

Repeating Ground Track Orbit Design - Wagner's Method

This *Numerit* program (`repeat3`) calculates the *mean* semimajor axis required for a repeating ground track orbit using an algorithm devised by Carl Wagner. The numerical method is described in "A Prograde Geosat Exact Repeat Mission?", *The Journal of the Astronautical Sciences*, Vol. 39, No. 3, July-September 1991, pp. 313-326. This computer program is valid for both circular and elliptical repeating ground track orbits.

This computer program starts with the following initial guess for the required mean semimajor axis

$$a_0 = \mathbf{m}^{1/3} \left[\left(\frac{R}{D} \right) \mathbf{w}_e \right]^{-2/3} \quad (1)$$

and iteratively improves this guess with the following update:

$$a_{i+1} = \mathbf{m}^{1/3} \left[\left(\frac{R}{D} \right) \mathbf{w}_e \right]^{-2/3} \left[1 - \frac{3}{2} J_2 \left(\frac{r_e}{a_i} \right)^2 \left(1 - \frac{3}{2} \sin^2 i \right) \right]^{2/3} \left[1 + J_2 \left(\frac{r_e}{a_i} \right)^2 \left\{ \frac{3}{2} \left(\frac{R}{D} \right) \cos i - \frac{3}{4} (5 \cos^2 i - 1) \right\} \right]^{2/3} \quad (2)$$

where

- R = integer number of orbits in repeat period
- D = integer number of synodic (or nodal) days
- J_2 = second gravity coefficient
- \mathbf{w}_e = inertial rotation rate of the Earth
- r_e = equatorial radius of the Earth
- i = orbital inclination
- \mathbf{m} = gravitational constant of the Earth

The value of \mathbf{w}_e used in this program is 7.2921151467e-5 radians/second.

The equation for the *nodal day* is given by

$$T_N = \frac{2\mathbf{p}}{\mathbf{w}_e - \dot{\Omega}} \quad (3)$$

where $\dot{\Omega}$ is the perturbation of the right ascension of the ascending node.

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The relationship between the repeat parameters R and D is as follows:

$$\frac{R}{D} = \frac{\tilde{n} + \mathbf{w}}{\mathbf{w}_e - \dot{\Omega}} \quad (4)$$

where \tilde{n} is the perturbed mean motion and \mathbf{w} is the perturbation in argument of perigee.

The following is a typical draft output created with this software.

```
program repeat3
< repeating ground track - Wagner's method >

mean semimajor axis          7192.2310562 kilometers
mean eccentricity            0
mean inclination              108 degrees
mean argument of perigee     0 degrees
mean raan                    0 degrees

number of orbits to repeat    271
number of solar days to repeat 19.0548179339

Keplerian period              101.170790859 minutes
nodal period                  101.250693081 minutes

length of nodal day          1444.15462236 minutes
fundamental interval         25.2398523985 degrees
```