

Time to Repeat a Ground Track

This *Numerit* program (`repeat1`) estimates the time required for a satellite to repeat its ground track. The satellite is propagated using Kozai's algorithm and the user can select a closure tolerance that controls how well the repeat time is predicted. In the discussion which follows all orbital elements are assumed to be Kozai "mean" orbital elements.

The algorithm begins by initializing the Earth-relative longitude of the ascending node and the total number of days to repeat according to

$$\begin{aligned} I_{an} &= 0 \\ \text{ndays} &= 0 \end{aligned} \tag{1}$$

The nodal period of the satellite is computed using the expression

$$t_n = \frac{2p}{(\tilde{n} + \tilde{w})} \tag{2}$$

where \tilde{n} is the "perturbed" mean motion and \tilde{w} is the perturbation of the argument of perigee due to Earth oblateness or J_2 . The nodal period is the time required for a satellite to travel from one ascending (or descending) node crossing to the next ascending (or descending) node crossing.

The delta-longitude at the ascending node per nodal period, sometimes called the *fundamental interval*, is given by

$$\Delta I = t_n (\mathbf{w}_e - \dot{\Omega}) \tag{3}$$

where \mathbf{w}_e is the inertial rotation rate of the Earth and $\dot{\Omega}$ is the perturbation of the right ascension of the ascending node due to Earth oblateness.

The current Earth relative longitude and number of orbits are incremented according to

$$\begin{aligned} I_{i+1} &= I_i + \Delta I \\ \text{norbits} &= \text{norbits} + 1 \end{aligned} \tag{4}$$

After each increment, convergence is checked. If $|I - 2p| \leq \mathbf{e}$ or $|I| \leq \mathbf{e}$ the method has satisfied the user-defined closure tolerance \mathbf{e} . The total number of days to repeat the ground track is determined from $\text{ndays} = \text{norbits} t_n / 86400$.

The software will prompt you for the semimajor axis, eccentricity, inclination and argument of perigee of a satellite's orbit. It will also ask you to input a closure tolerance in degrees. A number between 0.1 and 0.01 degrees is suggested.

Orbital Mechanics with Numerit

The following is a typical draft output created with this program.

```
program repeat1  
  
  < time to repeat ground track - Kozai orbit propagation >  
  
  mean semimajor axis          8000 kilometers  
  mean eccentricity            0  
  mean inclination             28.5 degrees  
  mean argument of perigee    0 degrees  
  mean raan                    100 degrees  
  
  number of orbits to repeat   2075  
  number of solar days to repeat 170.653126271  
  
  Keplerian period             118.684684295 minutes  
  nodal period                 118.429157509 minutes  
  length of nodal day          1420.46616856 minutes  
  fundamental interval         30.0144400807 minutes  
  
  closure tolerance            0.1 degrees  
  actual closure                0.0368324847497 degrees
```