Shadow Conditions of Satellites in Elliptical Earth Orbits

Accurate predictions of shadow conditions for any type of satellite orbit can be determined by using a combination of one-dimensional minimization and root-finding. The algorithm used in this ***Numerit*** program (`shadow2`) searches for minimum values of the separation angle between the shadow axis and the satellite's position vector as a function of time. If this angle lies within the penumbral, umbral or cylindrical shadow angle (selected by the user), the algorithm uses Brent's root-finding method to look backward and forward relative to this minimum time to find entrance and exit conditions. This computer program uses Kozai's method for the orbit propagation.

In this computer program the objective function we wish to minimize is defined by

$$f_m = \cos^{-1}\left(-\mathbf{U}_{sun} \bullet \mathbf{U}_{sat}\right) \tag{1}$$

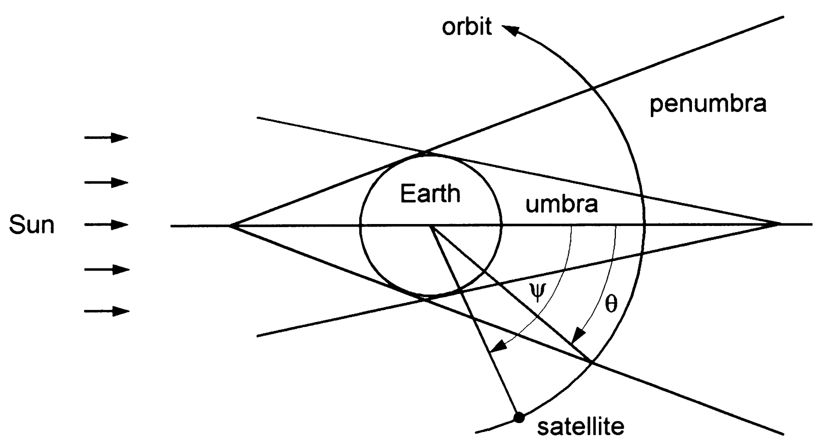
where $\mathbf{U}_{sun}$ and $\mathbf{U}_{sat}$ are the ECI unit position or *pointing* vectors of the Sun and satellite, respectively. This scalar value f_m is equal to the angle between the satellite and the anti-Sun vector.

During the root-finding calculations the objective function is given by

$$f_m = \cos^{-1}\left(-\mathbf{U}_{sun} \bullet \mathbf{U}_{sat}\right) - q \tag{2}$$

where q is either the penumbra, umbra or cylinder shadow angle. The software lets the user select which angle to use during these calculations. Before solving for the roots which define shadow entrance and exit, the solution is first bracketed using a *geometric acceleration* and *rectification* technique.

The following diagram illustrates the shadow and orbit geometry.



The shadow angles are the angles between the anti-Sun ECI unit pointing vector and the shadow boundary at the satellite's geocentric distance. The cylindrical shadow angle at the point of shadow entrance or exit is given by:

$$q_c = \sin^{-1}\left(\frac{r_{eq}}{r_{sat}}\right) \tag{3}$$

The angle of the umbra portion of the shadow at the satellite's location is determined from

$$q_u = \sin^{-1}\left(\frac{d_{sun} - r_{eq}}{r_{sun}}\right) - q_c \tag{4}$$

The penumbra shadow angle can be calculated from the following expression:

$$q_p = \sin^{-1}\left(\frac{d_{sun} + r_{eq}}{r_{sun}}\right) - q_c \tag{5}$$

In these equations, r_{eq} is the equatorial radius of the Earth, d_{sun} is the radius of the Sun, and r_{sat} and r_{sun} are the geocentric distances of the satellite and Sun, respectively.

The phase angle at shadow entrance and exit is the angle between the ECI vectors to the Sun and satellite. The phase angle at the entrance and exit points can be calculated with the equation given by

$$y = \cos^{-1}\left(\boldsymbol{U}_{sun} \bullet \boldsymbol{U}_{sat}\right) \tag{6}$$

where $\boldsymbol{U}_{sun}$ and $\boldsymbol{U}_{sat}$ are the ECI unit position vectors of the Sun and satellite, respectively. These unit vectors are evaluated at the points of shadow entrance and exit. The phase angle is an indication of the brightness or illumination of the satellite relative to an Earth observer. The actual brightness of a satellite is a function of its shape, reflective properties, and orientation or attitude in space. For a spherical satellite the illuminated fraction can be calculated from

$$I = \frac{1 + \cos y}{2} \tag{7}$$

The software will ask you for the initial calendar date and Universal Time. It will then ask you to input the classical orbital elements of the satellite and the simulation duration in days. The program allows you to calculate shadow conditions for the penumbra, umbra or cylindrical portion of the shadow. Finally, the user can elect to create a draft display of the shadow conditions. For this option and long simulation times, the program can create many pages of data.

The software will automatically create a graphics display of shadow conditions.

The following is part of a typical draft display created with this program.

```
program shadow2

< shadow conditions of satellites in elliptic Earth orbits >


enter shadow conditions

calendar date                 June 14, 1990
universal time                23 h 00 m 00 s
Julian date                   2448057.458

shadow angle                  75.10856437  degrees

exit shadow conditions

calendar date                 June 14, 1990
universal time                23 h 15 m 26.8623 s
Julian date                   2448057.469

shadow angle                  46.64050488  degrees

event duration                00 h 15 m 26.8622 s
```

The following is the companion graphics display for this example. Please note that the graphics option plots the shadow duration at the simulation time corresponding to minimum separation angle.

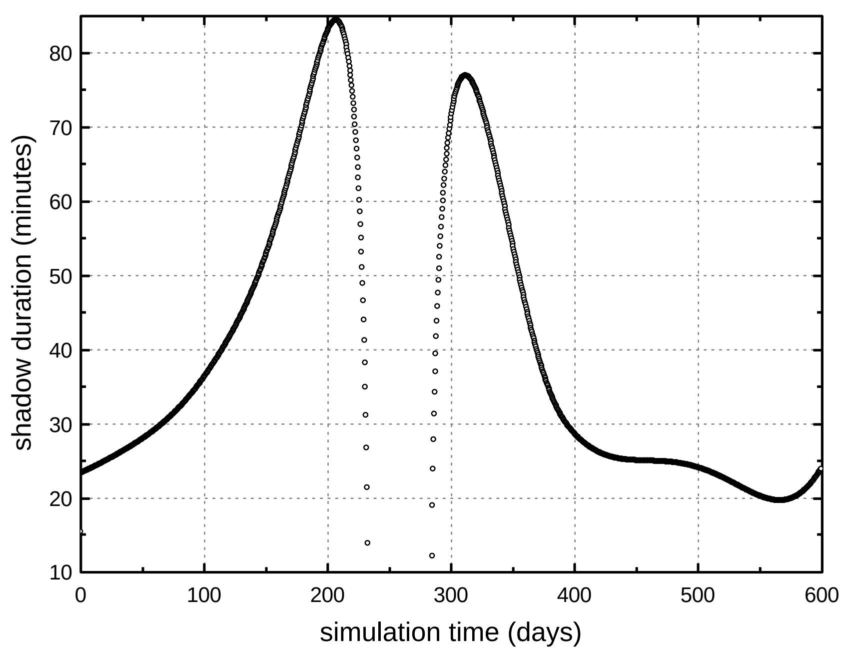


Figure 1. Shadow Duration versus Simulation Time