Sun Synchronous Orbit Design - J_2 + J_4 Solution

This ***Numerit*** program (`sunsync2`) calculates the *mean* orbital inclination required for a sun-synchronous orbit. It uses a $J_2 + J_4$ form of Kozai's method during the iterative solution process. The user can provide either the semimajor axis and eccentricity, or the perigee and apogee altitudes.

This computer program uses Brent's method to find a real root of the following nonlinear sun-synchronous *constraint equation*:

$$f(i) = \dot{l} - \dot{\Omega} = 0 \tag{1}$$

where $\dot{l} = 2p/365.2422$ is the orbital rate of the Earth around the Sun and $\dot{\Omega}$ is the first-order secular perturbation of the right ascension of the ascending node given by

$$\dot{\Omega} = \frac{d\Omega}{dt} = -\frac{3}{2} J_2 \tilde{n} \left(\frac{r_{eq}}{p}\right)^2 \cos i \left[1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{p}\right)^2 \left\{ \begin{array}{l} \frac{3}{2} + \frac{e^2}{6} - 2\sqrt{1-e^2} \\ -\left(\frac{5}{3} - \frac{5}{24} e^2 - 3\sqrt{1-e^2}\right) \sin^2 i \end{array} \right\} \right] \tag{2}$$

$$-\frac{35}{8} J_4 \left(\frac{r_{eq}}{p}\right)^4 \tilde{n} \left(1 + \frac{3}{2} e^2\right) \left(\frac{12 - 21 \sin^2 i}{14}\right) \cos i$$

The perturbed mean motion $\tilde{n}$ due to J_2 and J_4 is

$$\tilde{n} = \frac{dM}{dt} = n \left\{ \begin{array}{l} 1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{p}\right)^2 \sqrt{1-e^2} \left(1 - \frac{3}{2} \sin^2 i\right) \\ + \frac{3}{128} J_2^2 \left(\frac{r_{eq}}{p}\right)^4 \sqrt{1-e^2} \left[\begin{array}{l} 16\sqrt{1-e^2} + 25\left(1-e^2\right) - 15 \\ + \left[30 - 96\sqrt{1-e^2} - 90\left(1-e^2\right)\right] \cos^2 i \\ + \left[105 + 144\sqrt{1-e^2} + 25\left(1-e^2\right)\right] \end{array} \right] \\ - \frac{45}{128} J_4 \left(\frac{r_{eq}}{p}\right)^4 \sqrt{1-e^2}\, e^2 \left(3 - 30\cos^2 i + 35\cos^4 i\right) \end{array} \right\} \tag{3}$$

The orbital inclination bracketing interval for this algorithm is $i_0 - 1° \le i \le i_0 + 1°$ where i_0 is an orbital inclination *initial guess* given by

$$i_0 = \cos^{-1}\left\{ -\frac{2}{3} \left(\frac{p}{r_{eq}}\right)^2 \frac{\dot{l}}{n J_2} \right\} \tag{4}$$

where r_{eq} is the equatorial radius of the Earth and $p = a\left(1-e\right)$ is the semiparameter.

The following is a typical draft output created with this program. For this example the perigee and apogee altitudes were provided to the software.

```
program sunsync2

< sun-synchronous orbit design - j2 + j4 solution >

mean perigee altitude       350  kilometers
mean apogee altitude        1000  kilometers

mean semimajor axis         7053.14  kilometers
mean orbital eccentricity   0.0460787677545

mean orbital inclination    98.0306105692  degrees
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