inclination (deg)    argper (deg)
  1.14611304023     0.330645716884    1.40254066573       88.0201907089

    raan (deg)    true anomaly (deg)   arglat (deg)      period (years)
  338.374901322     271.979809291              0         1.22701361

delta-v and energy requirements

x-component of delta-v         -8.5638364 km/sec
y-component of delta-v          3.7184544 km/sec
z-component of delta-v         0.72979753 km/sec

total delta-v                   9.3647638 km/sec

energy                          87.698801 km^2/sec^2
```

The software will also allow you to create a two-dimensional plot of the planet orbits and transfer trajectory. If you elect this program option, you will be asked for a plot step size in days. A value between 5 and 10 days should be adequate for planets of the inner solar system. Larger step sizes can be input for the outer planets. The graphics display is a view from the north ecliptic pole looking down upon the ecliptic plane. For best results and proper perspective the user can manually adjust the width and height of the plot to be identical. The following is the companion graphics display for this example.

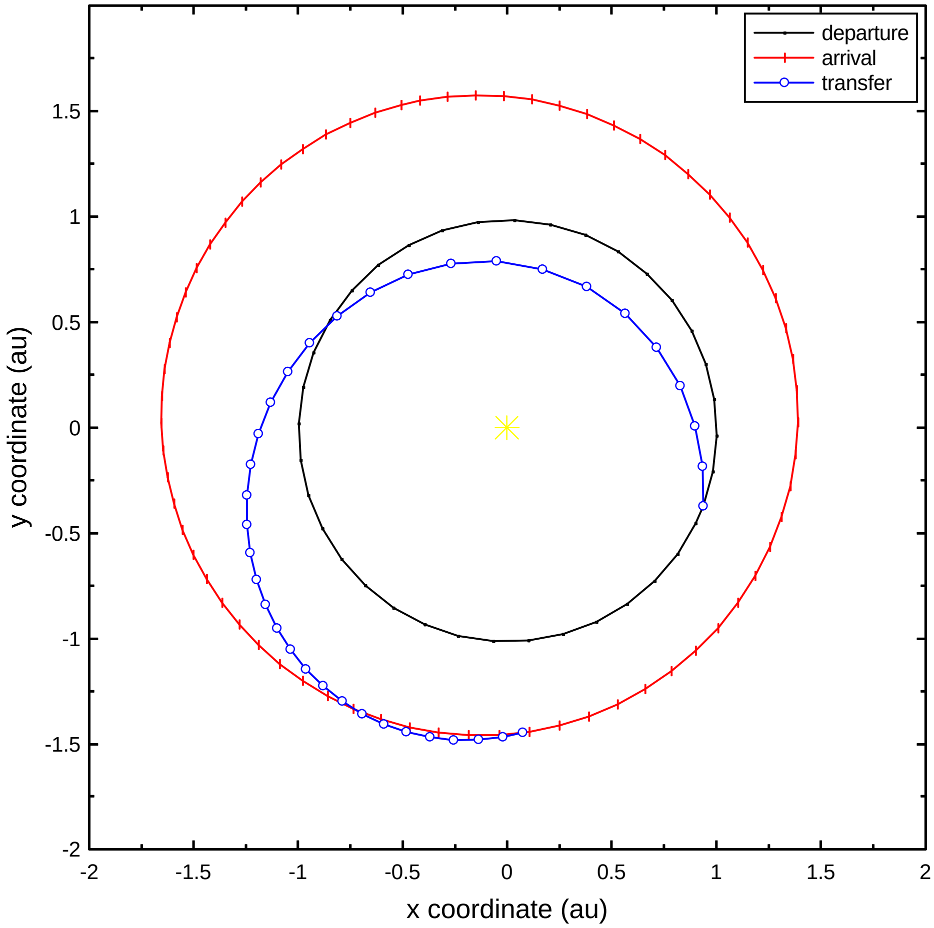

Figure 1. Patched-Conic Interplanetary Trajectory