Finite Burn Orbit Transfer

This ***Numerit*** program (`maneuvr3`) simulates a single finite burn for orbit transfer between two coplanar and coapsidal elliptical orbits. The user can choose from three types of thrust vector "steering" during the maneuver. This software assumes a constant thrust level and propellant flow rate. The single finite burn maneuver occurs at the mutual perigee of the initial and final orbits, and the burn is "centered" about perigee.

Tangential steering

For this type of steering the thrust pointing direction is tangential to the instantaneous radius vector of the spacecraft and in the direction of the orbital motion. This implies that both the pitch and yaw angles are always zero during the finite burn maneuver.

Gravity turn steering

For this type of steering the thrust pointing direction is always aligned with the instantaneous velocity vector of the space vehicle.

Linear pitch angle steering

For this third type of steering the pitch angle of the maneuver changes linearly during the burn. The angular range or arc over which the maneuver occurs is given by

$$a = 2p\left(\frac{t_d}{t}\right) \tag{1}$$

where t is the orbital period of the initial orbit and t_d is the thrust duration of the maneuver. The initial and final pitch angles of the maneuver are given by

$$q_i = -\frac{a}{2} \quad \text{and} \quad q_i = +\frac{a}{2}$$

The pitch angle at any time *t* is determined from the following expression

$$q(t) = q_i - \dot{q}\left(t - t_{ign}\right) \tag{2}$$

where the pitch rate is given by $\dot{q} = \left(q_i - q_f\right)/t_d$ and t_{ign} is the ignition time of the maneuver. In these expressions t_d is the thrust duration of the maneuver.

According to the *ideal rocket equation*, the thrust duration is calculated from:

$$t_d = \frac{g I_{sp} m_p}{T} \tag{3}$$

where I_{sp} is the specific impulse and m_p is the propellant mass required for the maneuver. The propellant mass required for the maneuver is also calculated from the ideal rocket equation. The product $g I_{sp}$ is called the *exhaust velocity*.

The "gravity loss" for a finite burn maneuver is given by:

$$\Delta V_g = \int_{t_{ign}}^{t_{bo}} g \sin g \, dt \tag{4}$$

where g is the scalar acceleration of gravity at the location of the maneuver, g is the instantaneous flight path angle, t_{ign} is the ignition time of the maneuver and t_{bo} is the burnout or termination time of the burn. The gravity loss occurs because the $g \sin g$ term turns the vehicle away from the optimal thrust direction during a finite burn maneuver.

The inertial delta-velocity added to the vehicle by the finite burn is given by

$$\Delta \boldsymbol{V}_i = \int_{t_{ign}}^{t_{bo}} T \, \boldsymbol{u}_{eci} \, dt \tag{5}$$

The total acceleration of the spacecraft is modelled in this software as a combination of "Keplerian" gravity and thrust acceleration according to the following expression:

$$\boldsymbol{a} = \boldsymbol{g} + \left(\frac{T}{m} \right) \boldsymbol{u}_{eci} \tag{6}$$

In this equation T is the thrust level, m is the instantaneous mass of the vehicle, $\boldsymbol{g}$ is the gravitational acceleration vector at the spacecraft's location and $\boldsymbol{u}_{eci}$ is the inertial unit pointing vector along which the thrust is applied. This program numerically integrates this system of nonlinear vector differential equations while accounting for the change in the spacecraft's mass due to propellant expenditure.

The transformation of a unit pointing vector in the *radial-tangential-normal* (rtn) coordinate system centered at the spacecraft, $\boldsymbol{u}_{rtn}$, to an *Earth-centered-inertial* (eci) unit pointing or thrust vector, $\boldsymbol{u}_{eci}$, is given by the following vector-matrix operation:

$$\boldsymbol{u}_{eci} = \begin{bmatrix} -h_x & (\boldsymbol{h} \times \boldsymbol{r})_x & r_x \\ -h_y & (\boldsymbol{h} \times \boldsymbol{r})_y & r_y \\ -h_z & (\boldsymbol{h} \times \boldsymbol{r})_y & r_z \end{bmatrix} \boldsymbol{u}_{rtn} \tag{7}$$

In this matrix $\boldsymbol{h}$ is the angular momentum vector and $\boldsymbol{r}$ is the inertial position vector of the spacecraft. Please note that the *x*, *y* and *z* components of these three matrix elements are unit vectors.

The rtn unit pointing vector at any mission time t is determined from the pitch and yaw angles as follows:

$$\mathbf{u}_{rtn}(t) = \begin{Bmatrix} \sin j\,(t) \cos q(t) \\ \cos j\,(t) \cos q(t) \\ \sin q(t) \end{Bmatrix} \tag{8}$$

For the coplanar orbit transfer simulated in this computer program, the yaw angle or *out-of-plane* angle j is always zero.

The following is a typical draft output created with this software. This display illustrates a finite burn orbit transfer that uses *linear pitch rate steering*.

```
program maneuvr3

< finite burn orbit transfer between elliptical orbits >

initial orbit

perigee altitude       300  kilometers
apogee altitude        300  kilometers
initial mass           4000  kilograms
thrust                 400  newtons
specific impulse       300  seconds
exhaust velocity       2941.995  meters/second
propellant flow rate   0.1359621617  kilograms/second

impulsive maneuver

delta-v                56.78159651  meters/second
thrust duration        562.3715185  seconds
propellant mass        76.46124735  kilograms

finite burn maneuver - linear pitch rate steering

delta-v                56.78158393  meters/second
gravity loss           5.778380649  meters/second

pitch rate             -0.06628393351  degrees/second

final orbit

perigee altitude       299.9788784  kilometers
apogee altitude        496.4739952  kilometers

     sma (km)         eccentricity    inclination (deg)    argper (deg)
   6776.366437       0.01449856045             0         0.05173617726

    raan (deg)     true anomaly (deg)   arglat (deg)       period (min)
            0      18.71117899         18.76291516        92.52413264
```

If this same maneuver was performed using *tangential steering*, the finite burn and final orbit results are as follows:

```
finite burn maneuver - tangential steering

delta-v                55.77901126  meters/second
gravity loss           7.769443023  meters/second
```

```
final orbit

perigee altitude       301.7275718  kilometers
apogee altitude        498.2204744  kilometers

     sma (km)         eccentricity    inclination (deg)    argper (deg)
   6778.114023       0.01449465898             0         0.08059757493

    raan (deg)     true anomaly (deg)   arglat (deg)       period (min)
            0      18.67499424         18.75559181        92.55992712
```

If this same maneuver was performed using *gravity turn steering*, the simulation results are as follows:

```
finite burn maneuver - gravity turn steering

delta-v                55.79309888  meters/second
gravity loss           7.783571337  meters/second

final orbit

perigee altitude       301.7153876  kilometers
apogee altitude        498.2330218  kilometers

     sma (km)         eccentricity    inclination (deg)    argper (deg)
   6778.114205       0.01449648297             0         0.03594816615

    raan (deg)     true anomaly (deg)   arglat (deg)       period (min)
            0      18.71961583          18.755564        92.55993083
```

As a check on the algorithm, let's run the first case with a very high thrust (400,000 newtons) and verify that the finite burn solution approaches the impulsive burn solution.

The results are as follows:

```
finite burn maneuver - linear pitch rate steering

delta-v                56.78159651  meters/second
gravity loss           5.967276374e-06  meters/second

pitch rate             -0.06628393351  degrees/second

final orbit

perigee altitude       300.0000001  kilometers
apogee altitude        499.9999965  kilometers

     sma (km)         eccentricity    inclination (deg)    argper (deg)
   6778.139998       0.01475330964             0         5.277241736e-05

    raan (deg)     true anomaly (deg)   arglat (deg)       period (min)
            0      0.0187218565         0.01877462892       92.56045918
```

Notice how close the perigee and apogee altitudes are to the "targeted" values. Notice also that the gravity loss is very small. For this example the solution is insensitive to the type of steering method.