Program ROCKET

A Computer Program for Predicting and Optimizing Single Stage Rocket Performance

Introduction

This document is the user's guide for a computer program called ROCKET that can be used to predict and optimize the performance of single-stage rockets. The software predicts either maximum altitude or the optimum launch mass that maximizes altitude.

The ROCKET computer program implements the following models and numerical methods:

(1) U.S. Standard 1976 atmosphere model

(2) Linear or cubic spline interpolation of rocket engine thrust-time tabular data

(3) Numerical solution of the differential equations of rocket flight

(4) Combination of root-finding with one-dimensional optimization

Running a ROCKET Simulation

A simulation can be run by simply typing **`rocket rocket1.dat`** from a DOS command line. In the command line method for running a simulation, `rocket1.dat` can be the name of any compatible input data file.

The software can also be run by simply double clicking on **`rocket.exe`** from Windows Explorer. This option will display the following screen asking the user to interactively specify the name of the input data file.

```
          ***********************************
          *          Program ROCKET         *
          *                                 *
          * Model Rocket Trajectory Analysis *
          *                                 *
          *     written by C. David Eagle    *
          *                                 *
          *          March 19, 2004         *
          ***********************************

please input the name of the simulation definition file
```

The next section describes the format of ROCKET input data files.

Input Data File

The ROCKET computer program is "data-driven" by two ASCII input file created by the user. This section describes a typical input data file for the software. In the following discussion the actual input file contents are in `courier` font and all explanations are in *times italic* font.

Each number or *command* within an ROCKET input file is preceded by one or more lines of *annotation* text. Do not delete any of these annotation lines or increase or decrease the number of lines reserved for each comment. You may change them to reflect your own explanation. The annotation line also includes the correct units and when appropriate, the valid range of the input. ASCII text input such as `rocket1.csv` is not case sensitive but must be spelled correctly.

The first five lines of any input file are reserved for user comments. These lines are ignored by the software. However the ROCKET input file must begin with five and only five initial text lines.

```
**************************
* rocket1.dat            *
* single stage test case *
* February 18, 2004      *
**************************
```

The first input defines the initial mass option. Option 1 lets the user specify the initial liftoff mass and option 2 tells the software to solve for the optimum initial mass that maximizes the rocket's vertical altitude.

```
type of simulation
------------------
1 = user-defined initial mass
2 = compute optimum initial mass
--------------------------------
1
```

The next input specifies the name of the rocket engine thrust-time data file. The format and contents of this file are described later in this document.

```
name of rocket engine thrust-time data file
b4.dat
```

The next program input defines the type of thrust-time interpolation to use during the simulation.

```
type of thrust-time interpolation
---------------------------------
1 = linear
2 = cubic spline
----------------
1
```

This next text input defines the name of the boost phase output file. This file contains trajectory information from liftoff to rocket engine burnout.

```
name of boost phase output file
rocket1.csv
```

The next input allows the user to specify the time step size of the data in the boost phase output file.

```
step size for boost phase data file (seconds)
0.01
```

This next text input defines the name of the complete trajectory output file. This file contains trajectory information from liftoff to maximum vertical altitude.

```
name of complete trajectory output file
rocket2.csv
```

The next input allows the user to specify the time step size of the data in the complete trajectory output file.

```
step size for complete trajectory data file (seconds)
0.01
```

This next input specifies the length of the launch rod or rail.

```
launch rod/rail length (meters)
1.0d0
```

This numeric input allows the user to define the launch angle, measured with respect to the "local horizon". A vertical launch is a launch angle of 90 degrees.

```
launch angle (degrees)
89.0d0
```

This input defines the total impulse of the rocket engine. For best results, this total impulse should be compatible with the type of thrust-time interpolation defined earlier.

```
rocket engine total impulse (newtons-second)
4.29d0
```

This input defines the thrust duration of the rocket engine. For best results, this number should also be compatible with the type of thrust-time interpolation defined earlier.

```
rocket engine thrust duration (seconds)
1.03d0
```

This input defines the propellant mass of the rocket engine(s).

```
rocket engine propellant mass (grams)
6.0d0
```

This input defines the initial or "liftoff" mass of the entire rocket.

```
initial vehicle mass (grams)
40.0d0
```

This input defines the frontal diameter of the rocket. This is the area used in the aerodynamic drag calculations.

```
frontal diameter (millimeters)
18.0d0
```

This input defines the drag coefficient of the rocket.

```
drag coefficient (non-dimensional)
0.321d0
```

The next four inputs define the initial flight conditions for the simulation.

```
**********************
* initial conditions *
**********************

initial x-component of distance (meters)
0.0d0

initial y-component of distance (meters)
0.0d0

initial x-component of velocity (meters/second)
0.0d0

initial y-component of velocity (meters/second)
0.0d0
```

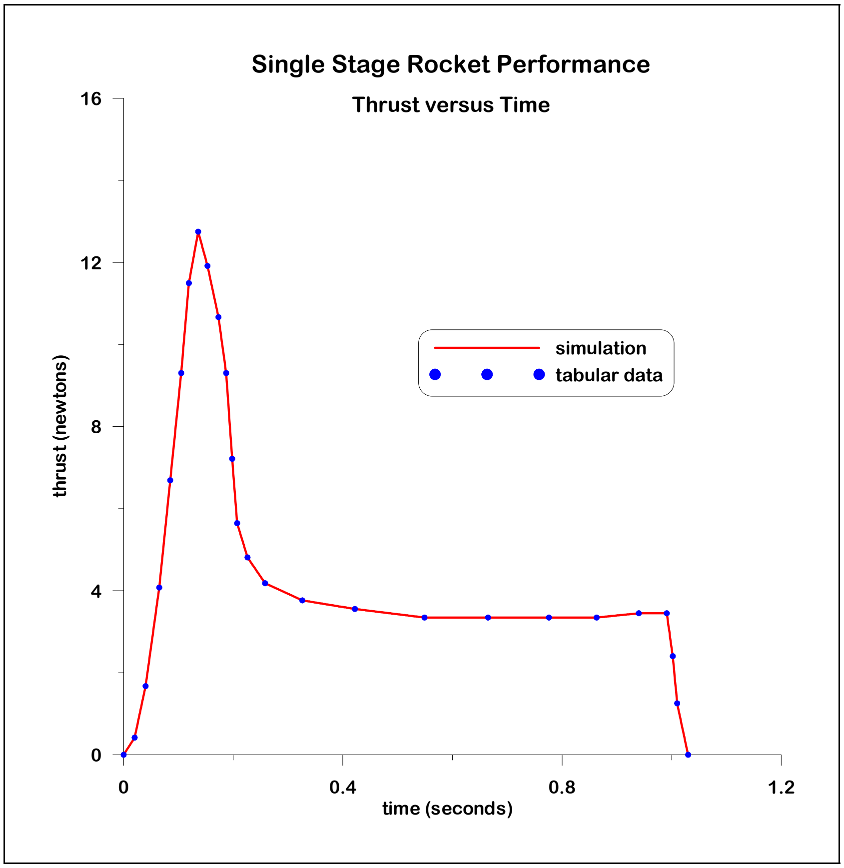
Rocket Engine Data File

The following is a data file for the Estes B4 rocket engine extracted from the ThrustCurve.org web site. The first line describes the type of rocket engine data and the second line is the number of thrust-time data pairs in the file. Do not remove these two lines. Column one is the time in seconds and column two is the thrust in Newtons.

```
Estes B4 thrust-time data
26
0.000  0.000
0.020  0.418
0.040  1.673
0.065  4.076
0.085  6.690
0.105  9.304
0.119 11.496
0.136 12.750
0.153 11.916
0.173 10.666
0.187  9.304
0.198  7.214
0.207  5.645
0.226  4.809
0.258  4.182
0.326  3.763
0.422  3.554
0.549  3.345
0.665  3.345
0.776  3.345
0.863  3.345
0.940  3.449
0.991  3.449
1.002  2.404
1.010  1.254
1.030  0.000
100.0  0.000
```

Please note that the final line of data is a large time with zero thrust.

The following is a typical plot of a thrust-time data file.



Output Files

The software will create two comma separated variable (csv) data files containing important trajectory information as a function of simulation time. These files contain such items as x position, y position, x velocity, y velocity, instantaneous mass, total impulse, thrust, aerodynamic drag, propellant flow rate, theta, flight path angle, load factor, density, Mach number, ideal velocity, drag loss, and gravity loss. The unit of each data column is described in the file header.

The following is a portion of an example output file created with this computer program.

```
      time (sec)        ,       x-position (m)     ,      y-position (m)     ,     x-velocity (mps)    , …
0.0000000000000000D+00, 0.0000000000000000D+00, 0.0000000000000000D+00, 0.0000000000000000D+00, …
0.1000000000000000D-01, 0.1519829147088023D-05, -.4032615464780154D-03, 0.4559519981310694D-03, …
0.2000000000000000D-01, 0.1215902968285041D-04, -.1264739656002221D-02, 0.1823907683294825D-02, …
0.3000000000000000D-01, 0.4408315431245311D-04, -.1887470280876662D-02, 0.5017481617568292D-02, …
0.4000000000000000D-01, 0.1216443467754249D-03, -.8763200406369668D-03, 0.1095182012355813D-01, …
0.5000000000000000D-01, 0.2746981690541766D-03, 0.3479135065153971D-02, 0.2035994685880751D-01, …
0.6000000000000000D-01, 0.5428810211037865D-03, 0.1344966286910407D-01, 0.3397953164287221D-01, …
0.7000000000000001D-01, 0.9686866747973188D-03, 0.3146972993136748D-01, 0.5201335956092009D-01, …
0.8000000000000000D-01, 0.1602033638821910D-02, 0.6039916569929499D-01, 0.7561848945260032D-01, …
```

These data files can be read into plotting software to create graphic displays of important trajectory characteristics.

Screen Display of Simulation Final Results

After a ROCKET simulation successfully executes, it will display a brief summary of the final solution. This display illustrates the predicted burnout and maximum altitude conditions for the user's simulation.

The following is a typical screen display created with the ROCKET software.

```
*********************************************
flight conditions - user-defined initial mass
*********************************************


burnout conditions
------------------

time                   1.03000000000000      seconds

altitude               60.2140279633387      meters

velocity               99.8826791626548      meters/second

mass                   33.9992469058002      grams

total impulse          4.29053846235286      newton-seconds

exhaust velocity       715.000000000000      meters/second

maximum altitude conditions
---------------------------

time                   8.43195806770460      seconds

horizontal distance    12.9570174127885      meters

altitude               372.810858261436      meters

horizontal velocity    1.36404103088020      meters/seconds

vertical velocity     4.898210901659159E-010 meters/second

ideal velocity         116.201574135786      meters/second

drag loss              33.5050679553395      meters/second

gravity loss           81.3311159864568      meters/second
```

The numerical value for vertical velocity indicates how well the ROCKET software has predicted maximum altitude.

Technical Discussion

The two-dimensional equations of rocket motion relative to a flat, non-rotating Earth are given by the following system of first-order nonlinear differential equations:

$$m\frac{dV_x}{dt} = T\sin\theta - D\sin\theta$$

$$m\frac{dV_y}{dt} = T\cos\theta - D\cos\theta - m\,g$$

$$\frac{dm}{dt} = -\frac{T}{V_{ex}}$$

where

$m =$ mass of rocket
$T =$ rocket engine thrust
$V_{ex} =$ rocket engine exhaust velocity $= I_t / m_p$
$I_t =$ rocket engine total impulse
$m_p =$ propellant mass
$V_x =$ horizontal component of velocity
$V_y =$ vertical component of velocity
$D = \frac{1}{2}\rho V^2 C_D S =$ aerodynamic drag force
$\rho =$ atmospheric density
$V = \sqrt{V_x^2 + V_y^2} =$ total velocity
$C_D =$ drag coefficient
$S =$ aerodynamic reference area
$g =$ acceleration of gravity
$\theta =$ flight path angle relative to vertical

Since $\theta = 90^\circ - \gamma$, where γ is the flight path angle, the first two equations of motion can also be written in the following form:

$$m\frac{dV_x}{dt} = T\cos\gamma - D\cos\gamma$$

$$m\frac{dV_y}{dt} = T\sin\gamma - D\sin\gamma - m\,g$$

In these equations,

$$\sin\theta = \frac{V_x}{V} = \cos\gamma$$

$$\cos\theta = \frac{V_y}{V} = \sin\gamma$$

The *ideal velocity* created by a rocket's propulsive system can be determined from the following expression

$$V_{ideal} = \int_{t_i}^{t_b} \frac{T}{m}\,dt = \int_{t_i}^{t_b} a_T\,dt$$

The *delta-v losses* associated with rocket flight are given by the following series of equations.

thrust vector

$$\Delta V_{TV} = \int_{t_i}^{t_b} \frac{T}{m}\left(1-\cos\alpha\right)dt$$

aerodynamic drag

$$\Delta V_{drag} = \int_{t_i}^{t_b} \frac{D}{m}\,dt$$

gravity

$$\Delta V_g = \int_{t_i}^{t_b} g\sin\gamma\,dt$$

These equations are included in the equations of motion and numerically integrated during the rocket's flight.

The load factor experience by the rocket is determined from

$$n = \frac{T-D}{mg}$$

Numerical Methods

This section describes several of the numerical methods used in the ROCKET computer program.

During the simulation, atmospheric density and speed of sound are determined using linear interpolation of a tabular representation of the U.S. Standard 1976 atmosphere. The instantaneous thrust of the rocket is determined using either linear or cubic spline interpolation of the tabular thrust-time data. The drag coefficient and reference area are assumed constant in this computer program.

The ROCKET computer program uses a Runge-Kutta-Fehlberg (RKF78) variable step size numerical method to solve the system of differential equations. The truncation error tolerance for this algorithm is *hardwired* to a value of `1.0d-12`.

The maximum altitude is determined using Brent's one-dimensional root-finding algorithm to find the time at which the vertical velocity component V_y is equal to zero within a hardwired tolerance of `1.0d-10`. During the search for maximum altitude, the rocket's equations of motion are predicted using the RKF78 method.

The optimum initial mass is determined using a one-dimensional minimization numerical method that searches for the initial liftoff mass that maximizes the final vertical altitude. During the search for the best initial mass, the rocket's equations of motion are predicted using the RKF78 method and the time of maximum altitude is predicted using Brent's method. Therefore, this algorithm consists of a nonlinear programming method (NLP) with an embedded root-finder which in turn uses an embedded numerical integrator.

Additional information about rocket flight and numerical methods can be found in the classic book, *Topics in Advanced Model Rocketry* by G. J. Caporaso, G. K. Mandell and W. P. Bengen, MIT Press, 1971.