

Sun-synchronous Orbit Design - Osculating Inclination

This *Numerit* program (`sunsync3`) calculates the osculating orbital inclination required for a sun-synchronous orbit. The orbit is propagated by numerical integration during the solution process, and the software allows the user to specify the degree (zonals) and order (tesserals) of the gravity model to use during the simulation. This software can also include the point mass gravity perturbation of the Sun during the solution process. This feature permits precision orbit design of a sun-synchronous mission orbit and helps to minimize the number and magnitude of orbit maintenance maneuvers.

The numerical method used in this computer program involves a combination of nested root-finding and numerical integration. The software is "hardwired" to use the EGM96 gravity model. The user can change this by editing the file name prior to the call to the `readgrav` function. This *Numerit* function can accommodate gravity model ASCII data files up to degree and order 18.

The bracketing interval for the required osculating orbital inclination i is equal to

$$i_0 - 5^\circ \leq i \leq i_0 + 5^\circ \quad (1)$$

where i_0 is the initial inclination guess provided by the user.

The main nonlinear equation solved by this algorithm is given by

$$f(t) = \mathbf{1} - \left\{ \frac{\Omega(t_0) - \Omega(t)}{t - t_0} \right\} = 0 \quad (2)$$

where the quantity in the braces is the *forward finite-difference* numerical derivative of RAAN. By solving Equation (2), we are trying to match the required heliocentric orbital rate of the Earth, $\mathbf{1} = 2\mathbf{p}/365.2422$, with the perturbed motion of the orbit plane, Ω , due to the non-spherical gravity of the Earth and the point-mass gravity of the Sun.

While solving for the roots of Equation (2), the algorithm also propagates the orbit to the times of ascending node crossings by solving the following equation

$$g(t) = r_z = 0 \quad (3)$$

subject to the *mission constraint* $v_z > 0$. This geometric constraint ensures that the satellite is ascending in its orbit as it crosses the equator.

The simulation duration of this application is defined by the user in terms of an integer number of nodal periods. More nodal periods will provide "better" orbit design at the expense of longer computer run times. A value of at least 10 is recommended. The software will display both the Keplerian and nodal period of the solution.

Orbital Mechanics with Numerit

The following is a typical draft display created with this program.

```
program sunsync3

< sun-synchronous orbit design - osculating inclination >

semimajor axis          8000 kilometers
eccentricity            0.015
inclination             102.610503413 degrees
argument of perigee    270 degrees
raan                    0 degrees

keplerian period        118.684684295 minutes
nodal period            118.608779162 minutes

degree of gravity model 4
order of gravity model  4

simulation includes solar perturbations
```