Repeating Ground Track Orbit Design
(Required Osculating Semimajor Axis)

This ***Numerit*** program (`repeat4`) calculates the osculating semimajor axis required for a repeating ground track orbit. The orbit is propagated using numerical integration during the solution process. This version of the software uses first-order equations of motion that include J_2. However, the code can be modified to use equations of motion that include higher order gravity terms and other perturbations such as aerodynamic drag, solar radiation pressure and so forth.

The program begins by setting the initial right ascension of the ascending node (RAAN) to zero and the initial true anomaly to the ascending node $(n = -w)$. The bracketing interval for the required osculating semimajor axis a is set equal to

$$a_0 - 100 \le a \le a_0 + 100 \tag{1}$$

where a_0 is the initial semimajor axis guess provided by the user.

The main nonlinear equation solved by this script is given by

$$f(t) = FI - l_{an} = 0 \tag{2}$$

where l_{an} is the east longitude of the nodal crossing and FI is called the *fundamental interval* which is equal to

$$FI = 2p\,\frac{ndays}{norbits} \tag{3}$$

In this equation *ndays* is the integer number of days in the user-defined repeat cycle and *norbits* is the integer number of orbits in the cycle.

By solving Equation (2), we are trying to minimize the difference between the required fundamental interval and the east longitude change caused by the Earth's rotation for one nodal period. While solving for the root of Equation (2), the algorithm also propagates the orbit to the next ascending node crossing by solving the following equation:

$$g(t) = r_z = 0 \tag{4}$$

subject to the mission constraint $v_z > 0$. The complete process involves a "nested" root-finding numerical technique.

The east longitude of the ascending node at any time t given by

$$l\,(t) = \tan^{-1}\left(r_{y_{ecf}}, r_{x_{ecf}}\right) \tag{5}$$

The Earth-centered-fixed (ECF) components of the position vector are given by

$$r_{x_{ecf}} = r_{x_{eci}} \cos q + r_{y_{eci}} \sin q$$
$$r_{y_{ecf}} = r_{y_{eci}} \cos q - r_{x_{eci}} \sin q \qquad (6)$$

where $q = w_e (t - t_0)$ is the right ascension of the satellite relative to the initial crossing. In these equations $r_{x_{eci}}$ and $r_{y_{eci}}$ are the inertial components of the satellite's position vector.

The following is a typical draft output created by this program.

```
program repeat4

< repeating ground track orbit design - osculating semimajor axis >

semimajor axis                7200.54264407  kilometers
eccentricity                  0
inclination                   108  degrees
argument of perigee           0  degrees
raan                          0  degrees

number of orbits to repeat    271
number of days to repeat      19

Keplerian period              101.346216149 minutes
nodal period                  101.251007402  minutes

fundamental interval          25.2398523985  degrees
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