Orbital Periods of a Satellite

In orbital mechanics we are concerned with the following orbital periods of a satellite:

- *Keplerian* - the unperturbed or two-body period
- *Nodal* - the time interval from one ascending (or descending) node to the next
- *Anomalistic* - the time interval from one perigee to the next
- *Sidereal* - the time interval from one value of argument of latitude to the next identical value

This ***Numerit*** program (`period1`) can be used to estimate these orbital periods. Each algorithm is based on the *osculating* orbital elements of a satellite and includes the perturbation due to J_2.

The Keplerian or unperturbed orbital period of a satellite is given by

$$t_k = 2p\sqrt{\frac{a^3}{m}} \qquad (1)$$

The anomalistic period of a satellite based on osculating orbital elements at perigee is given by

$$t_a = 2p\sqrt{\frac{a^3}{m}}\left\{1 - \frac{3J_2 r_{eq}^2}{2a^2\left(1-e^2\right)^3}\left(1 - 3\sin^2 i \sin^2 w\right)\right\} \qquad (2)$$

The nodal period of a satellite based on osculating orbital elements of the satellite at the ascending node is

$$t_n = 2p\sqrt{\frac{a^3}{m}}\left\{1 - \frac{3J_2\left(4 - 5\sin^2 i\right)}{4\left(\frac{a}{r_{eq}}\right)^2\sqrt{1-e^2}\left(1 + e\cos w\right)^2} - \frac{3J_2\left(1 + e\cos w\right)^3}{2\left(\frac{a}{r_{eq}}\right)^2\left(1-e^2\right)^3}\right\} \qquad (3)$$

The sidereal period of a satellite based on osculating orbital elements at perigee of the orbit is given by

$$t_s = 2p\sqrt{\frac{a^3}{m}}\left\{1 - \frac{3}{2}J_2\left(\frac{a\,r_{eq}^2}{r^3}\right)\left(1 - 3\sin^2 i \sin^2 u\right) - \frac{3}{4}J_2\left(\frac{r_{eq}}{a}\right)^2\frac{1}{\sqrt{1-e^2}}\left[\frac{4 - 5\sin^2 i}{\left(1 + e\cos u\right)}\right]\right\} \qquad (4)$$

In these equations

$a =$ semimajor axis
$e =$ orbital eccentricity
$i =$ orbital inclination
$w =$ argument of perigee
$u =$ true anomaly
$u =$ argument of latitude $= w + u$
$r =$ geocentric distance
$r_{eq} =$ equatorial radius of the Earth
$m =$ gravitational constant of the Earth
$J_2 =$ oblateness gravity coefficient

The following is a typical draft output created with this program.

```
program period1

< orbital periods - j2 analytic solution >

semimajor axis          8000  kilometers
eccentricity            0.015
inclination             28.5  degrees
argument of perigee     270  degrees
true anomaly            30  degrees

Keplerian period        118.68468  minutes

nodal period            118.38678  minutes

anomalistic period      118.64405  minutes

sidereal period         118.45169  minutes
```