

Rise and Set of a Satellite - Numerical Integration Method

This *Numerit* application (`riseset3`) can be used to determine rise and set conditions of Earth satellites relative to a ground site or observer on an oblate Earth. The user can specify an minimum elevation or "mask" angle for each simulation. The numerical method uses root-finding while numerically integrating the satellite's J_2 equations of motion to calculate visibility conditions. The equations of motion can be expanded to include other perturbations such as aerodynamic drag, higher-order gravity effects, third body attractions and so forth.

The topocentric elevation angle of a satellite relative to an observer or ground site on an oblate Earth is calculated from

$$E = \sin^{-1}\left(\frac{r_{z_{topo}}}{r_{topo}}\right) \quad (1)$$

where the components of the topocentric position vector \mathbf{r}_{topo} are determined from the following transformation matrix and the ECI position vector \mathbf{r}_{eci} of the satellite:

$$\mathbf{r}_{topo} = [T]\mathbf{r}_{eci} = \begin{bmatrix} \sin f \cos q & \sin f \sin q & -\cos f \\ -\sin q & \cos q & 0 \\ \cos f \cos q & \cos f \sin q & \sin f \end{bmatrix} \mathbf{r}_{eci} \quad (2)$$

In this transformation f is the geodetic latitude of the ground site and q is the local sidereal time at the ground site.

The local sidereal time at a ground site is given by

$$\mathbf{q}(t) = \mathbf{q}_{g0} + \mathbf{w}_e t + \mathbf{I}_e \quad (3)$$

where \mathbf{q}_{g0} is the Greenwich sidereal time at 0 hours universal time, \mathbf{w}_e is the inertial rotation rate of the Earth, t is the elapsed time since 0 hours universal time and \mathbf{I}_e is the east longitude of the ground site.

The ECI position vector used in this transformation is the position of the satellite relative to the observer or ground site. It is determined from the ECI position vectors of the observer \mathbf{r}_{obs} and satellite \mathbf{r}_{sat} according to

$$\mathbf{r}_{eci} = \mathbf{r}_{sat} - \mathbf{r}_{obs} \quad (4)$$

The scalar slant range from the observer to the satellite is computed from the components of this vector according to

$$p = \sqrt{x_{eci}^2 + y_{eci}^2 + z_{eci}^2} \quad (5)$$

The topocentric azimuth angle is calculated from the x and y components of the topocentric position vector using the following expression

$$A = \tan^{-1}(r_{y_{topo}}, -r_{x_{topo}}) \quad (6)$$

The topocentric elevation angle is calculated from the z component of the topocentric unit position vector with this next expression

$$E = \sin^{-1}(r_{z_{topo}}) \quad (7)$$

Azimuth is measured positive clockwise from north (90° is east, 180° is south, etc.) and elevation is positive above the local horizontal or tangent plane at the observer's geographic location or ground site.

The ECI range-rate vector $\dot{\mathbf{p}}_{eci}$ of the satellite relative to the observer is determined from

$$\dot{\mathbf{p}}_{eci} = \mathbf{v}_{sat} - \mathbf{w} \times \mathbf{r}_{sat} \quad (8)$$

where $\mathbf{w} = \mathbf{w}_e(0, 0, 1)^T$ is the inertial rotation vector of the Earth and \mathbf{v}_{sat} is the ECI velocity vector of the satellite.

The topocentric range-rate vector is computed from the transformation

$$\dot{\mathbf{p}}_{topo} = [T]\dot{\mathbf{p}}_{eci} \quad (9)$$

The derivative of slant range or range-rate is given by

The azimuth and elevation rates are determined from the x , y and z components of the topocentric range and range-rate vectors as follows

$$\begin{aligned} \dot{A} &= \frac{\dot{p}_{topo_x} p_{topo_y} - \dot{p}_{topo_y} p_{topo_x}}{p_x^2 + p_y^2} \\ \dot{E} &= \frac{\dot{p}_{topo_z} - \dot{p} \sin E}{\sqrt{p_x^2 + p_y^2}} \end{aligned} \quad (10)$$

The objective function used by this numerical method to find visibility is equal to

$$f(t) = E - E_{\min} \quad (11)$$

where E_{\min} is a minimum elevation angle constraint or "mask". Whenever two consecutive values of this function change sign, the satellite is either rising (- to +) or setting (+ to -), and the root-finder solves for the time at which Equation (11) is zero.

Orbital Mechanics with Numerit

The program will interactively prompt you for the following information:

- (1) initial calendar date
- (2) initial universal time
- (3) orbital elements of the satellite
- (4) geographic coordinates of the ground site
- (5) the simulation duration
- (6) minimum elevation angle constraint

The following is part of a typical draft output created with this software.

```
program riseset3
< ground site-to-satellite visibility - numerical integration method >

rise conditions

calendar date          January 1, 1998
universal time         7 h 44 m 29.2008 s
Julian date            2450814.823

topocentric azimuth angle  176.2342547 degrees
topocentric elevation angle    5 degrees
topocentric slant range     4297.445719 kilometers

set conditions

calendar date          January 1, 1998
universal time         7 h 55 m 40.0817 s
Julian date            2450814.83

topocentric azimuth angle  115.4239813 degrees
topocentric elevation angle    5 degrees
topocentric slant range     4309.244185 kilometers

event duration         00 h 11 m 10.8809 s

rise conditions

calendar date          January 1, 1998
universal time         9 h 46 m 22.5019 s
Julian date            2450814.907

topocentric azimuth angle  217.0158161 degrees
topocentric elevation angle    5 degrees
topocentric slant range     4301.834585 kilometers

set conditions

calendar date          January 1, 1998
universal time         10 h 5 m 44.6471 s
Julian date            2450814.921

topocentric azimuth angle  99.82861378 degrees
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Orbital Mechanics with Numerit

topocentric elevation angle 5 degrees
topocentric slant range 4312.296171 kilometers

event duration 00 h 19 m 22.1451 s

rise conditions

calendar date January 1, 1998
universal time 11 h 51 m 46.551 s
Julian date 2450814.994

topocentric azimuth angle 242.6572615 degrees
topocentric elevation angle 5 degrees
topocentric slant range 4308.685918 kilometers

set conditions

calendar date January 1, 1998
universal time 12 h 13 m 5.7648 s
Julian date 2450815.009

topocentric azimuth angle 105.3580888 degrees
topocentric elevation angle 5 degrees
topocentric slant range 4307.766664 kilometers

event duration 00 h 21 m 19.2138 s

rise conditions

calendar date January 1, 1998
universal time 13 h 58 m 27.5527 s
Julian date 2450815.082

topocentric azimuth angle 257.5883163 degrees
topocentric elevation angle 5 degrees
topocentric slant range 4312.457782 kilometers

set conditions

calendar date January 1, 1998
universal time 14 h 19 m 30.009 s
Julian date 2450815.097

topocentric azimuth angle 123.4429902 degrees
topocentric elevation angle 5 degrees
topocentric slant range 4299.365135 kilometers

event duration 00 h 21 m 2.45636 s