

Low Thrust Orbit Transfer Between Circular Earth Orbits

This *Numerit* program (ltoct) determines the characteristics of low-thrust orbital transfer between two coplanar or inclined circular orbits. The numerical method used in this script is described in Chapter 14 of the book *Orbital Mechanics* by V. Chobotov and the technical paper "The Reformulation of Edelbaum's Low-thrust Transfer Problem Using Optimal Control Theory" by J. A. Kechichian, AIAA-92-4576-CP. This algorithm is valid for total inclination changes Δi given by $0 < \Delta i < 114.6^\circ$. This computer program assumes that the thrust acceleration magnitude is constant during the orbit transfer.

The initial thrust vector yaw angle \mathbf{b}_0 is given by the following expression

$$\tan \mathbf{b}_0 = \frac{\sin\left(\frac{\mathbf{p}}{2} \Delta i\right)}{\frac{V_0}{V_f} - \cos\left(\frac{\mathbf{p}}{2} \Delta i\right)} \quad (1)$$

where the speed on the initial circular orbit is $V_0 = \sqrt{\mathbf{m}/r_0}$ and the speed on the final circular orbit is $V_f = \sqrt{\mathbf{m}/r_f}$. In these equations $r_0 = r_{eq} + h_0$ is the geocentric radius of the initial orbit, $r_f = r_{eq} + h_f$ is the geocentric radius of the final orbit, r_{eq} is the radius of the Earth and \mathbf{m} is the gravitational constant of the Earth. The initial altitude is h_0 and the final altitude is h_f .

The total velocity change required for a low-thrust orbit transfer is given by

$$\Delta V = V_0 \cos \mathbf{b}_0 - \frac{V_0 \sin \mathbf{b}_0}{\tan\left(\frac{\mathbf{p}}{2} \Delta i + \mathbf{b}_0\right)} \quad (2)$$

The total transfer time is given by $t = \Delta V/f$ where f is the thrust acceleration.

The time evolution of the yaw angle, speed and inclination change are given by the following three expressions:

$$\begin{aligned} \mathbf{b}(t) &= \tan^{-1}\left(\frac{V_0 \sin \mathbf{b}_0}{V_0 \cos \mathbf{b}_0 - f t}\right) \\ V(t) &= \sqrt{V_0^2 - 2V_0 f t \cos \mathbf{b}_0 + f^2 t^2} \\ \Delta i(t) &= \frac{2}{\mathbf{p}} \left[\tan^{-1}\left(\frac{f t V_0 \cos \mathbf{b}_0}{V_0 \sin \mathbf{b}_0}\right) + \frac{\mathbf{p}}{2} - \mathbf{b}_0 \right] \end{aligned} \quad (3)$$

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The software will prompt you for the initial and final circular orbit altitudes, orbital inclinations, and the thrust acceleration. The program prompts also specify the proper units for each user input.

The following is a typical draft display created with this program. It illustrates an orbital transfer from a low Earth orbit (LEO) with an inclination of 28.5° to a geosynchronous Earth orbit (GSO) with an orbital inclination of 0° . The thrust acceleration for this example is $3.5\text{E-}7$ kilometers/second².

```
program ltot
< low-thrust orbit transfer >

initial orbit altitude      621.86 kilometers
initial orbit inclination   28.5 degrees
initial orbit velocity      7546.05384101 meters/second

final orbit altitude       35787.86 kilometers
final orbit inclination    0 degrees
final orbit velocity       3074.59358959 meters/second

total inclination change   28.5 degrees

total delta-v             5783.77506286 meters/second

thrust duration           191.262402872 days

initial yaw angle         21.9849695836 degrees
```

The following plots illustrate the time evolution of the yaw angle, orbital inclination, velocity and semimajor axis during the orbit transfer.

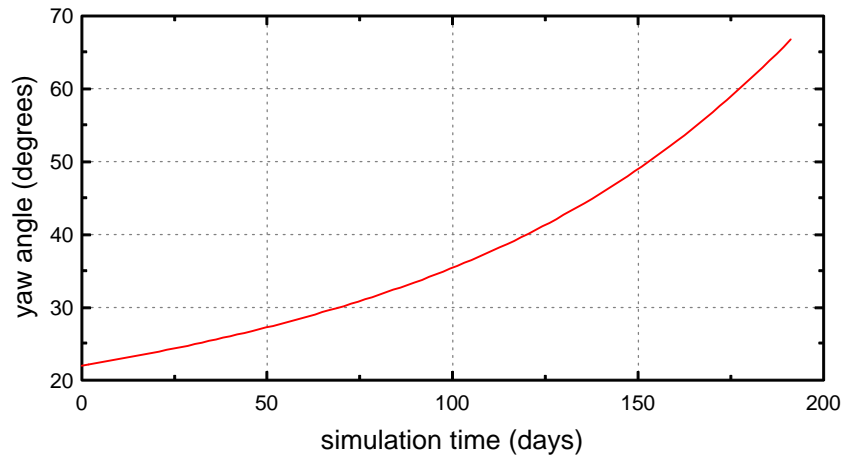


Figure 1. Yaw Angle

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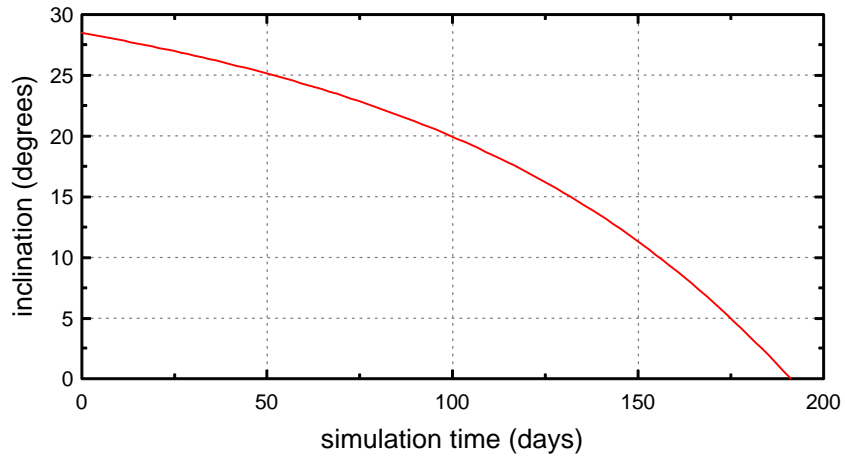


Figure 2. Orbital Inclination

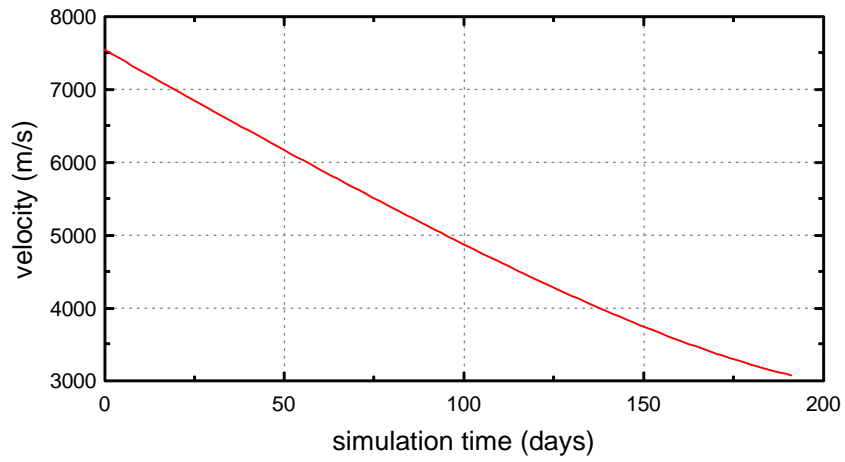


Figure 3. Velocity

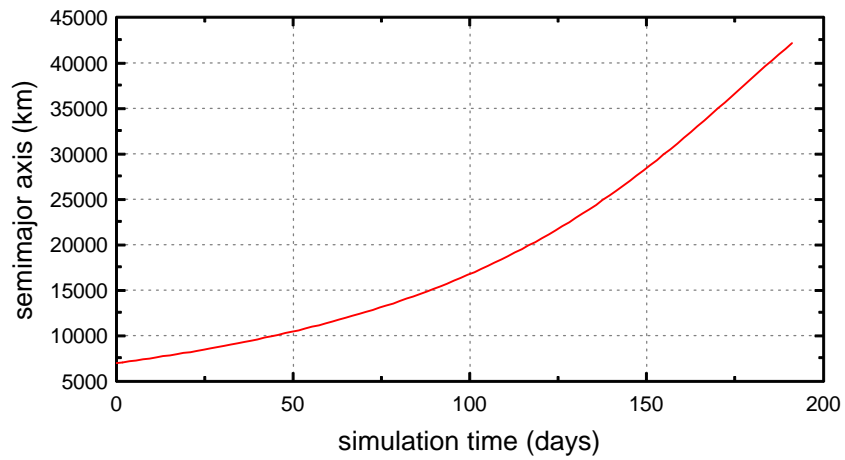


Figure 4. Semimajor Axis