

Long-Term Prediction of View Periods

This *Numerit* program (`vperiod`) which can be used to determine the long-term view periods between a satellite in a circular Earth orbit and a ground site on a spherical Earth. The mathematics of this algorithm are described in "The Long-Term Forecast of Station View Periods", by Martin W. Lo, JPL TDA Progress Report 42-118, August 15, 1994. This technical report also contains an error analysis comparing this closed-form solution and the classical method of perturbed orbit propagation.

This technique is valid for orbits and ground sites which satisfy the following conditions:

- Non-equatorial, circular Earth orbits
- Orbits perturbed by the secular effect of J_2 only
- Non-repeating ground track
- Ground stations not at the north or south pole

The *view period ratio* is defined by the following expression:

$$\mathbf{r} = \lim_{T \rightarrow \infty} \frac{P(T)}{T} \quad (1)$$

where T is the total elapsed or simulation time and $P(T)$ is the total station view period of a single satellite during the simulation time T .

The view period function is given by the following expression:

$$f(\mathbf{f}) = \frac{\cos \mathbf{f} \cos^{-1} \left(\frac{\cos \mathbf{q} - \sin \mathbf{f} \sin \mathbf{f}_s}{\cos \mathbf{f}_s \cos \mathbf{f}} \right)}{p^2 \sqrt{\sin^2 i - \sin^2 \mathbf{f}}} \quad (2)$$

where \mathbf{f}_s is the latitude of the ground site and \mathbf{q} is the Earth central angle. The Earth central angle is a function of the satellite's altitude h_{sat} and the minimum elevation angle e_{min} as follows:

$$\mathbf{q} = \cos^{-1} \left(\frac{r_e}{r_{sat}} \cos e_{min} \right) - e_{min} \quad (3)$$

where r_e is the radius of the Earth and $r_{sat} = r_e + h_{sat}$.

The maximum ground site latitude which can be viewed by this satellite is equal to the sum of the orbital inclination i and the Earth central angle \mathbf{q} .

Orbital Mechanics with Numerit

The view period ratio for any ground site is calculated using the following definite integral:

$$\mathbf{r} = \int_{f_1}^{f_2} f(\mathbf{f}) d\mathbf{f} \quad (4)$$

where the finite limits of integration are given by

$$\begin{aligned} \mathbf{f}_1 &= \max(\mathbf{f}_s - \mathbf{q}, -L_i) \\ \mathbf{f}_2 &= \max(\mathbf{f}_s + \mathbf{q}, L_i) \end{aligned} \quad (5)$$

These limits depend upon the orbital inclination i , the ground site latitude of interest \mathbf{f}_s , the Earth central angle \mathbf{q} and the value of L_i which is given by

$$L_i = \begin{cases} i & \text{if } i \leq 90^\circ \\ 180 - i & \text{if } i > 90^\circ \end{cases} \quad (6)$$

From the view period ratio, we can determine the total view period during any time interval of interest T using

$$\mathbf{r}T = \text{total view period during the elapsed time } T \quad (7)$$

Furthermore,

$$\begin{aligned} \mathbf{r}24 \text{ hours} &= \text{average daily total view period} \\ \mathbf{r}7 \text{ days} &= \text{average weekly total view period} \end{aligned} \quad (8)$$

Another application is the maximum average contact time possible with the K ground stations and a single satellite during a 24 hour interval given by:

$$C = (\mathbf{r}_1 + \mathbf{r}_2 + \cdots + \mathbf{r}_K) 24 \text{ hours} \quad (9)$$

The extension of this technique to constellations of satellites is straight-forward.

Numerical Solution

The view period integral is solved numerically using a Gauss-Kronrod quadrature algorithm due to T. Patterson with enhancements by F. Krogh and W. Snyder. A description of this numerical method can be found in "Algorithm 699: A New Representation of Patterson's Quadrature Formulae", Fred T. Krogh and W. Van Synder, *ACM Transactions on Mathematical Software*, Vol. 17, No. 4, December 1991, Pages 457-461. This algorithm was ported from Fortran to *Numerit*.

Orbital Mechanics with Numerit

The syntax of this *Numerit* function is as follows:

```
function kquad(a, b, tol, result, k, icheck)
` one-dimensional quadrature
` method due to T. Patterson with modifications
` by F. Krogh and W. Snyder (ACM Algorithm 699)
` input
` a      = lower integration limit
` b      = upper integration limit
` tol    = convergence tolerance
` output
` result = array of solution(s)
` k      = index of result array which best satisfies tol
` icheck = 0 ==> convergence
`        = 1 ==> non-convergence
```

This program can determine the view period ratio for either a single ground site or range of ground sites. The software will prompt you for the satellite's altitude and orbital inclination. It will also ask you for a minimum elevation constraint.

The following is a plot of the view period ratio as a function of the ground site latitude for a satellite at an altitude of 200 kilometers and 28.5° inclination. The minimum elevation angle for this example was 0° .

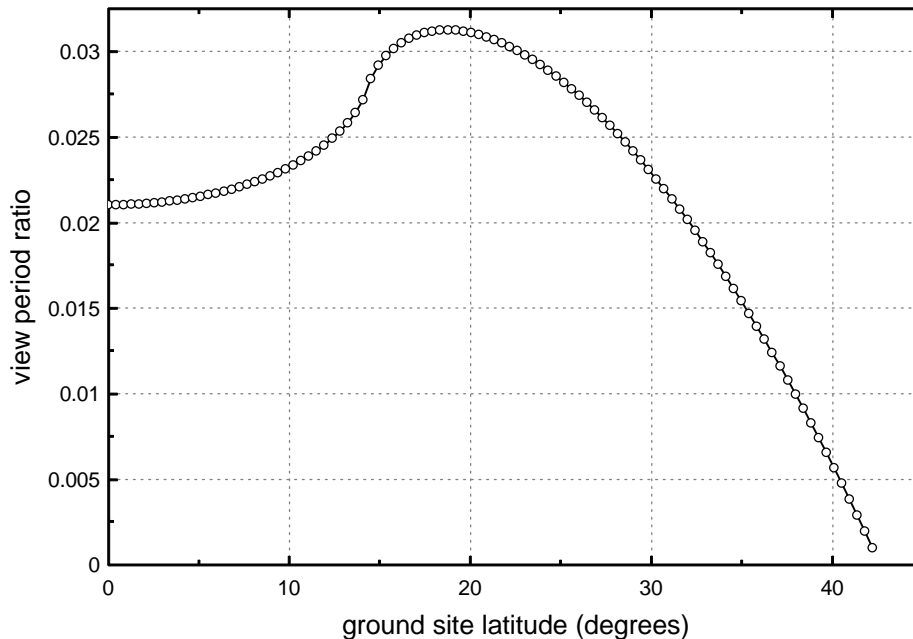


Figure 1. Long Term Prediction of Station View Periods