Primer Vector Analysis of Interplanetary Trajectories

This ***Numerit*** program (`primer1`) demonstrates how to use *primer vector theory* to analyze the performance of two impulse interplanetary orbit transfers. The term primer vector was invented by Derek F. Lawden and represents the adjoint vector for velocity. A technical discussion about primer theory can be found in Professor Lawden's classic text, *Optimal Trajectories for Space Navigation*, Butterworths, London, 1963. Another excellent resource is "Primer Vector Theory and Applications", Donald J. Jezewski, NASA TR R-454, November 1975.

As shown by Lawden the following four necessary conditions must be satisfied in order for an impulsive orbital transfer to be optimal:

(1) the primer vector and its first derivative are everywhere continuous

(2) whenever a velocity impulse occurs, the primer vector is aligned with the impulse and has unit magnitude

(3) the magnitude of the primer vector may not exceed unity on a coasting arc

(4) the time derivative of the magnitude of the primer vector is zero at all interior junction points (intermediate impulse) separating coasting arcs

Let's begin our discussion by partitioning the state transition matrix as follows:

$$\Phi(t, t_0) = \begin{bmatrix} \dfrac{\partial \boldsymbol{r}}{\partial \boldsymbol{r}_0} & \dfrac{\partial \boldsymbol{r}}{\partial \mathbf{v}_0} \\ \dfrac{\partial \mathbf{v}}{\partial \boldsymbol{r}_0} & \dfrac{\partial \mathbf{v}}{\partial \mathbf{v}_0} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \tag{1}$$

where

$$\Phi_{11} = \left[\frac{\partial \boldsymbol{r}}{\partial \boldsymbol{r}_0} \right] = \begin{bmatrix} \partial x / \partial x_0 & \partial x / \partial y_0 & \partial x / \partial z_0 \\ \partial y / \partial x_0 & \partial y / \partial y_0 & \partial y / \partial z_0 \\ \partial z / \partial x_0 & \partial z / \partial y_0 & \partial z / \partial z_0 \end{bmatrix}$$

and so forth.

The value of the primer vector at any point along a two body trajectory is given by

$$\boldsymbol{p}(t) = \Phi_{11}(t, t_0)\boldsymbol{p}_0 + \Phi_{11}(t, t_0)\dot{\boldsymbol{p}}_0 \tag{2}$$

and the value of the primer vector derivative at any elapsed time t is

$$\dot{\boldsymbol{p}}(t) = \Phi_{21}(t, t_0)\boldsymbol{p}_0 + \Phi_{22}(t, t_0)\dot{\boldsymbol{p}}_0 \tag{3}$$

The primer vector boundary conditions at the initial and final impulses are as follows:

$$\boldsymbol{p}(t_0) = \boldsymbol{p}_0 = \frac{\Delta \mathbf{v}_0}{|\Delta \mathbf{v}_0|}$$
$$\boldsymbol{p}(t_f) = \boldsymbol{p}_f = \frac{\Delta \mathbf{v}_f}{|\Delta \mathbf{v}_f|} \tag{4}$$

These two conditions illustrate that at the locations of velocity impulses, the primer vector is a unit vector in the direction of the impulses.

Finally, the value of the primer vector derivative at the initial time is

$$\dot{\boldsymbol{p}}(t_0) = \dot{\boldsymbol{p}}_0 = \Phi_{12}^{-1}(t, t_0)\left\{\boldsymbol{p}_f - \Phi_{11}(t, t_0)\boldsymbol{p}_0\right\} \tag{5}$$

The software will prompt you for a departure calendar date guess and planet. It will also ask you for an arrival calendar date guess and planet. Each planet is defined by a *body number* with 1 = Mercury, 2 = Venus, 3 = Earth and so forth. The user can calculate the minimum launch, arrival or total ΔV required for interplanetary missions. A "perform no optimization" option is also available.

The following is a typical draft output created with this software. This example determines the minimum total delta-v for a typical Earth-to-Mars trajectory.

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program primer1

< primer vector analysis of interplanetary trajectories >

optimum launch + arrival delta-v

departure planet              Earth
departure calendar date       September 8, 1990
departure universal time      17 h 42 m 27.0652 s
departure julian date         2448143.23781

arrival planet                Mars
arrival calendar date         April 24, 1991
arrival universal time        1 h 9 m 54.8401 s
arrival julian date           2448370.54855

transfer time                 227.310738136  days

heliocentric ecliptic orbital elements of the departure planet

     sma (au)       eccentricity    inclination (deg)    argper (deg)
   1.000001018       0.01671253405            0          102.7772036

    raan (deg)     true anomaly (deg)   arglat (deg)     period (years)
            0      243.0347351         345.8119387        1.000020406

heliocentric ecliptic orbital elements of the arrival planet

     sma (au)       eccentricity    inclination (deg)    argper (deg)
   1.523679342       0.09339275536    1.84977833         286.4092384
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```
   raan (deg)     true anomaly (deg)   arglat (deg)      period (years)
  49.49098923      159.8403105         86.24954884         1.880825777

heliocentric ecliptic orbital elements of the transfer orbit

    sma (au)        eccentricity     inclination (deg)    argper (deg)
  1.34324983       0.2533459148     3.679673832          348.0499856

   raan (deg)     true anomaly (deg)   arglat (deg)      period (years)
  345.8119387      11.95001441                  0         1.556837882


departure delta-v and energy requirements

x-component of delta-v        2.6436656 km/sec
y-component of delta-v        2.9609367 km/sec
z-component of delta-v        2.1275116 km/sec

total delta-v                 4.5036007 km/sec

energy                         20.28242 km^2/sec^2

arrival delta-v and energy requirements

x-component of delta-v       0.40412791 km/sec
y-component of delta-v        2.3577386 km/sec
z-component of delta-v       -1.1238908 km/sec

total delta-v                 2.6429872 km/sec

energy                        6.9853812 km^2/sec^2

mission total delta-v         7.1465879 km/sec

mission total energy          27.267801 km^2/sec^2
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This application will also plot the behavior of the primer vector magnitude for user-defined patched-conic interplanetary trajectories. The following is the companion graphics display for this example. From Lawden's necessary conditions we can see that this is an optimal two impulse orbital transfer.

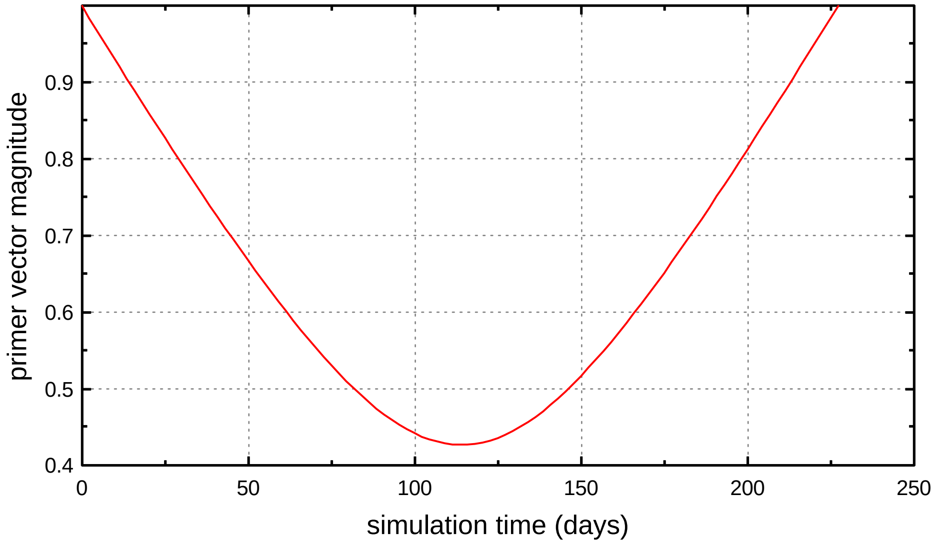

Figure 1. Primer Vector Magnitude