Perturbed Earth Orbit Lambert Problem

This ***Numerit*** program (`lambert3`) can be used to solve the J_2 perturbed form of Lambert's problem for spacecraft in Earth orbit. Lambert's problem is concerned with the determination of an orbit which passes between two orbital positions within a specified time-of-flight. This classic astrodynamic problem is also known as the orbital *two-point boundary value problem* (TPBVP). More sophisticated equations of motion that account for other types of perturbations can easily be implemented using this software.

At each iteration the initial delta-velocity update is given by

$$\Delta \boldsymbol{v} = \left[\Phi_{12}\right]^{-1} \Delta \boldsymbol{r} \tag{1}$$

where the error in the solution is determined from the difference between the two body final position vector $\boldsymbol{r}_{tb}$ and the final position vector predicted by numerical integration $\boldsymbol{r}_{int}$ of the equations of orbital motion as follows:

$$\Delta \boldsymbol{r} = \boldsymbol{r}_{tb} - \boldsymbol{r}_{int} \tag{2}$$

The new initial velocity can be calculated from

$$\boldsymbol{v}_{n+1} = \boldsymbol{v}_n + \Delta \boldsymbol{v}_n \tag{3}$$

The sub-matrix Φ_{12} of the full state transition matrix is as follows:

$$\Phi_{12} = \left[\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{r}_0}\right] = \begin{bmatrix} \partial x/\partial \dot{x}_0 & \partial x/\partial \dot{y}_0 & \partial x/\partial \dot{z}_0 \\ \partial y/\partial \dot{x}_0 & \partial y/\partial \dot{y}_0 & \partial y/\partial \dot{z}_0 \\ \partial z/\partial \dot{x}_0 & \partial z/\partial \dot{y}_0 & \partial z/\partial \dot{z}_0 \end{bmatrix} \tag{4}$$

This sub-matrix consists of the partial derivatives of the rectangular components of the final position vector with respect to the initial velocity vector.

This iterative *shooting* method is initialized with the Keplerian Lambert solution which is described in the ***Numerit*** document "Earth Orbit Lambert Problem". The algorithm will terminate if either the scalar magnitude of the final position error $\Delta \boldsymbol{r}$ is less than 1.0e-7 or the number of iterations exceeds 10.

The software defaults to direct transfer orbits (`direct = 1`) and zero complete revolutions (`revmax = 0`). This can be changed by the user to find other possible transfer orbit solutions.

The software will ask for the classical orbital elements of the initial and final orbits. It will also ask you to input the transfer time in hours. The computer program will display both

the Keplerian and perturbed solutions. These solutions include the classical orbital elements of the transfer orbit and the initial delta-v required.
The following is a typical draft output created with this software.

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program lambert3

< perturbed Earth orbit Lambert problem >

orbital elements of the initial orbit

     sma (km)         eccentricity    inclination (deg)    argper (deg)
          8000                 0           28.5                    0

    raan (deg)     true anomaly (deg)   arglat (deg)       period (min)
           100                30               30        118.6846843

orbital elements of the Keplerian transfer orbit

     sma (km)         eccentricity    inclination (deg)    argper (deg)
   8936.3507741      0.23038055247    53.115954015         92.835325025

    raan (deg)     true anomaly (deg)   arglat (deg)       period (min)
   116.27932412      284.51896871      17.354293736        140.11996165

orbital elements of the perturbed transfer orbit

     sma (km)         eccentricity    inclination (deg)    argper (deg)
   8941.7153868      0.23062587125    52.930132319         92.764342048

    raan (deg)     true anomaly (deg)   arglat (deg)       period (min)
   116.20653681      284.63373286      17.398074911        140.24615446

Keplerian delta-v requirements

x-component of delta-v       2799.9689068 meters/second
y-component of delta-v       -563.01052291 meters/second
z-component of delta-v       2260.2729081 meters/second

total delta-v                3642.2026779 meters/second

perturbed delta-v requirements

x-component of delta-v       2781.8920941 meters/second
y-component of delta-v       -572.99697456 meters/second
z-component of delta-v       2246.6518045 meters/second

total delta-v                3621.4214732 meters/second

transfer time                          1 hours
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