Lunar Trajectory Design and Analysis

This ***Numerit*** program (`lunar`) can be used to design and analyze transfer trajectories from the Earth to the Moon. The software calculates the optimum trans-lunar injection (TLI) impulsive ΔV and park orbit departure conditions. This computer program uses the SLP96 ephemeris to determine the geocentric coordinates of the Moon.

This software optimizes the following aspects of the park orbit

- Julian date of the TLI
- park orbit right ascension of the ascending node
- park orbit true anomaly at TLI

while solving the two body Lambert problem between the Earth and Moon.

The user must provide the following inputs:

- transfer time from the park orbit to the Moon
- initial guess for the calendar date of the TLI
- orbital elements of the park orbit

The user must also provide targeting conditions to a point on a Moon-centered or *selenocentric* sphere. This information consists of the radius of the sphere and the declination and right ascension of a point on this sphere. These angular coordinates are specified with respect to an Earth-centered inertial or ECI coordinate system.

The target position vector is calculated from the radius of the target sphere r_t and angular coordinates $\boldsymbol{a}_t$ and $\boldsymbol{d}_t$ according to

$$\boldsymbol{r}_t = \begin{Bmatrix} r_{t_x} \\ r_{t_y} \\ r_{t_z} \end{Bmatrix} = r_t \begin{Bmatrix} \cos\boldsymbol{d}_t \cos\boldsymbol{a}_t \\ \cos\boldsymbol{d}_t \sin\boldsymbol{a}_t \\ \sin\boldsymbol{d}_t \end{Bmatrix} \tag{1}$$

The final geocentric position vector of the transfer trajectory used during the Lambert solution is given by

$$\boldsymbol{r}_f = \boldsymbol{r}_{e-m} + \boldsymbol{r}_t \tag{2}$$

where $\boldsymbol{r}_{e-m}$ is the geocentric inertial position vector of the Moon.

The following is a typical draft display created with this software. For this example the radius of the targeting sphere is 10000 kilometers and the right ascension and declination of the point on the sphere is 180 and 0 degrees, respectively.

```
program lunar

< lunar trajectory analysis >

departure calendar date       January 1, 2002

departure universal time      00 h 15 m 44.0632 s

orbital elements of the park orbit at the TLI

     sma (km)         eccentricity    inclination (deg)    argper (deg)
         8000                0          28.5                     0

    raan (deg)     true anomaly (deg)   arglat (deg)       period (min)
   8.214788308      345.7124265         345.7124265         118.6846843

orbital elements of the transfer orbit at the TLI

     sma (km)         eccentricity    inclination (deg)    argper (deg)
   246442.2186       0.9675380297    28.49733269         345.7183113

    raan (deg)     true anomaly (deg)   arglat (deg)       period (min)
   8.216212011      359.9928639         345.7111753         20292.35088

TLI delta-v and transfer time

x-component of delta-v       347.7712242
y-component of delta-v       2496.401312
z-component of delta-v       1314.006432

total delta-v                2842.459716 meters/second
total delta-v                9325.655536 feet/second

transfer time                        72 hours
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Although this computer program solves the lunar Lambert problem in a two-body or Keplerian sense, the user can graphically examine the orbital motion subject to the perturbations of an oblate Earth and the point mass gravity of the Moon.

To create the graphics information, the software numerically integrates the spacecraft equations of motion due to the oblateness of the Earth and the point-mass gravity of the Moon. The acceleration vector of the Moon represented by a point mass is given by

$$\boldsymbol{a}_m(r,t) = -m_m\left(\frac{\boldsymbol{r}_{m-s}}{\left|\boldsymbol{r}_{m-s}\right|^3} + \frac{\boldsymbol{r}_{e-m}}{\left|\boldsymbol{r}_{e-m}\right|^3}\right) \tag{3}$$

where

$m_m =$ gravitational constant of the Moon
$\boldsymbol{r}_{m-s} =$ position vector from the Moon to the satellite
$\boldsymbol{r}_{e-m} =$ position vector from the Earth to the Moon

These position vectors are related by $\boldsymbol{r}_{m-s} = \boldsymbol{r}_{e-s} - \boldsymbol{r}_{e-m}$ where $\boldsymbol{r}_{e-s}$ is the geocentric inertial (ECI) position vector of the satellite.

Prior to creating the graphics data, the software will prompt you for the "graphics simulation duration" in hours. In order to view the "real world" effects on the lunar transfer trajectory, this number should be larger than the transfer time.

The following is an Earth-centered inertial (ECI) graphics display of the lunar transfer trajectory. In this view we are looking down on a plane parallel to the Earth's equator from the Earth's north pole or spin axis. The small circle at the lower right is the park orbit and the large circle at the left is the Moon's sphere-of-influence (SOI). The radius of the lunar SOI is 64,000 kilometers. The motion of the Moon during the transfer is the orbital arc which begins in the middle of the display. The location of the Moon is marked at 4 hour increments and the distance units are in Earth radii (6378.14 kilometers).

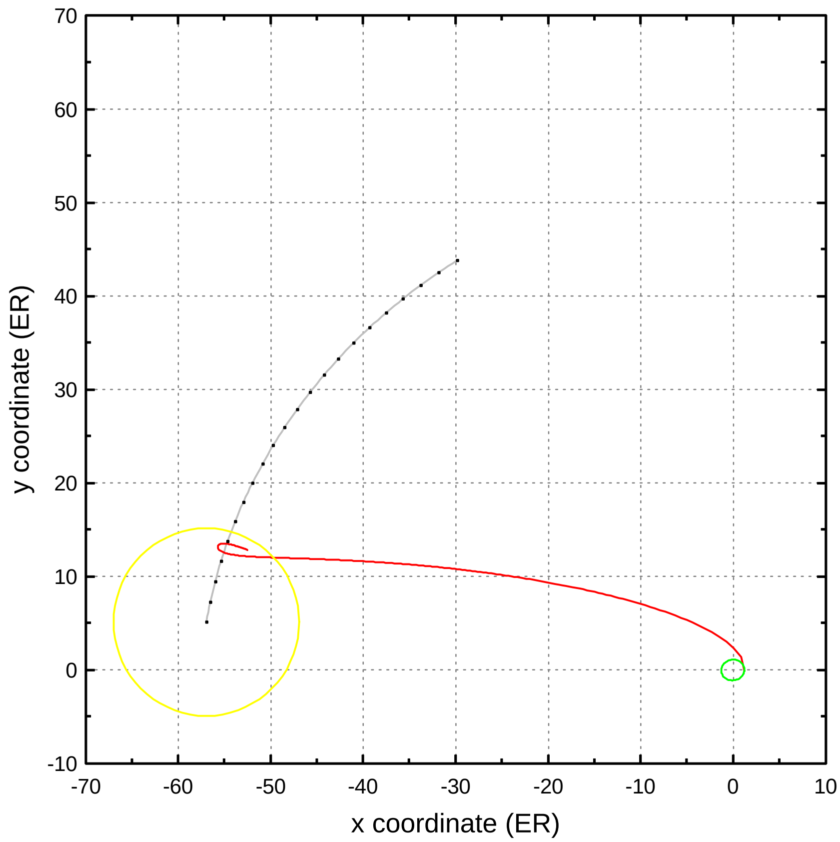

Figure 1. Geocentric Trajectory

This next picture illustrates the selenocentric motion of the spacecraft within the Moon's SOI. The star symbol indicates entrance into the SOI, the small circle is the Moon and the large circle is the lunar SOI. The distance units in this graph are kilometers. This view is also from the north pole and the *x*-*y* plane is parallel to the Earth's equator. From this view we can see that the trajectory of the spacecraft relative to the Moon is a hyperbola.

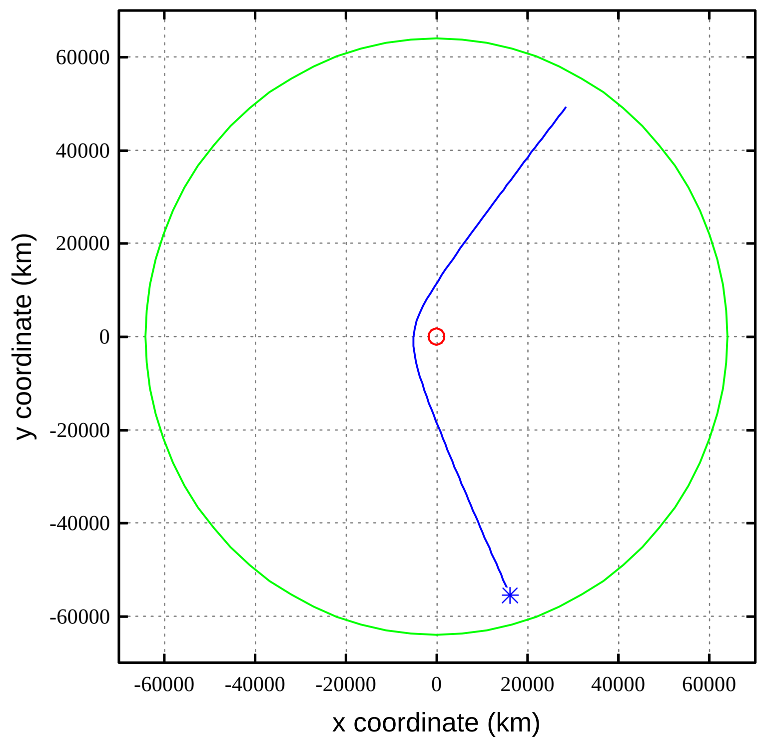

Figure 2. Selenocentric Trajectory

After the graphics are complete, the software will calculate and display the selenocentric orbital elements of the spacecraft's orbit. The fundamental plane for these elements is parallel to the Earth's equator.

The following is the draft display for this example.

```
lunar orbit

     sma (km)          eccentricity    inclination (deg)    argper (deg)
   -4558.739511        2.270453526    154.4666221          359.6557977

    raan (deg)      true anomaly (deg)   arglat (deg)        period (min)
   176.1461465       252.0273            251.667825                   0
```

The fact that the orbital eccentricity is greater than one confirms that the selenocentric trajectory is a hyperbola. Since a hyperbola is not a "closed" orbit there is no orbital period.