Circular Orbit Plane Change Maneuver

This ***Numerit*** program (`maneuvr1`) can be used to calculate *impulsive* orbital maneuvers that modify the orbital inclination and right ascension of the ascending node (RAAN) of <u>circular</u> Earth orbits.

The following diagram illustrates the geometry of this type of maneuver.

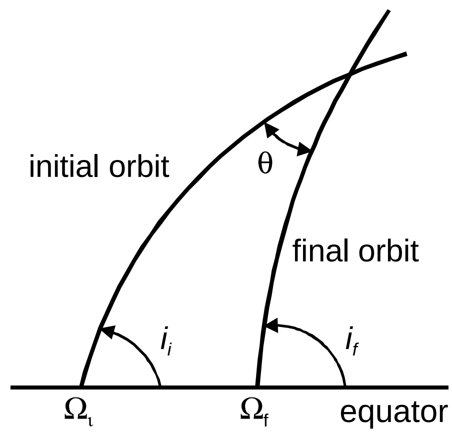

Figure 1. Orbital Geometry

In this picture the orbital inclinations of the initial and final orbits are i_i and i_f, respectively. The RAAN of the initial orbit is Ω_i and Ω_f is the RAAN of the final orbit. The right ascension of the ascending node of an orbit is measured from the inertial *x*-axis along the equator in the direction of the Earth's rotation. From spherical trigonometry relationships q is the angle between the two orbit planes.

The next diagram illustrates the possible points of intersection. From this *ground track* schematic we can see that there are two sets of pairs of orbit intersections on both the initial and final orbits which depend on the relative RAAN between these two orbits.

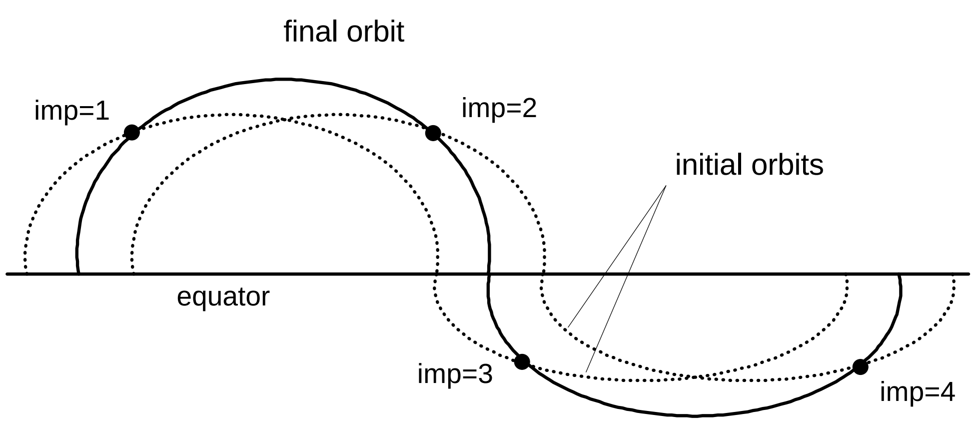

Figure 2. Orbital Intersections

The total plane change angle due to the modification of inclination and RAAN can be expressed as

$$q = \cos^{-1}\left[\sin i_i \sin i_f \cos\left(\Omega_f - \Omega_i\right) + \cos i_i \cos i_f\right] \tag{1}$$

We can define an index *imp* which depends on the *sign* of the right ascension of the ascending node change $\Delta\Omega = \Omega_f - \Omega_i$ as follows:

If $\Delta\Omega > 0$ then *imp* = 1 and 3

or

If $\Delta\Omega < 0$ then *imp* = 2 and 4.

It is convenient to define the location of impulses by their argument of latitude. The argument of latitude is the angle from the ascending node, measured along the orbital plane, to the point of interest. The argument of latitude is equal to the sum of the argument of perigee and true anomaly. Since for circular orbits there is no argument of perigee, argument of latitude and true anomaly are identical.

The two possible arguments of latitude on the initial orbit depend on the values of *imp* according to the following expression:

$$u_i = \text{integer}(imp/2)\,p - (-1)^{imp}\,u \tag{2}$$

where u is the impulse argument of latitude on the initial orbit which is given by

$$u = \cos^{-1}\left(\frac{\cos i_i \sin i_f \cos\Delta\Omega}{\sin i_i \sin q}\right) \tag{3}$$

We can determine the argument of latitude of an impulse on the final orbit by forming the unit position vectors from the ascending node to the impulse. The first argument of latitude on the final orbit is given by

$$u = \cos^{-1}\left(\boldsymbol{U}_1 \bullet \boldsymbol{U}_2\right) \tag{4}$$

where $\boldsymbol{U}_1$ is the unit position vector of the impulse on the initial orbit and $\boldsymbol{U}_2$ is the unit position vector to the ascending node of the final orbit. The argument of latitude of the second impulse opportunity on the final orbit is equal to 180 degrees plus this value.

The maneuver $\Delta\boldsymbol{V}$ vector is given by the vector difference between the velocity vectors of the initial and final orbits as follows:

$$\Delta\boldsymbol{V} = \boldsymbol{V}_i - \boldsymbol{V}_f \tag{5}$$

These velocity vectors are evaluated at the points of orbital intersection. The scalar magnitude of the impulsive maneuver is determined from the components of this vector according to

$$\Delta V = \sqrt{\Delta V_x^2 + \Delta V_y^2 + \Delta V_z^2} \tag{6}$$

For the case where there is no RAAN change, the two impulse locations occur at the ascending and descending nodes of both the initial and final orbits. The arguments of latitude of these two orbital points are 0 and 180 degrees, respectively.

The software will prompt you for the (common) altitude, inclination and right ascension of the ascending node of both the initial and final orbits.

The following is a typical draft output created with this program.

```
program maneuvr1

< one impulse transfer between circular orbits >

circular orbit altitude      185  kilometers

initial inclination          28.5  degrees
initial RAAN                 100  degrees

final inclination            45  degrees
final RAAN                   120  degrees

solution # 1

initial orbit true anomaly   44.44926982  degrees
final orbit true anomaly     28.19997057  degrees
delta-V required             2733.788177  meters/second

solution # 2

initial orbit true anomaly   224.4492698  degrees
final orbit true anomaly     208.1999706  degrees
delta-V required             2733.788177  meters/second
```