Sun-synchronous Orbit Design

A sun synchronous orbit has a nodal regression rate which matches the Earth's orbital rate around the Sun. The orbit plane of a sun synchronous orbit maintains a fixed geometry with respect to the Earth-Sun line. This requirement implies that a sun synchronous orbit is always retrograde with an orbital inclination greater than 90°. Sun synchronous orbits are useful because they provide near-constant Sun angles relative to a satellite viewing a fixed location on the Earth's surface.

The sun synchronous governing equation is

$$\cos i + \left\{ -\frac{2}{3} \left(\frac{p}{r_{eq}} \right)^2 \frac{\dot{l}}{n J_2} \right\} = 0 \tag{1}$$

where

$\dot{l} =$ orbital rate of the Earth $(\approx 0.985 \text{ degrees/day})$
$a =$ semimajor axis
$e =$ orbital eccentricity
$p = a\left(1 - e^2\right) =$ semiparameter
$i =$ orbital inclination
$m =$ gravitational constant of the Earth
$r_{eq} =$ equatorial radius of the Earth
$J_2 =$ second zonal gravity harmonic of the Earth
$n = \sqrt{m/a^3} =$ mean motion

This orbit design equation has been implemented in a ***Numerit*** program called `sunsync1` which calculates the mean orbital inclination required for a sun synchronous orbit. The orbital perturbations are based on Kozai's method and the orbital inclination should be considered a Kozai mean value.

This algorithm starts with an initial guess for the orbital inclination given by

$$i_0 = \cos^{-1} \left\{ -\frac{2}{3} \left(\frac{p}{r_{eq}} \right)^2 \frac{\dot{l}}{n J_2} \right\} \tag{2}$$

It then computes the perturbed mean motion based on this value of inclination using the following expression:

$$\tilde{n} = \frac{dM}{dt} = n \left\{ 1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{p} \right)^2 \sqrt{1 - e^2} \left(1 - \frac{3}{2} \sin^2 i_0 \right) \right\} \tag{3}$$

A new guess for the orbital inclination is

$$i_{n+1} = \cos^{-1}\left\{-\frac{2}{3}\left(\frac{p}{r_{eq}}\right)^2 \frac{\dot{l}}{\tilde{n}\,J_2}\right\} \tag{4}$$

This iteration continues until convergence, $\left|i_{n+1} - i_n\right| \le e$. In the software this convergence criterion is "hardwired" to a value of 1.0e-8.

The software will prompt you for either the semimajor axis and eccentricity, or the perigee and apogee altitudes. The following is a typical draft output created with this computer program.

```
program sunsync1

< sun synchronous orbits - Kozai j2 solution >

mean perigee altitude      350  kilometers
mean apogee altitude       1000  kilometers
mean semimajor axis        7053.14  kilometers

mean orbital eccentricity  0.0460787677545
mean orbital inclination   98.0570610205  degrees

number of iterations       2
```