Transformation to and from the lunar ME and PA Coordinate Systems

According to JPL D-32296, "Lunar Constants and Models Document" and the IAU 2000 resolutions, the constant transformation matrix from the lunar Mean Earth/polar axis (ME) system to the lunar Principal axis (PA) system is given by the following (1-2-3) rotation sequence;

$$[PA] = R_z\left(63.8986''\right) R_y\left(79.0768''\right) R_x\left(0.1462''\right) [ME]$$

The transformation matrix from the PA system to the ME system is given by

$$R_x\left(-0.1462''\right) R_y\left(-79.0768''\right) R_z\left(-63.8986''\right)$$

The numerical components of the ME-to-PA transformation matrix are as follows;

```
  0.999999878527094       3.097894216177013E-004 -3.833748976184077E-004
 -3.097891271165531E-004  0.999999952015005       8.275630251118771E-007
  3.833751355924360E-004 -7.087975496937868E-007  0.999999926511499
```

The PA-to-ME transformation is the transpose of this matrix.

These numbers have been confirmed using the following Fortran call to the `sxform` subroutine of the JPL Spice library;

```
      call sxform('moon_me', 'moon_pa', eph_time, xform1)
```

The following Fortran data statement can be used to include the ME-to-PA transformation matrix in an analysis program.

```
c     ME-to-PA transformation matrix

      data tm_me2pa
     &              / 0.9999998785527094d0, -3.097891271165531d-4,
     &                3.833751355924360d-4,
     &                3.097894216177013d-4, 0.999999952015005d0,
     &               -7.087975496937868d-7,
     &               -3.833748976184077d-4, 8.275630251118771d-7,
     &                0.999999926511499d0 /
```

where `tm_me2pa` is dimensioned 3, 3.

The conversion from arc-seconds to radians is given by $\frac{\pi}{648000}$.

Approximate lunar pole right ascension, declination and prime meridian in the PA system

Page 7 of the JPL document also provides the following "tweaks" to the orientation of the moon in order to approximate the orientation in the PA system.

$$\alpha_{PA} = \alpha_{IAU} + 0.0553\cos W_p + 0.0034\cos\left(W_p + E1\right)$$

$$\delta_{PA} = \delta_{IAU} + 0.0220\sin W_p + 0.0007\sin\left(W_p + E1\right)$$

$$W_{PA} = W_{IAU} + 0.01775 - 0.0507\cos W_p - 0.0034\cos\left(W_p + E1\right)$$

where W_p is the polynomial part of the prime meridian equation given by

$$W_p = 38.3213 + \dot{W}d - 1.4 \bullet 10^{-12} d^2$$

and

$$\begin{aligned}\alpha_{IAU} &= 269.9949 + 0.0031T - 3.8787\sin E1 - 0.1204\sin E2 \\ &+0.0700\sin E3 - 0.0172\sin E4 + 0.0072\sin E6 \\ &-0.0052\sin E10 + 0.0043\sin E13\end{aligned}$$

$$\begin{aligned}\delta_{IAU} &= 66.5392 + 0.0130T + 1.5419\cos E1 + 0.0239\cos E2 \\ &-0.0278\cos E3 + 0.0068\cos E4 - 0.0029\cos E6 \\ &+0.0009\cos E7 + 0.0008\cos E10 - 0.0009\cos E13\end{aligned}$$

$$\begin{aligned}W_{IAU} &= W_p + 3.5610\sin E1 + 0.1208\sin E2 + \\ &-0.0642\sin E3 + 0.0158\sin E4 + 0.0252\cos E5 \\ &-0.0066\sin E6 - 0.0047\sin E7 - 0.0046\cos E8 \\ &+0.0028\sin E9 + 0.0052\sin E10 + 0.0040\sin E11 \\ &+0.0019\sin E12 - 0.0044\sin E13\end{aligned}$$

In these equations, $\dot{W} = 13.17635815$ degrees/day and T is the time in Julian centuries given by $T = \left(JD - 2451545.0\right)/36525$ and JD is the TDB Julian Date.

The trigonometric arguments, in degrees, for these equations are

$$E1 = 125.045 - 0.0529921d$$
$$E2 = 250.089 - 0.1059842d$$
$$E3 = 260.008 + 13.0120009d$$
$$E4 = 176.625 + 13.3407154d$$
$$E5 = 357.529 + 0.9856003d$$
$$E6 = 311.589 + 26.4057084d$$
$$E7 = 134.963 + 13.0649930d$$
$$E8 = 276.617 + 0.3287146d$$
$$E9 = 34.226 + 1.7484877d$$
$$E10 = 15.134 - 0.1589763d$$
$$E11 = 119.743 + 0.0036096d$$
$$E12 = 239.961 + 0.1643573d$$
$$E13 = 25.053 + 12.9590088d$$

where $d = JD - 2451545$ is the number of days since January 1.5, 2000. These equations are given in "Report of the IAU/IAG Working Group on Cartographic Coordinates and Rotational Elements of the Planets and Satellites: 2000", *Celestial Mechanics and Dynamical Astronomy*, **82**: 83-110, 2002.