Rise and Set of a Satellite - SGP4 Method

This ***Numerit*** application (`riseset2`) can be used to determine rise and set conditions of Earth satellites relative to a ground site or observer on an oblate Earth. The user can specify an minimum elevation or "mask" angle for each simulation. The numerical method uses a combination of one-dimensional minimization and root-finding to calculate visibility conditions. Additional information about this technique can be found in the application "Predicting Orbital Events: Part 2". In this example, the satellite's orbit is propagated using the NORAD SGP4 method. The software will read a Two Line Element (TLE) database and extract the orbital elements of a user-specified satellite.

The topocentric elevation angle of a satellite relative to an observer or ground site on an oblate Earth is calculated from

$$E = \sin^{-1}\left(r_{z_{topo}}\right) \tag{1}$$

where the components of the topocentric position vector $\boldsymbol{r}_{topo}$ are determined from the following transformation matrix and the ECI position vector $\boldsymbol{r}_{eci}$ of the satellite:

$$\boldsymbol{r}_{topo} = [T]\boldsymbol{r}_{eci} = \begin{bmatrix} \sin f \cos q & \sin f \sin q & -\cos f \\ -\sin q & \cos q & 0 \\ \cos f \cos q & \cos f \sin q & \sin f \end{bmatrix} \boldsymbol{r}_{eci} \tag{2}$$

In this transformation f is the geodetic latitude of the ground site and q is the local sidereal time at the ground site.

The local sidereal time at a ground site is given by

$$q(t) = q_{g0} + w_e t + l_e \tag{3}$$

where q_{g0} is the Greenwich sidereal time at 0 hours universal time, w_e is the inertial rotation rate of the Earth, t is the elapsed time since 0 hours universal time and l_e is the east longitude of the ground site.

The ECI position vector used in this transformation is the position of the satellite relative to the observer or ground site. It is determined from the ECI position vectors of the observer $\boldsymbol{r}_{obs}$ and satellite $\boldsymbol{r}_{sat}$ according to

$$\boldsymbol{r}_{eci} = \boldsymbol{r}_{sat} - \boldsymbol{r}_{obs} \tag{4}$$

The scalar slant range from the observer to the satellite is computed from the components of this vector according to

$$p = \sqrt{x_{eci}^2 + y_{eci}^2 + z_{eci}^2} \tag{5}$$

The topocentric azimuth angle is calculated from the x and y components of the topocentric position vector using the following expression

$$A = \tan^{-1}\left(r_{y_{topo}},\ -r_{x_{topo}}\right) \tag{6}$$

The topocentric elevation angle is calculated from the z component of the topocentric unit position vector with this next expression

$$E = \sin^{-1}\left(r_{z_{topo}}\right) \tag{7}$$

Azimuth is measured positive clockwise from north (90° is east, 180° is south, etc.) and elevation is positive above the local horizontal or tangent plane at the observer's geographic location or ground site.

The ECI range-rate vector $\dot{\boldsymbol{p}}_{eci}$ of the satellite relative to the observer is determined from

$$\dot{\boldsymbol{p}}_{eci} = \boldsymbol{v}_{sat} - \boldsymbol{w} \times \boldsymbol{r}_{sat} \tag{8}$$

where $\boldsymbol{w} = w_e(0, 0, 1)^T$ is the inertial rotation vector of the Earth and $\boldsymbol{v}_{sat}$ is the ECI velocity vector of the satellite.

The topocentric range-rate vector is computed from the transformation

$$\dot{\boldsymbol{p}}_{topo} = [T]\dot{\boldsymbol{p}}_{eci} \tag{9}$$

The derivative of slant range or range-rate is given by

The azimuth and elevation rates are determined from the x, y and z components of the topocentric range and range-rate vectors as follows

$$\dot{A} = \frac{\dot{p}_{topo_x} p_{topo_y} - \dot{p}_{topo_y} p_{topo_x}}{p_x^2 + p_y^2}$$

$$\dot{E} = \frac{\dot{p}_{topo_z} - \dot{p}\sin E}{\sqrt{p_x^2 + p_y^2}} \tag{10}$$

The objective function used to calculate visibility conditions is equal to

$$f(t) = -E + E_{\min} \tag{11}$$

where $E_{\min}$ is a minimum elevation angle constraint or "mask". Notice that this is actually a maximization problem since we are using the *negative* value of this function.

The program will interactively prompt you for the following information:

(1) the name of the Two Line Element (TLE) database file
(2) the name of a satellite in the database
(3) initial calendar date
(4) initial universal time
(5) geographic coordinates of the ground site
(6) the simulation duration
(7) minimum elevation angle constraint

The following is a typical draft output created with this software.

```
program riseset2

< ground site-to-satellite visibility - sgp4 >

tle epoch

June 19, 1998
10 h 40 m 40.9109 s

rise conditions

calendar date                June 21, 1998
universal time               00 h 00 m 00 s
Julian date                  2450985.5

topocentric azimuth angle    110.5081463  degrees
topocentric elevation angle  39.03853656  degrees
topocentric slant range      565.8089107  kilometers

maximum elevation conditions

calendar date                June 21, 1998
universal time               00 h 00 m 00 s
Julian date                  2450985.5

topocentric azimuth angle    110.5081463  degrees
topocentric elevation angle  39.03853656  degrees
topocentric slant range      565.8089107  kilometers

set conditions

calendar date                June 21, 1998
universal time               00 h 2 m 55.9589 s
Julian date                  2450985.502

topocentric azimuth angle    122.9628418  degrees
topocentric elevation angle  5.000020532  degrees
topocentric slant range      1714.796819  kilometers

event duration               00 h 2 m 55.9589 s
```

```
rise conditions

calendar date                June 21, 1998
universal time               1 h 31 m 56.4297 s
Julian date                  2450985.564

topocentric azimuth angle    270.7290444  degrees
topocentric elevation angle  5.000000002  degrees
topocentric slant range      1719.810002  kilometers

maximum elevation conditions

calendar date                June 21, 1998
universal time               1 h 34 m 23.5995 s
Julian date                  2450985.566

topocentric azimuth angle    231.4924133  degrees
topocentric elevation angle  10.04630467  degrees
topocentric slant range      1354.628295  kilometers

set conditions

calendar date                June 21, 1998
universal time               1 h 36 m 50.321 s
Julian date                  2450985.567

topocentric azimuth angle    192.1575178  degrees
topocentric elevation angle  5.000000002  degrees
topocentric slant range      1711.066877  kilometers

event duration               00 h 4 m 53.8913 s
```