

Rise and Set of an Earth Satellite (Kozai Orbit Propagation Method)

This *Numerit* application (`riseset1`) can be used to determine rise and set conditions of Earth satellites relative to a ground site or observer on an oblate Earth. The user can specify an minimum elevation or "mask" angle for each simulation. The numerical method uses a combination of one-dimensional minimization and root-finding to calculate visibility conditions. Additional information about this technique can be found in the application "Predicting Orbital Events: Part 2". In this example, the satellite's orbit is propagated using Kozai's method.

The topocentric elevation angle of a satellite relative to an observer or ground site on an oblate Earth is calculated from

$$E = \sin^{-1}\left(r_{z_{topo}}\right) \quad (1)$$

where the components of the topocentric position vector \mathbf{r}_{topo} are determined from the following transformation matrix and the ECI position vector \mathbf{r}_{eci} of the satellite:

$$\mathbf{r}_{topo} = [T]\mathbf{r}_{eci} = \begin{bmatrix} \sin\phi \cos\theta & \sin\phi \sin\theta & -\cos\phi \\ -\sin\theta & \cos\theta & 0 \\ \cos\phi \cos\theta & \cos\phi \sin\theta & \sin\phi \end{bmatrix} \mathbf{r}_{eci} \quad (2)$$

In this transformation ϕ is the geodetic latitude of the ground site and θ is the local sidereal time at the ground site.

The local sidereal time at a ground site is given by

$$\theta(t) = \theta_{g_0} + \omega_e t + \lambda_e \quad (3)$$

where θ_{g_0} is the Greenwich sidereal time at 0 hours universal time, ω_e is the inertial rotation rate of the Earth, t is the elapsed time since 0 hours universal time and λ_e is the east longitude of the ground site.

The ECI position vector used in this transformation is the position of the satellite relative to the observer or ground site. It is determined from the ECI position vectors of the observer \mathbf{r}_{obs} and satellite \mathbf{r}_{sat} according to

$$\mathbf{r}_{eci} = \mathbf{r}_{sat} - \mathbf{r}_{obs} \quad (4)$$

The scalar slant range from the observer to the satellite is computed from the components of this vector according to

$$p = \sqrt{x_{eci}^2 + y_{eci}^2 + z_{eci}^2} \quad (5)$$

The topocentric azimuth angle is calculated from the x and y components of the topocentric position vector using the following expression

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$$A = \tan^{-1}(r_{y_{topo}}, -r_{x_{topo}}) \quad (6)$$

The topocentric elevation angle is calculated from the z component of the topocentric unit position vector with this next expression

$$E = \sin^{-1}(r_{z_{topo}}) \quad (7)$$

Azimuth is measured positive clockwise from north (90° is east, 180° is south, etc.) and elevation is positive above the local horizontal or tangent plane at the observer's geographic location or ground site.

The ECI range-rate vector $\dot{\mathbf{p}}_{eci}$ of the satellite relative to the observer is determined from

$$\dot{\mathbf{p}}_{eci} = \mathbf{v}_{sat} - \boldsymbol{\omega} \times \mathbf{r}_{sat} \quad (8)$$

where $\boldsymbol{\omega} = \omega_e(0, 0, 1)^T$ is the inertial rotation vector of the Earth and \mathbf{v}_{sat} is the ECI velocity vector of the satellite.

The topocentric range-rate vector is computed from the transformation

$$\dot{\mathbf{p}}_{topo} = [T]\dot{\mathbf{p}}_{eci} \quad (9)$$

The derivative of slant range or range-rate is given by

The azimuth and elevation rates are determined from the x , y and z components of the topocentric range and range-rate vectors as follows

$$\begin{aligned} \dot{A} &= \frac{\dot{p}_{topo_x} p_{topo_y} - \dot{p}_{topo_y} p_{topo_x}}{p_x^2 + p_y^2} \\ \dot{E} &= \frac{\dot{p}_{topo_z} - \dot{p} \sin E}{\sqrt{p_x^2 + p_y^2}} \end{aligned} \quad (10)$$

The objective function used to calculate visibility conditions is equal to

$$f(t) = -E + E_{\min} \quad (11)$$

where E_{\min} is a minimum elevation angle constraint or "mask". Notice that this is actually a maximization problem since we are using the negative value of this function.

The program will interactively prompt you for the following information:

- (1) initial calendar date
- (2) initial universal time
- (3) orbital elements of the satellite

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- (4) geographic coordinates of the ground site
- (5) the simulation duration
- (6) minimum elevation angle constraint

The following is a typical draft output created with this software. It illustrates the first few visibility events for this example.

```
program riseset1

< ground site-to-satellite visibility >

rise conditions

calendar date          January 1, 1998
universal time         18 h 34 m 12.8674 s
Julian date            2450815.27376004

topocentric azimuth angle  224.2328 degrees
topocentric elevation angle  4.999998 degrees
topocentric slant range    4303.912 kilometers

maximum elevation conditions

calendar date          January 1, 1998
universal time         18 h 35 m 35.5934 s
Julian date            2450815.27471752

topocentric azimuth angle  216.9662 degrees
topocentric elevation angle  5.242341 degrees
topocentric slant range    4278.779 kilometers

set conditions

calendar date          January 1, 1998
universal time         18 h 36 m 58.3837 s
Julian date            2450815.27567574

topocentric azimuth angle  209.6895 degrees
topocentric elevation angle  4.999998 degrees
topocentric slant range    4301.094 kilometers

event duration         00 h 2 m 45.5162 s

rise conditions

calendar date          January 2, 1998
universal time         7 h 39 m 23.5901 s
Julian date            2450815.81902303

topocentric azimuth angle  179.8097 degrees
topocentric elevation angle  5.000011 degrees
topocentric slant range    4298.664 kilometers
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maximum elevation conditions

calendar date	January 2, 1998
universal time	7 h 45 m 30.3178 s
Julian date	2450815.82326757
topocentric azimuth angle	146.2183 degrees
topocentric elevation angle	10.63634 degrees
topocentric slant range	3792.587 kilometers

set conditions

calendar date	January 2, 1998
universal time	7 h 51 m 36.239 s
Julian date	2450815.82750277
topocentric azimuth angle	112.8029 degrees
topocentric elevation angle	5.000011 degrees
topocentric slant range	4310.008 kilometers
event duration	00 h 12 m 12.6489 s